Name:	
Class:	

Standardised Competence-Oriented
Written School-Leaving Examination

AHS

12th January 2017

Mathematics

Part 1 Tasks

BMB
Bundesministerium
für Bildung

Advice for Completing the Tasks

Dear candidate,

The following booklet for Part 1 contains 24 tasks. The tasks can be completed independently of one another. You have *120 minutes* available in which to work through this booklet.

Please use a blue or black pen that cannot be rubbed out. You may use a pencil for tasks that require you to draw a graph, vectors or a geometric construction.

Please do all of your working out solely in this booklet. Write your name on the first page of the booklet in the space provided.

All answers must be written in this booklet. In the assessment of your work, everything that is not crossed out will be considered. Your solutions must be clearly marked. If a solution is not clearly marked or if more than one solution is given, the task will be considered to be unsolved. Draw a line through any notes you make.

You may use a pre-approved formula book as well as your usual electronic device(s).

Please hand in the task booklet at the end of the examination.

Assessment

Every task in Part 1 will be awarded either 0 points or 1 point. Every sub-task in Part 2 will be awarded 0, 1 or 2 points. The tasks marked with an A will be awarded either 0 points or 1 point.

– If at least 16 of the 24 tasks in Part 1 are solved correctly, you will pass the examination.
– If fewer than 16 of the 24 tasks in Part 1 are solved correctly, then the tasks marked with an A from Part 2 may compensate for the shortfall (as part of the "range of essential skills" outlined by the LVBO).
  If, including the tasks marked with an A from Part 2, at least 16 tasks are solved correctly, you will pass the examination.
  If, including the tasks marked with an A from Part 2, fewer than 16 tasks are solved correctly, you will not be awarded enough points to pass the examination.
– If at least 16 tasks are solved correctly (including the compensation tasks marked with an A from Part 2), a grade will be awarded as follows:

Pass	16–23 points
Satisfactory	24–32 points
Good	33–40 points
Very Good	41–48 points

Explanation of the Task Types

Some tasks require a ***free answer***. For these tasks, you should write your answer directly underneath each task in the task booklet. Other task types used in the examination are as follows:

Matching tasks: For this task type you will be given a number of statements, tables or diagrams, which will appear alongside a selection of possible answers. To correctly answer these tasks, you will need to match each statement, table or diagram to its corresponding answer. You should write **the letter of the correct answer** next to the statement, table or diagram in the space provided.

Example:
You are given two equations.

Task:
Match the two equations to their corresponding description (from A to D).

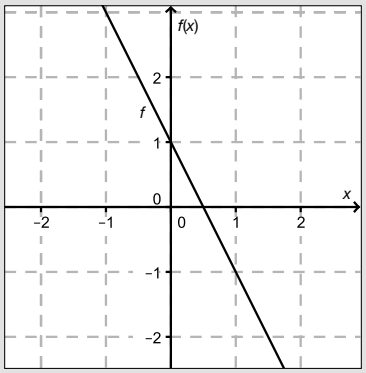
$1 + 1 = 2$	A
$2 \cdot 2 = 4$	C

A	Addition
B	Division
C	Multiplication
D	Subtraction

Construction tasks: This task type requires you to draw points, lines and/or curves in the task booklet.

Example:
Below you will see a linear function f where $f(x) = k \cdot x + d$.

Task:
On the axes provided below, draw the graph of a linear function for which $k = -2$ and $d > 0$.



Multiple-choice tasks of the form "1 out of 6": This task type consists of a question and six possible answers. Only **one answer** should be selected. You should put a cross next to the only correct answer in the space provided.

Example:
Which equation is correct?

Task:
Put a cross next to the correct equation.

1 + 1 = 1	☐
2 + 2 = 2	☐
3 + 3 = 3	☐
4 + 4 = 8	☒
5 + 5 = 5	☐
6 + 6 = 6	☐

Multiple-choice tasks of the form "2 out of 5": This task type consists of a question and five possible answers, of which **two answers** should be selected. You should put a cross next to each of the two correct answers in the space provided.

Example:
Which equations are correct?

Task:
Put a cross next to each of the two correct equations.

1 + 1 = 1	☐
2 + 2 = 4	☒
3 + 3 = 3	☐
4 + 4 = 8	☒
5 + 5 = 5	☐

Multiple-choice tasks of the form "x out of 5": This task type consists of a question and five possible answers, of which **one, two, three, four *or* five answers** may be selected. The task will require you to: "Put a cross next to each correct statement/equation …". You should put a cross next to each correct answer in the space provided.

Example:
Which of the equations given are correct?

Task:
Put a cross next to each correct equation.

1 + 1 = 2	☒
2 + 2 = 4	☒
3 + 3 = 6	☒
4 + 4 = 4	☐
5 + 5 = 10	☒

Gap-fill: This task type consists of a sentence with two gaps, i.e. two sections of the sentence are missing and must be completed. For each gap you will be given the choice of three possible answers. You should put a cross next to **each of the two answers** that are necessary to complete the sentence correctly.

Example:
Below you will see 3 equations.

Task:
Complete the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

The operation in equation _____ ① _____ is known as summation or _____ ② _____ .

①	
1 – 1 = 0	☐
1 + 1 = 2	☒
1 · 1 = 1	☐

②	
Multiplication	☐
Subtraction	☐
Addition	☒

Changing an answer for a task that requires a cross:

1. Fill in the box that contains the cross for your original answer.
2. Put a cross in the box next to your new answer.

1 + 1 = 3	☐
2 + 2 = 4	☒
3 + 3 = 5	☐
4 + 4 = 4	☐
5 + 5 = 9	■

In this instance, the answer "5 + 5 = 9" was originally chosen. The answer was later changed to be "2 + 2 = 4".

Selecting an answer that has been filled in:

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

1 + 1 = 3	☐
2 + 2 = 4	(■)
3 + 3 = 5	☐
4 + 4 = 4	■
5 + 5 = 9	☐

In this instance, the answer "2 + 2 = 4" was filled in and then selected again.

If you still have any questions now, please ask your teacher.

Good Luck!

Task 1

Value Added Tax for Audio Books

Since 2015, certain audio books in Germany have been taxed at a reduced rate of 7 % Value Added Tax (VAT) instead of 19 %.

Task:

Suggest a formula to calculate the reduced price € *y* of an audio book including 7 % VAT, that originally cost € *x* including 19 % VAT.

Task 2

Quadratic Equation

A quadratic equation is given by $a \cdot x^2 + 10 \cdot x + 25 = 0$ where $a \in \mathbb{R}, a \neq 0$.

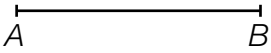
Task:

Determine the value(s) of a for which the equation has exactly one real solution.

$a =$ _____

Task 3

Dividing a Line Segment

The line segment AB: 

is internally split at the point T at a ratio of $3:2$.

Task:

Write down a formula to calculate the point T.

$T =$ _____

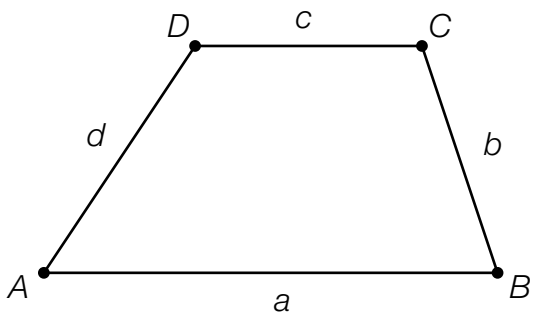
Task 4

Trapezium

The coordinates of the vertices of a trapezium *ABCD* are:

$A = (2,-6)$
$B = (10,-2)$
$C = (9,2)$
$D = (3,y)$

The sides $a = AB$ and $c = CD$ are parallel.



Task:

Determine the value of the coordinate y of the point D.

$y =$ _____

Task 5

Parallel Lines

The line g has vector equation g: $X = \begin{pmatrix} 1 \\ -2 \end{pmatrix} + s \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

The line h is parallel to g and goes through the origin.

Task:

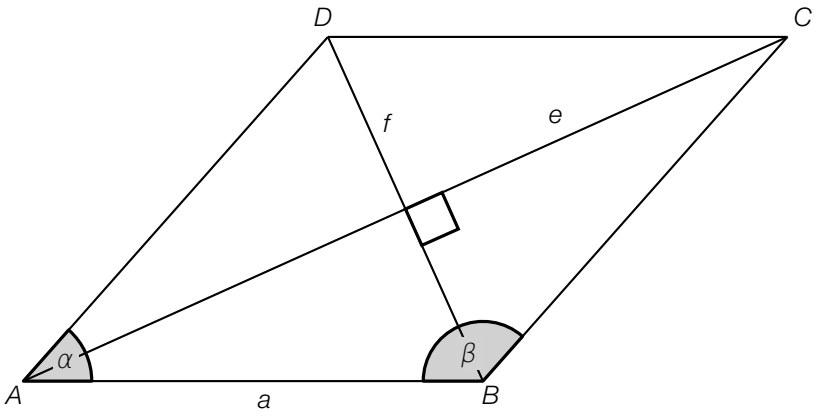
Determine the equation of the line h in the form $a \cdot x + b \cdot y = c$ where $a, b, c \in \mathbb{R}$.

h: _____

Task 6

Rhombus

In a rhombus of side length a, the diagonals $e = \overline{AC}$ and $f = \overline{BD}$ bisect one another. The diagonal e bisects the angle $\alpha = \sphericalangle DAB$ and the diagonal f bisects the angle $\beta = \sphericalangle ABC$.



Task:

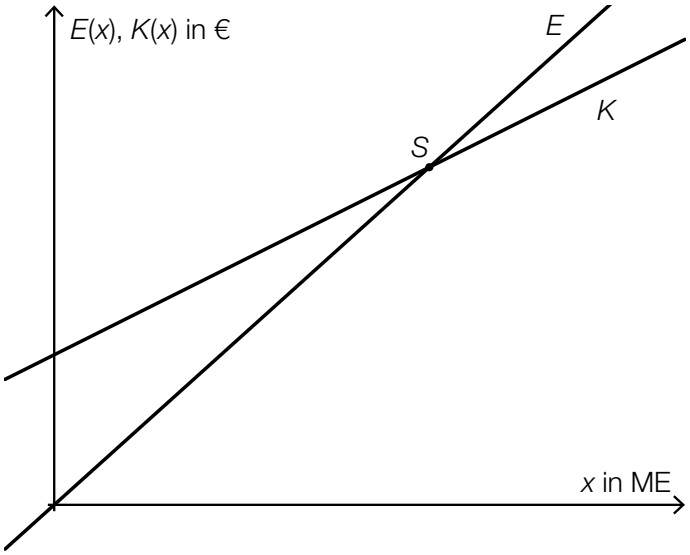
The side length a and angle β are known.
Write down a formula to calculate f in terms of a and β .

$f =$ _____

Task 7

Point of Intersection

The function E describes the revenue, $E(x)$, and the function K the costs, $K(x)$, in euros for a production volume x. The production volume x is given in units of quantity (ME). The graphs of both functions are represented in the diagram below:



Task:

Interpret both of the coordinates of the point of intersection S of the graphs of the functions in the given context.

Task 8

Increasing Function

Below, you will see five functions.

Task:

Which of the functions f below are strictly monotonically increasing in each interval $[x_1, x_2]$ where $0 < x_1 < x_2$? Put a cross next to each of the two correct functions.

A linear function f with equation $f(x) = a \cdot x + b$ $(a > 0, b > 0)$	☐
A power function f with equation $f(x) = a \cdot x^n$ $(a < 0, n \in \mathbb{N}, n > 0)$	☐
A sine function f with equation $f(x) = a \cdot \sin(b \cdot x)$ $(a > 0, b > 0)$	☐
An exponential function f with equation $f(x) = a \cdot e^{k \cdot x}$ $(a > 0, k < 0)$	☐
An exponential function f with equation $f(x) = c \cdot a^x$ $(a > 1, c > 0)$	☐

Task 9

Electric Resistance

The electric resistance R of a cylindrical conductor with radius r and length l can be calculated using the formula $R = \rho \cdot \frac{l}{r^2 \cdot \pi}$. The size of the specific resistivity, ρ, is dependent on the material and temperature of the conductor.

Task:

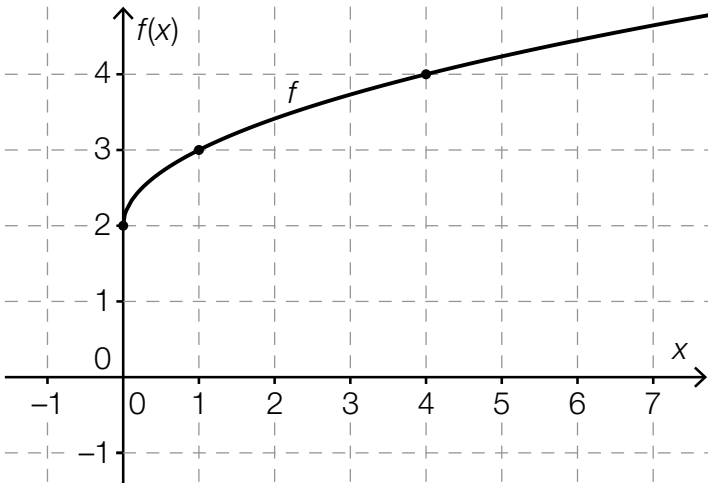
In the table below, relationships that can be derived from the formula for electric resistance are shown.
Which of the equations listed below describe(s) a linear function? Put a cross next to each of the correct equations.

$R(l) = \rho \cdot \frac{l}{r^2 \cdot \pi}$ where ρ, r constant	☐
$l(R) = \frac{R}{\rho} \cdot r^2 \cdot \pi$ where ρ, r constant	☐
$R(\rho) = \rho \cdot \frac{l}{r^2 \cdot \pi}$ where l, r constant	☐
$R(r) = \rho \cdot \frac{l}{r^2 \cdot \pi}$ where ρ, l constant	☐
$l(r) = \frac{R}{\rho} \cdot r^2 \cdot \pi$ where R, ρ constant	☐

Task 10

Function

The diagram below shows the graph of a function f where $f(x) = a \cdot x^{\frac{1}{2}} + b$ ($a, b \in \mathbb{R}, a \neq 0$).
The points shown in bold have integer coordinates.



Task:

Determine the values of a and b.

a = _____

b = _____

Task 11

Population Growth

The size of a population can be described dependant of the time by a function N, where $N(t) = N_0 \cdot e^{0.1188 \cdot t}$. The time, t, is measured in hours. In this function, N_0 represents the size of the population at time $t = 0$ and $N(t)$ the size of the population at the time $t \geq 0$.

Task:

Determine the percentage p by which the population increases per hour.

$p \approx$ _____ %

Task 12

Trigonometric Functions

Let f and g be functions where $f(x) = -\sin(x)$ and $g(x) = \cos(x)$.

Task:

Determine the value $b \in [0, 2\pi]$ by which the graph of f would need to be translated to be identical to the graph of g, i. e. so that $-\sin(x + b) = \cos(x)$ holds.

Task 13

Fertility

On the Statistik Austria website, the following information can be found under the heading Fertility:

“The total fertility rate in 2014 was 1.46 children per woman. Thus, if the age-specific fertility rates remain constant, a woman in Austria who is 15 years old today will, statistically speaking, have 1.46 children by her 50th birthday. This average lies significantly below the “maintenance level” of around 2 children per woman.”

Source: http://www.statistik.at/web_de/statistiken/menschen_und_gesellschaft/bevoelkerung/demographische_indikatoren/index.html [23.02.2016].

Task:

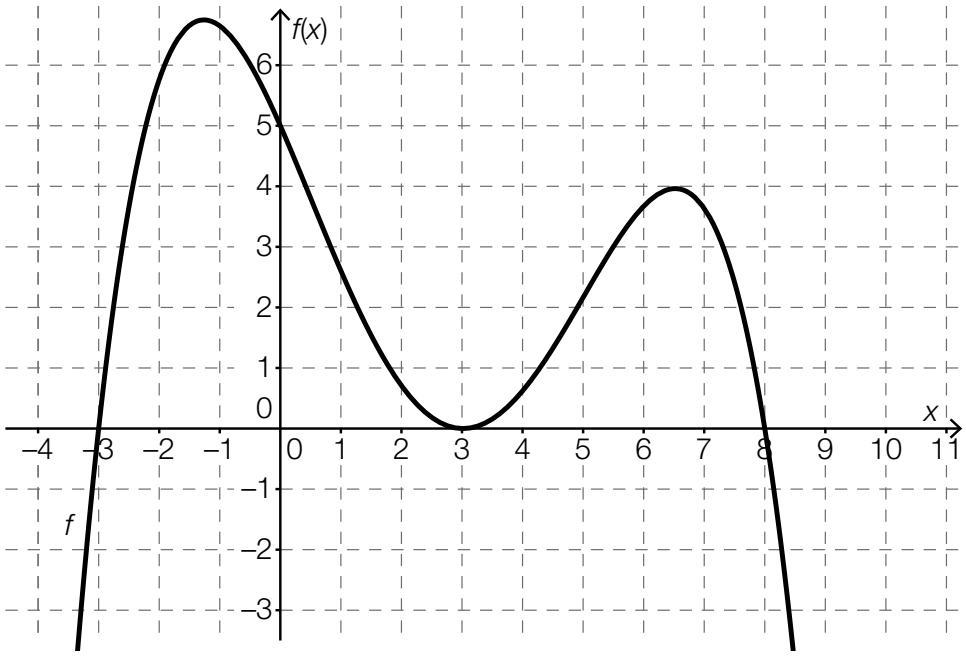
Determine the percentage by which the total fertility rate for 2014 of 1.46 children per woman would have to increase to meet the “maintenance level”.

percentage increase: _____ %

Task 14

Rates of Change of a Polynomial Function

The diagram below shows the graph of a polynomial function f.



Task:

Put a cross next to each of the two correct statements.

The differential quotient at the point where $x = 6$ is larger than the differential quotient at the point where $x = -3$.	☐
The differential quotient at the point where $x = 1$ is negative.	☐
The difference quotient in the interval $[-3, 0]$ is 1.	☐
The average rate of change is not 0 in any interval.	☐
The difference quotient in the interval $[3, 6]$ is positive.	☐

Task 15

Derivative and Antiderivative

Let f be a polynomial function and F one of its antiderivatives.

Task:

Put a cross next to each of the two correct statements.

A function F is an antiderivative of the function f if $f(x) = F(x) + c$ $(c \in \mathbb{R})$.	☐
A function f' is a derivative of the function f if $\int f(x)\mathrm{d}x = f'(x)$ holds.	☐
If the function f is defined at x_0, then $f'(x_0)$ gives the gradient of the tangent of the graph of f at this point.	☐
The function f has infinitely many antiderivatives that only differ by an additive constant.	☐
If the antiderivative F is integrated once, the function f is obtained.	☐

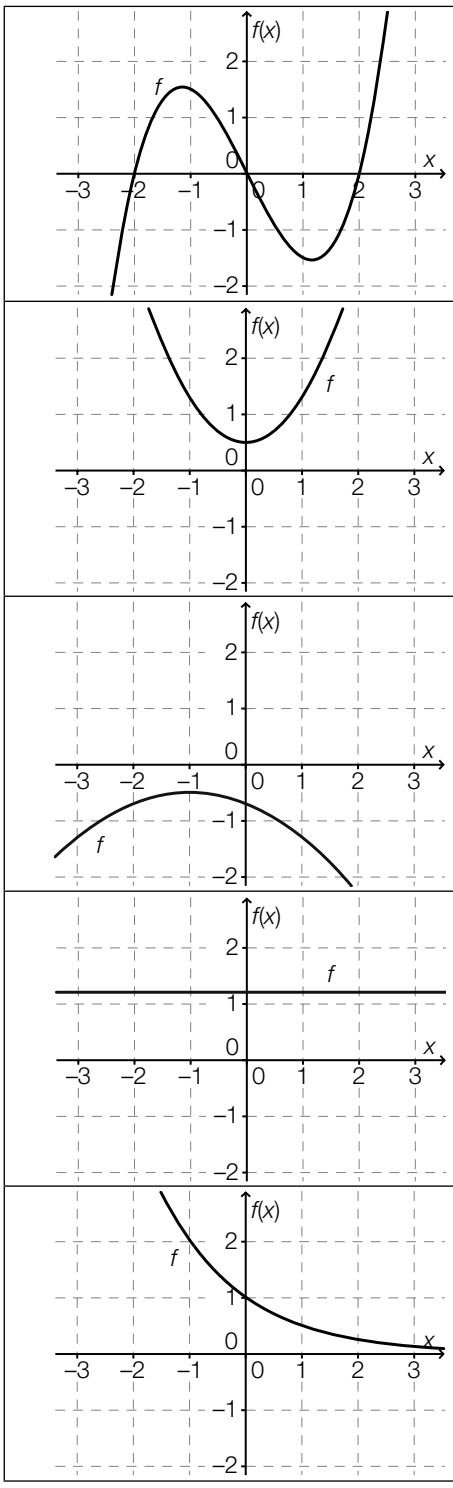
Task 16

Properties of the Second Derivative

Graphs of five real functions are given below.

Task:

For which of the functions given does $f''(x) > 0$ hold in the interval [–1, 1]?
Put a cross next to each of the two correct graphs.

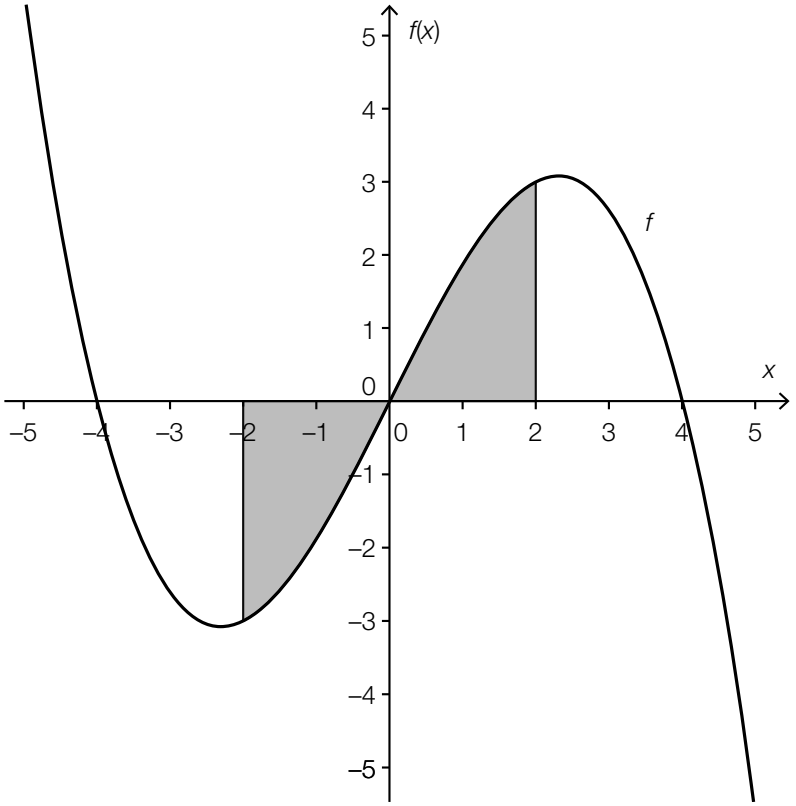


Task 17

Area

A section of the graph of the polynomial function f where $f(x) = -\frac{x^3}{8} + 2 \cdot x$ is shown below.

The area between the graph of the function f and the x-axis in the interval [–2, 2] is shaded in grey.



Task:

Determine the area of the region shaded in grey.

Task 18

Tachograph

Using a tachograph, the velocity of a vehicle in relation to time can be recorded. Let $v(t)$ be the velocity at time t.
The time is measured in hours (h) and the velocity in kilometres per hour (km/h).
A vehicle sets off at time $t = 0$.

Task:

Write down the meaning of the equation $\int_0^{0.5} v(t)\,\mathrm{d}t = 40$ by using the correct units in the given context.

Task 19

Average Number of Missed Lessons

In a school there are four sport classes: $S1$, $S2$, $S3$ and $S4$. The table below gives an overview of the number of students per class as well as the corresponding mean of the number of missed lessons in the first semester of a school year.

Class	Number of Students	Mean Number of Missed Lessons
$S1$	18	45.5
$S2$	20	63.2
$S3$	16	70.5
$S4$	15	54.6

Task:

Determine $\bar{x}_{ges}$, the mean number of missed lessons of all four sport classes for the given time period.

Task 20

Coin Toss

In a random experiment, a coin with a number on one side and a crest on the other is thrown twice.

Task:

Write down all possible results of this experiment. In your answer, *crest* can be abbreviated by *W* and *number* by *Z*.

Task 21

Online Gambling

A man plays the same online gambling game regularly over a long period. The game has a constant probability of winning. Out of 768 games, the man wins 162 times.

Task:

What is the probability of the man winning the next game? Put a cross next to the correct estimate for this probability.

0.162 %	☐
4.74 %	☐
16.2 %	☐
21.1 %	☐
7.68 %	☐
76.6 %	☐

Task 22

Soft and Hard-Boiled Eggs

At the breakfast buffet of a hotel there are 10 eggs in a basket. From the outside, the eggs appear to be identical. During preparation, one hard-boiled egg was accidentally put with nine soft-boiled eggs.

Task:

A woman takes an egg at random out of the full basket. Determine the probability that the next guest randomly chooses a hard-boiled egg.

Task 23

Random Experiment

In a random experiment that is repeated 25 times, the results “favourable” and “unfavourable” are possible. The random variable X describes how often the result “favourable” comes up. X is binomially distributed with an expectation value of 10.

Task:

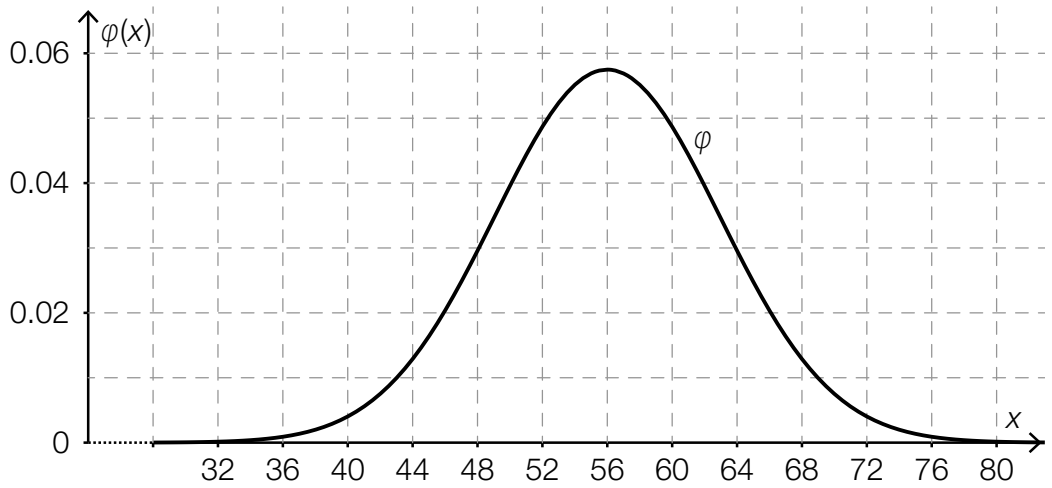
Two of the statements below can be deduced from this information. Put a cross next to each of the two correct statements.

$P(X = 25) = 10$	☐
If the random experiment is repeated 25 times, 10 of the results will definitely be “favourable”.	☐
The probability that a single random experiment is “favourable” is 40%.	☐
If the random experiment is conducted 50 times, the expectation value for the number of “favourable” results is 20.	☐
$P(X > 10) > P(X > 8)$	☐

Task 24

Blood Group

In Europe, the probability of being born with blood group B is around 0.14. For a study, n people who were born in Europe are chosen at random. The random variable X describes the number of people with blood group B. The distribution of X can be approximated by a normal distribution whose probability density function is shown in the diagram below.



Task:

From the diagram above, estimate the sample size n of this study.

$n \approx$ _____