Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

May 2017

Mathematics

Supplementary Examination 6
Examiner's Version

BMB
Bundesministerium
für Bildung

Instructions for the supplementary examination

The following supplementary examination is comprised of five tasks that can be completed independently of one another.

Each task contains two parts: The statement of the task requires the candidate to demonstrate core competencies, and the guiding question that follows it requires the candidate to show their ability to communicate their ideas.

In the following document, the examiner will find the tasks as well as the expected solutions and the answer key.

The preparation time is to be at least 30 minutes and the examination time is to be at most 25 minutes.

Assessment

Each task can be awarded zero, one or two points. There is one point available for each demonstration of core competencies as well as for each guiding question. A maximum of 10 points can be achieved.

For the grading of the examination the following scale should be used:

Grade	Minimum number of points
Pass	4 points for the core competencies + 0 points for the guiding questions 3 points for the core competencies + 1 point for the guiding questions
Satisfactory	5 points for the core competencies + 0 points for the guiding questions 4 points for the core competencies + 1 point for the guiding questions 3 points for the core competencies + 2 points for the guiding questions
Good	5 points for the core competencies + 1 point for the guiding questions 4 points for the core competencies + 2 points for the guiding questions 3 points for the core competencies + 3 points for the guiding questions
Very good	5 points for the core competencies + 2 points for the guiding questions 4 points for the core competencies + 3 points for the guiding questions

The examination board will decide on the final grade based on the candidate's performance in the supplementary examination as well as the result of the written examination.

Evaluation grid for the supplementary examination

This evaluation grid may be used to assist the examiner in assessing the candidate’s performance.

	Point for core competencies reached	Point for the guiding question reached
Task 1		
Task 2		
Task 3		
Task 4		
Task 5		

Task 1

Equivalent Transformations

Below, you will see two equations that hold for $x \in \mathbb{R}$:

- $3 - \frac{2x}{5} = -1$
- $\frac{3x}{5} + 1 = x - 3$

Task:

Determine whether these two equations are equivalent.

If the equations are equivalent, show a series of possible equivalent transformations that transform the first equation into the second.
If the equations are not equivalent, justify why this is the case.

Guiding question:

With specific reference to the example given below, explain why the rearrangement shown does not give rise to an equivalent equation. The equation is defined over the set of all real numbers.

$(x - 2)^2 = 25 \;\; | \;\; \sqrt{}$
$x - 2 = 5$

Solution of Task 1

Equivalent Transformations

Expected solution of the statement of the task:

The two equations are equivalent.

Possible equivalent transformations:
By subtracting the number 2, the equation $1 - \frac{2x}{5} = -3$ is obtained.
Then, by adding x, the equation becomes $1 + \frac{3x}{5} = -3 + x$, which is the same as the second equation.

Answer key:

The point for the core competencies is to be awarded if the equations are determined to be equivalent and possible equivalent transformations have been given correctly.

Expected solution of the guiding question:

The first equation has solutions -3 and 7; however, the second equation has only one solution, $x = 7$. Therefore, the two equations do not have the same set of solutions and are not equivalent.

Answer key:

The point for the guiding question is to be awarded if it has been correctly explained why the two equations are not equivalent.

Task 2

Cooling

At time $t_0 = 0$, a container with hot water is put outside where the ambient temperature is 0 °C. The temperature of the water, $T(t)$ (in °C), is dependent on the time t (in minutes) and can be described by the function T where $T(t) = 90 \cdot e^{-0.2 \cdot t}$.

Task:

Determine the half-life of this cooling process and explain the significance of this result within the context given.

Guiding question:

Show that the instantaneous rate of change of the temperature of the water, $T'(t)$, is directly proportional to the instantaneous temperature of the water at time t. Determine the constant of proportionality, k.

$k =$ ____

Explain the meaning of the absolute value T' in the context of the cooling process.

Solution of Task 2

Cooling

Expected solution of the statement of the task:

$45 = 90 \cdot e^{-0{,}2 \cdot t} \;\Rightarrow\; t \approx 3.5$

The temperature of the water has reduced to half of its starting temperature (from 90 ºC to 45 ºC) after approximately 3.5 minutes.

Answer key:

The point for the core competencies is to be awarded if the half-life has been correctly determined and correctly interpreted within the context of the question.

Expected solution of the guiding question:

$T'(t) = 90 \cdot (-0.2) \cdot e^{-0.2 \cdot t} = -0.2 \cdot T(t)$
$k = -0.2$

The absolute value of T' gives the speed of the cooling process.

Answer key:

The point for the guiding question is to be awarded if the directly proportional relationship has been shown and the constant of proportionality has been correctly determined ($k = -5$ is also to be accepted as a correct answer, as $T(t) = -5 \cdot T'(t)$). Also, the meaning of T' needs to have been given correctly.

Task 3

Crude Oil Price

In December 2015, the price of crude oil tended to fall daily. The price of crude oil is given by the barrel in US dollars. One barrel contains 159 litres.

On the 1^{st} December 2015 at 12:00 noon, the crude oil price was 41.70 US dollars per barrel. On the 11^{th} December 2015 at 12:00 noon, the price was 37.94 US dollars per barrel.

Task:

Determine the absolute and relative (percentage) change of the crude oil price per barrel for the given time period.

Guiding question:

Determine the average rate of change of the crude oil price per litre for the given time period (in days) and interpret your result in the given context.

Determine the price of 1 litre of crude oil on the 16^{th} December 2015 if the crude oil price from the 11^{th} December 2015 had continued to develop with the same average rate of change per day.

Solution of Task 3

Crude Oil Price

Expected solution of the statement of the task:

Absolute change: –3.76 US dollars per barrel
Relative change: –9 % or –0.09

Answer key:

The point for the core competencies is to be awarded if both values are given correctly. Positive values (3.76 US dollars and 9 %) are also to be accepted if the candidate explains verbally that these values represent a reduction.

Expected solution of the guiding question:

Possible solution:

Average rate of change: $\frac{\frac{37.94}{159} - \frac{41.7}{159}}{10} \approx -0.00236$

The crude oil price per litre reduced by an average of approximately 0.00236 US dollars per day in this time period.

The price per litre on 16.12.2015 if the development had continued:
$\frac{37.94}{159} - 5 \cdot 0.00236 \approx 0.2268$
The price per litre would have been approximately 0.2268 US dollars.

Answer key:

The point for the guiding question is to be awarded if the average rate of change of the crude oil price per litre has been given and interpreted correctly. Also, the crude oil price per litre on the 16.12.2015 needs to have been correctly given.
Tolerated range for the average rate of change: [–0.0024, –0.002]
Tolerated range for the price per litre: [0.22, 0.23]

Task 4

Integral

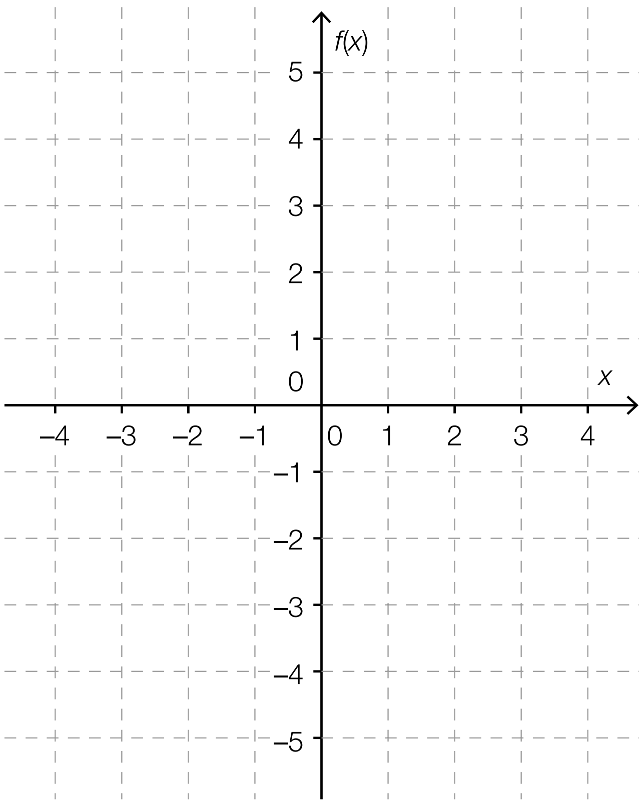
Let f be a linear function where $f(x) = -2 \cdot x + 2$.

Task:

Find the equation of the antiderivative of the function f, F, for which $F(2) = 1$ holds. Show your method.

Guiding question:

Find the value of the definite integral $\int_0^3 f(x)\,\mathrm{d}x$. Show your method.
Draw the graph of the function f in the coordinate system given below and explain why, in this case, the value of the definite integral does not correspond with the area enclosed by the graph of the function and the x-axis in the range [0, 3].



Solution of Task 4

Integral

Expected solution of the statement of the task:

Possible approach:

The equation $F(x) = -x^2 + 2 \cdot x + c$ holds for all antiderivatives.
As $F(2) = 1$, $-2^2 + 2 \cdot 2 + c = 1 \Rightarrow c = 1$.
Hence: $F(x) = -x^2 + 2 \cdot x + 1$.

Answer key:

The point for the core competencies is to be awarded if a correct equation for F has been given and a correct method has been explained.

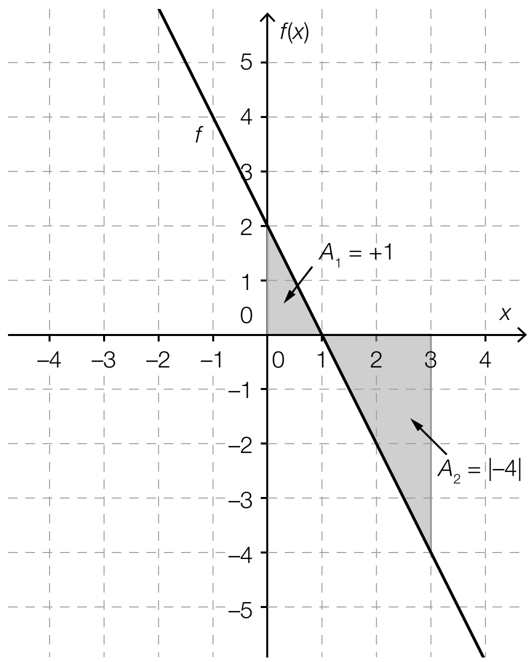
Expected solution of the guiding question:

Possible methods:
$\int_0^3 f(x)\,dx = \int_0^3 (-2 \cdot x + 2)\,dx = (-x^2 + 2 \cdot x)\Big|_0^3 = -3$ and/or $F(3) - F(0) = -2 - 1 = -3$

Possible explanation: When calculating areas, it needs to be considered that the integral of the function for regions that lie underneath the x-axis is negative. Therefore, the calculation has to be carried out in sections corresponding to the positions of the regions.

$\int_0^3 f(x)\,dx = 1 + (-4) = -3$

Area:
$A_1 + A_2 = 1 + |-4| = 5$



Answer key:

The point for the guiding question is to be awarded if the value of the integral has been determined correctly, a correct method has been used, the graph has been drawn correctly, and a correct explanation has been given.

Task 5

Cone

A cone that is thrown can either land on its curved surface or on its base.

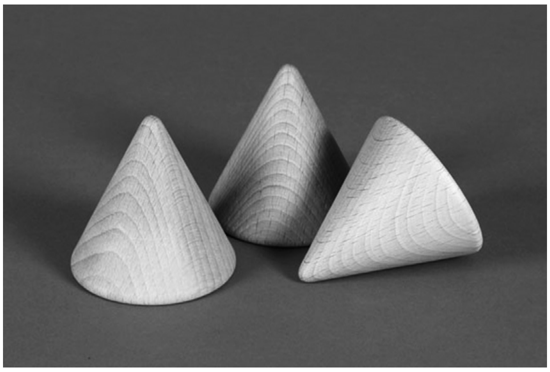


Image source: http://www.holzbausteine.at/images/Spitzkegel60.jpg [28.04.2016].

Task:

Throwing a cone of this kind can be seen as a random experiment. At first, the cone is thrown 50 times. In 12 of the throws, the cone lands on its base.

Felix carries out the following calculation:

$\left(\frac{12}{50}\right)^2 = \frac{144}{2{,}500} = 0.0576 = 5.76\ \%$

Interpret the result in the given context.

Guiding question:

Selin says that actually, the probability of the cone landing on its base is unknown.

Suggest an argument Selin can use to support her statement and how the experiment would need to be changed to calculate this probability as accurately as possible.

Solution of Task 5

Cone

Expected solution of the statement of the task:

The probability of the cone landing on its base twice in two throws is 5.76 % (given that the probability of the cone landing on its base is $\frac{12}{50}$).

Answer key:

The point for the core competencies is to be awarded if a correct interpretation in the given context has been supplied.

Expected solution of the guiding question:

The relative frequency only gives an estimate for the probability.
If an experiment is only conducted 50 times, then the probability calculated by the relative frequency is a very inaccurate approximation.
The experiment would have to be conducted much more often to be able to obtain a more accurate estimate for the probability.

Answer key:

The point for the guiding question is to be awarded if an answer corresponding to one of the suggested solutions has been given.