Name:	
Class:	

Standardised Competence-Oriented
Written School-Leaving Examination

AHS

28th September 2017

Mathematics

Part 2 Tasks

BMB
Bundesministerium
für Bildung

Advice for Completing the Tasks

Dear candidate,

The following booklet for Part 2 contains four tasks, each of which contains between two and four sub-tasks. All sub-tasks can be completed independently of one another. You have *150 minutes* available in which to work through this booklet.

Please use a blue or black pen that cannot be rubbed out. You may use a pencil for tasks that require you to draw a graph, vectors or a geometric construction.

When completing these tasks please use this booklet and the paper provided. Write your name on each piece of paper you use as well as on the first page of this task booklet in the space provided. Please label your work clearly to show which sub-task each answer relates to.

In the assessment of your work, everything that is not crossed out will be considered. Your solutions must be clearly marked. If a solution is not clearly marked or if more than one solution is given, the task will be considered to be unsolved. Draw a line through any notes you make.

You may use a pre-approved formula book as well as your usual electronic device(s).

Please hand in both the task booklet and the separate sheets you have used at the end of the examination.

Assessment

Every task in Part 1 will be awarded either 0 points or 1 point. Every sub-task in Part 2 will be awarded 0, 1 or 2 points. The tasks marked with an [A] will be awarded either 0 points or 1 point.

– If at least 16 of the 24 tasks in Part 1 are solved correctly, you will pass the examination.
– If fewer than 16 of the 24 tasks in Part 1 are solved correctly, then the tasks marked with an [A] from Part 2 may compensate for the shortfall (as part of the "range of essential skills" outlined by the LVBO).
  If, including the tasks marked with an [A] from Part 2, at least 16 tasks are solved correctly, you will pass the examination.
  If, including the tasks marked with an [A] from Part 2, fewer than 16 tasks are solved correctly, you will not be awarded enough points to pass the examination.
– If at least 16 tasks are solved correctly (including the compensation tasks marked with an [A] from Part 2), a grade will be awarded as follows:

Pass	16–23 points
Satisfactory	24–32 points
Good	33–40 points
Very Good	41–48 points

Explanation of the Task Types

Some tasks require a *free answer*. For these tasks, you should write your answer directly underneath each task in the task booklet or on the provided paper. Other task types used in the examination are as follows:

Matching tasks: For this task type you will be given a number of statements, tables or diagrams, which will appear alongside a selection of possible answers. To correctly answer these tasks, you will need to match each statement, table or diagram to its corresponding answer. You should write **the letter of the correct answer** next to the statement, table or diagram in the space provided.

Example:
You are given two equations.

Task:
Match the two equations to their corresponding description (from A to D).

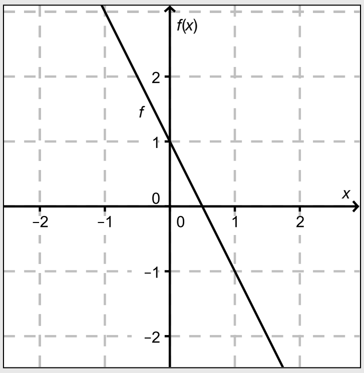
1 + 1 = 2	A
$2 \cdot 2 = 4$	C

A	Addition
B	Division
C	Multiplication
D	Subtraction

Construction tasks: This task type requires you to draw points, lines and/or curves in the task booklet.

Example:
Below you will see a linear function f where $f(x) = k \cdot x + d$.

Task:
On the axes provided below, draw the graph of a linear function for which $k = -2$ and $d > 0$.



Multiple-choice tasks of the form "1 out of 6": This task type consists of a question and six possible answers. Only one answer should be selected. You should put a cross next to the only correct answer in the space provided.

Example:
Which equation is correct?

Task:
Put a cross next to the correct equation.

1 + 1 = 1	☐
2 + 2 = 2	☐
3 + 3 = 3	☐
4 + 4 = 8	☒
5 + 5 = 5	☐
6 + 6 = 6	☐

Multiple-choice tasks of the form "2 out of 5": This task type consists of a question and five possible answers, of which two answers should be selected. You should put a cross next to each of the two correct answers in the space provided.

Example:
Which equations are correct?

Task:
Put a cross next to each of the two correct equations.

1 + 1 = 1	☐
2 + 2 = 4	☒
3 + 3 = 3	☐
4 + 4 = 8	☒
5 + 5 = 5	☐

Multiple-choice tasks of the form "x out of 5": This task type consists of a question and five possible answers, of which one, two, three, four *or* five answers may be selected. The task will require you to: "Put a cross next to each correct statement/equation …". You should put a cross next to each correct answer in the space provided.

Example:
Which of the equations given are correct?

Task:
Put a cross next to each correct equation.

1 + 1 = 2	☒
2 + 2 = 4	☒
3 + 3 = 6	☒
4 + 4 = 4	☐
5 + 5 = 10	☒

Gap-fill: This task type consists of a sentence with two gaps, i.e. two sections of the sentence are missing and must be completed. For each gap you will be given the choice of three possible answers. You should put a cross next to **each of the two answers** that are necessary to complete the sentence correctly.

Example:
Below you will see 3 equations.

Task:
Complete the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

The operation in equation ____①____ is known as summation or ____②____ .

①	
1 – 1 = 0	☐
1 + 1 = 2	☒
1 · 1 = 1	☐

②	
Multiplication	☐
Subtraction	☐
Addition	☒

Changing an answer for a task that requires a cross:

1. Fill in the box that contains the cross for your original answer.
2. Put a cross in the box next to your new answer.

1 + 1 = 3	☐
2 + 2 = 4	☒
3 + 3 = 5	☐
4 + 4 = 4	☐
5 + 5 = 9	■

In this instance, the answer "5 + 5 = 9" was originally chosen. The answer was later changed to be "2 + 2 = 4".

Selecting an answer that has been filled in:

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

1 + 1 = 3	☐
2 + 2 = 4	(■)
3 + 3 = 5	☐
4 + 4 = 4	■
5 + 5 = 9	☐

In this instance, the answer "2 + 2 = 4" was filled in and then selected again.

If you still have any questions now, please ask your teacher.

Good Luck!

Task 1

Radioactivity and Carbon Dating

When a radioactive substance decays, the number of nuclei still to decay decreases exponentially and can be approximated by the function N where $N(t) = N_0 \cdot e^{-\lambda \cdot t}$. In this function, N_0 is the number of atomic nuclei at time $t = 0$, $N(t)$ gives the number of nuclei still to decay at time $t \geq 0$, and λ is the so-called decay constant.

The activity, $A(t)$, is given by the absolute value of the instantaneous rate of change of the function N at time t. It is measured in Becquerels (Bq). An activity of 1 Bq corresponds to one decay per second.

For radioactive substances, the activity also decreases exponentially and can be modelled by a function A, where $A(t) = A_0 \cdot e^{-\lambda \cdot t}$. In this function, A_0 gives the activity at time $t = 0$ and $A(t)$ gives the activity at time $t \geq 0$.

Task:

a) Write down a formula using which the original number of atomic nuclei, N_0, can be calculated from the observed activity, A_0.

A sample of ^{238}U (uranium 238) has an activity of 17 Bq at time $t = 0$. The decay constant of ^{238}U is $\lambda \approx 4.92 \cdot 10^{-18}$ per second.
Determine the number of ^{238}U nuclei in the sample at time $t = 0$.

b) Using the amount of the carbon isotope ^{14}C contained in a sample, the age of the sample can be determined. Due to the metabolic process, an equilibrium concentration of ^{14}C and/or an activity of about 0.267 Bq per gram of carbon had adjusted between the formation and the decay of the isotope, in the atmosphere as well as in living organisms. With the dying of an organism (e.g. a tree), the absorption of ^{14}C ends. The ^{14}C-amount decreases from this point of time exponentially (with a decay constant of $\lambda \approx 1.21 \cdot 10^{-4}$ per year) and with it, the activity also begins to decrease exponentially.

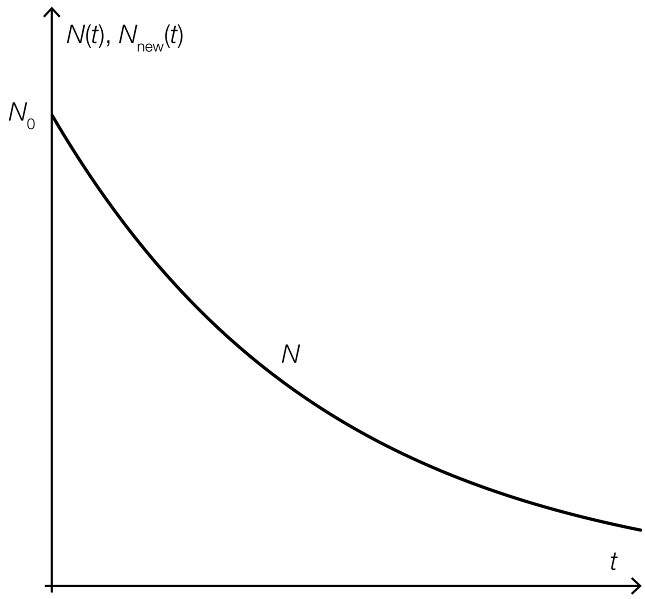
A wooden artefact contains 25 grams of carbon and has an activity of around 4 Bq.
Determine how many years ago this wood has died.

Determine whether more or fewer than half of the original number of ^{14}C atoms had decayed by the time the artefact was found and justify your answer.

c) The function N can also be written in the form $N(t) = N_0 \cdot 0.5^{\frac{t}{c}}$, where $c \in \mathbb{R}^+$.

[A] Determine the relationship between the constant c and the half-life of a radioactive substance, τ.

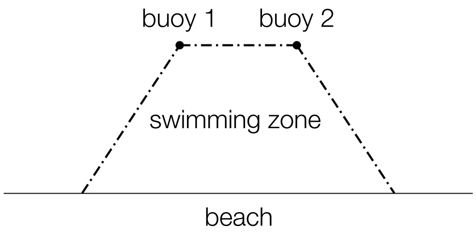
In the diagram below, the graph of a function N is shown, where $N(t) = N_0 \cdot 0.5^{\frac{t}{c}}$ with $c \in \mathbb{R}^+$.
On this diagram, draw the graph of a function N_{new}, where $N_{new}(t) = N_0 \cdot 0.5^{\frac{t}{c_{new}}}$ with $c_{new} \in \mathbb{R}^+$ given that $c_{new} < c$.



Task 2

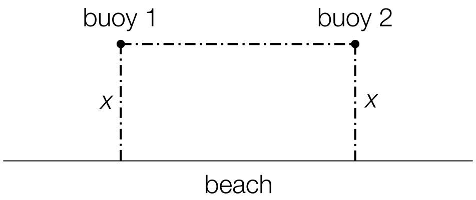
Swimming Zones

Due to the large number of motorboats, jet-skis etc., some beaches have introduced swimming zones.
All of the swimming zones dealt with in this task are created using two buoys and a 180 meter-long rope that is attached to a beach. The beach can be approximated by a straight line.



Task:

a) A rectangular swimming zone is shown below (x is measured in metres).



[A] Show that the equation the area $A(x)$ of this type of swimming zone can be given by $A(x) = 180 \cdot x - 2 \cdot x^2$.

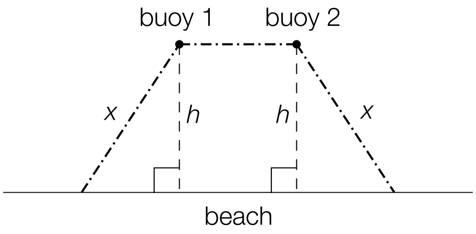
Determine the length, the width and the area of the swimming zone that has the largest possible area.

Length = ______________________________ m

Width = ______________________________ m

Area = ______________________________ m^2

b) A swimming zone in the shape of a trapezium is shown below (x and h are measured in metres).

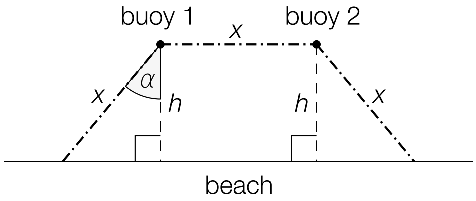


In order to calculate the area of a trapezium-shaped swimming zone, the following formula can be used: $A(x, h) = h \cdot \left(180 - 2 \cdot x + \sqrt{x^2 - h^2}\right)$.

Determine the possible values of x when h is 40 m long.

Determine the possible values of h when x is 50 m long.

c) A swimming zone in the shape of a trapezium is shown below in which the three sections of rope are the same length (x and h are measured in metres).

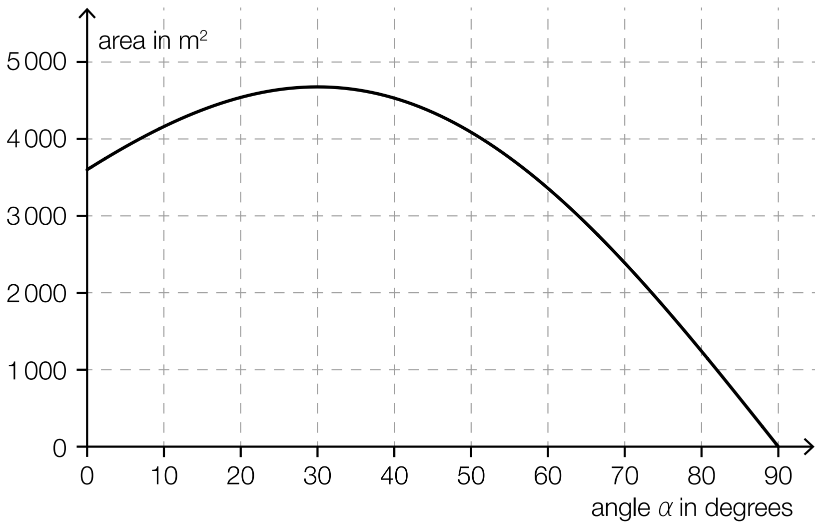


The area of a swimming zone of this type, $A(\alpha)$, can be described as a function of the angle α (where $A(\alpha)$ is measured in m^2 and α is measured in degrees).

Write down a formula that can be used to calculate the area of such a swimming zone given the angle α.

$A(\alpha) =$ ______________________________

In the diagram below, the values of the areas for various angles α are shown.



A swimming zone with the largest possible area is to be constructed. Using the diagram above, determine the length of the section of the beach from which the swimming zone with the largest possible area can be accessed.

Task 3

Brazil

Brazil is the largest and most populous country in South America.

In 2014, Brazil had 202.74 million inhabitants.

From censuses, the following inhabitant data is known:

Year	Number of Inhabitants
1970	94 508 583
1980	121 150 573
1991	146 917 459
2000	169 590 693
2010	190 755 799

Task:

a) [A] Write down the meaning of the values shown below in the context of the development of the number of inhabitants.

$$\sqrt[10]{\frac{121\,150\,573}{94\,508\,583}} \approx 1.02515$$

$$\sqrt[9]{\frac{169\,590\,693}{146\,917\,459}} \approx 1.01607$$

Using the values shown, justify why the development of the number of inhabitants within the complete time period between 1970 and 2010 cannot suitably be described by an exponential function.

b) Assuming that the trend follows linear growth, use the values for the numbers of inhabitants in 1991 and 2010 to write down a function f that describes the number of inhabitants. In this function, the time t is measured in years and the time $t = 0$ corresponds to the year 1991.

Determine the percentage by which the prediction given by the linear model for 2014 deviates from the actual value given in the introduction.

c) For the time period from 2010 to 2015, there was a constant birth rate of $b = 14.6$ and a constant death rate of $d = 6.6$ in Brazil. These figures mean that there were 14.6 births per 1 000 inhabitants and 6.6 deaths per 1 000 inhabitants per year.

The development of the number of inhabitants can be described by the difference equation $x_{n+1} = x_n + x_n \cdot \frac{1}{1000} \cdot (b - d) + m_n$, where x_n gives the number of inhabitants in year n and m_n gives the difference between the number of immigrants and emigrants. This difference is known as the net migration.

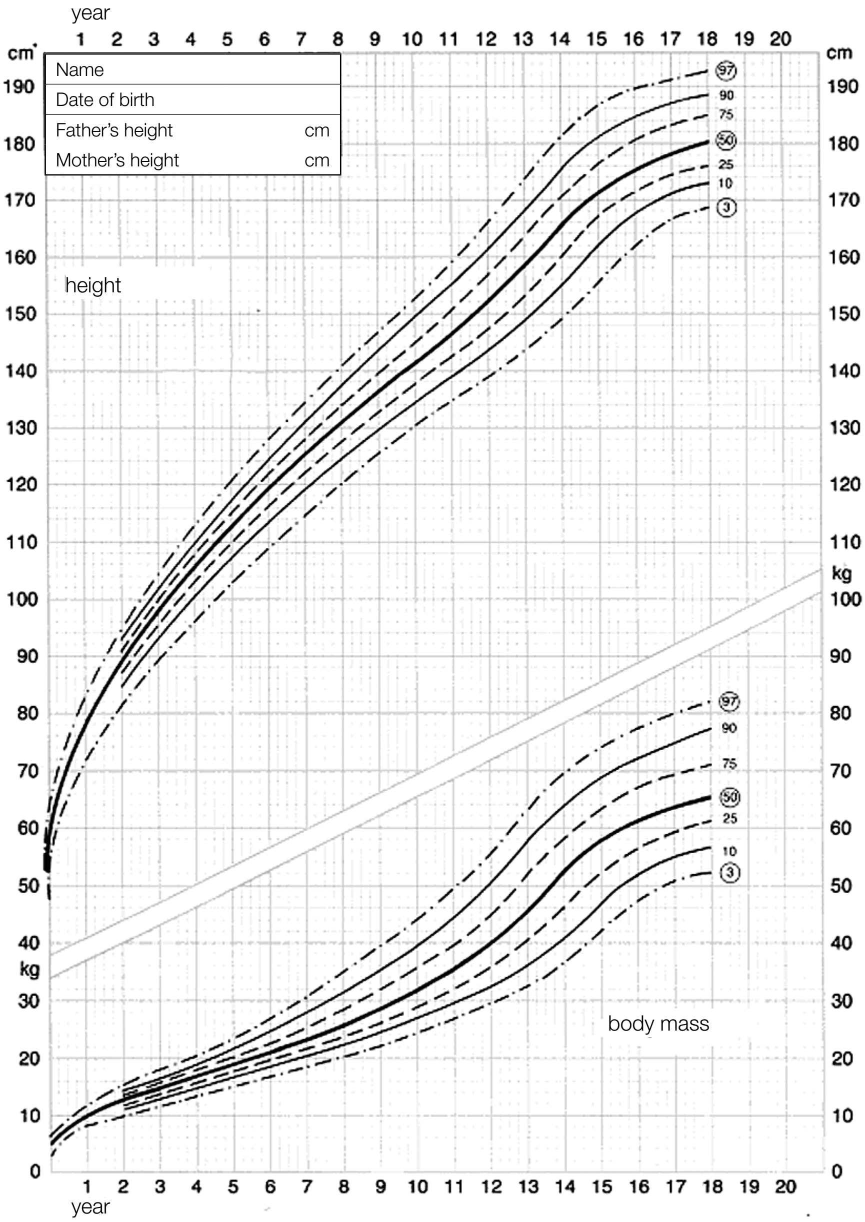
Write down the meaning of the expression $x_n \cdot \frac{1}{1000} \cdot (b - d)$ in the context of the development of the number of inhabitants.

Determine the largest possible value for net migration if the number of inhabitants in 2015 in comparison to the previous year has increased by a maximum of 1 %.

Task 4

Children’s Growth

In order to monitor the development of a child’s height and mass, the percentile curves for height (in cm) and mass (in kg) are provided in the baby’s ‘Developmental Record Book’. Percentiles split the heights and masses of children into percentage-areas. If a value lies on the 10th percentile curve for height, this means that 10 % of children at this age are shorter than or the same height as this value, and 90 % are taller than or the same height as this value.
It is common to describe all values between the 3rd and 97th percentiles as “normal”. The following diagram shows the height and body mass curves for boys aged between 0 and 18 years:



Source: http://www.grosswuchs.de/WachstumTabelleJ.htm [21.05.2014] (adapted).

Task:

a) A school doctor is examining a random sample of 8 year-old boys from his district and records, among other things, their masses (in kg). Using the results of these measurements, he constructs the symmetrical confidence interval [0.8535, 0.9465] with confidence level $\gamma = 0.95$ for the proportion of 8 year-old boys from his district whose mass lies in the "normal range" of [20 kg, 35 kg].

   Determine the difference in percentage points between the proportion of the sample with a body mass in the "normal range" according to the calculation and the proportion of all 8 year-old boys with a body mass in the "normal range" according to the diagram.

   Determine the number of 8 year-old boys that were included in this random sample.

b) The height of a particular child on their first, second, third (etc.) birthday is given by $g(1)$, $g(2)$, $g(3)$, … respectively. Write down either in words or as a formula, how the average rate of growth of this child in the three year period between its 6^{th} and 9^{th} birthday can be determined.

   Consider the growth curve for the 50^{th} percentile after a child's 8^{th} birthday. Determine the approximate age at which the instantaneous speed of growth is the highest.

c) [A] State which statistical value can be read from the 50^{th} percentile.

   Describe the difficulties that would arise if you were to attempt to construct a box plot to represent the heights of 8 year-old boys from the diagram given.