

Name:	
Class:	



Standardised Competence-Oriented  
Written School-Leaving Examination

AHS

28<sup>th</sup> September 2017

Mathematics

Part 1 Tasks

**BMB**

Bundesministerium  
für Bildung



## Advice for Completing the Tasks

Dear candidate,

The following booklet for Part 1 contains 24 tasks. The tasks can be completed independently of one another. You have 120 minutes available in which to work through this booklet.

Please use a blue or black pen that cannot be rubbed out. You may use a pencil for tasks that require you to draw a graph, vectors or a geometric construction.

Please do all of your working out solely in this booklet. Write your name on the first page of the booklet in the space provided.

All answers must be written in this booklet. In the assessment of your work, everything that is not crossed out will be considered. Your solutions must be clearly marked. If a solution is not clearly marked or if more than one solution is given, the task will be considered to be unsolved. Draw a line through any notes you make.

You may use a pre-approved formula book as well as your usual electronic device(s).

Please hand in the task booklet at the end of the examination.

### Assessment

Every task in Part 1 will be awarded either 0 points or 1 point. Every sub-task in Part 2 will be awarded 0, 1 or 2 points. The tasks marked with an **A** will be awarded either 0 points or 1 point.

- If at least 16 of the 24 tasks in Part 1 are solved correctly, you will pass the examination.
- If fewer than 16 of the 24 tasks in Part 1 are solved correctly, then the tasks marked with an **A** from Part 2 may compensate for the shortfall (as part of the “range of essential skills” outlined by the LVBO). If, including the tasks marked with an **A** from Part 2, at least 16 tasks are solved correctly, you will pass the examination.  
If, including the tasks marked with an **A** from Part 2, fewer than 16 tasks are solved correctly, you will not be awarded enough points to pass the examination.
- If at least 16 tasks are solved correctly (including the compensation tasks marked with an **A** from Part 2), a grade will be awarded as follows:

Pass	16–23 points
Satisfactory	24–32 points
Good	33–40 points
Very Good	41–48 points

### Explanation of the Task Types

Some tasks require a *free answer*. For these tasks, you should write your answer directly underneath each task in the task booklet. Other task types used in the examination are as follows:

**Matching tasks:** For this task type you will be given a number of statements, tables or diagrams, which will appear alongside a selection of possible answers. To correctly answer these tasks, you will need to match each statement, table or diagram to its corresponding answer. You should write **the letter of the correct answer** next to the statement, table or diagram in the space provided.

#### Example:

You are given two equations.

$1 + 1 = 2$	<i>A</i>
$2 \cdot 2 = 4$	<i>C</i>

A	Addition
B	Division
C	Multiplication
D	Subtraction

#### Task:

Match the two equations to their corresponding description (from A to D).

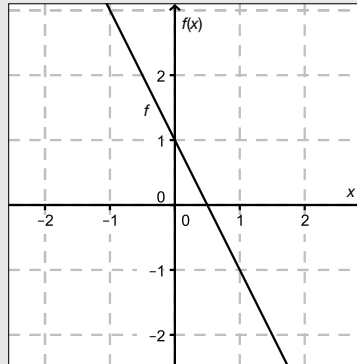
**Construction tasks:** This task type requires you to draw points, lines and/or curves in the task booklet.

**Example:**

Below you will see a linear function  $f$  where  $f(x) = k \cdot x + d$ .

**Task:**

On the axes provided below, draw the graph of a linear function for which  $k = -2$  and  $d > 0$ .



**Multiple-choice tasks of the form “1 out of 6”:** This task type consists of a question and six possible answers. Only **one** answer should be selected. You should put a cross next to the correct answer in the space provided.

**Example:**

Which equation is correct?

**Task:**

Put a cross next to the correct equation.

$1 + 1 = 1$	<input type="checkbox"/>
$2 + 2 = 2$	<input type="checkbox"/>
$3 + 3 = 3$	<input type="checkbox"/>
$4 + 4 = 8$	<input checked="" type="checkbox"/>
$5 + 5 = 5$	<input type="checkbox"/>
$6 + 6 = 6$	<input type="checkbox"/>

**Multiple-choice tasks of the form “2 out of 5”:** This task type consists of a question and five possible answers, of which **two** answers should be selected. You should put a cross next to each of the two correct answers in the space provided.

**Example:**

Which equations are correct?

**Task:**

Put a cross next to each of the two correct equations.

$1 + 1 = 1$	<input type="checkbox"/>
$2 + 2 = 4$	<input checked="" type="checkbox"/>
$3 + 3 = 3$	<input type="checkbox"/>
$4 + 4 = 8$	<input checked="" type="checkbox"/>
$5 + 5 = 5$	<input type="checkbox"/>

**Multiple-choice tasks of the form “x out of 5”:** This task type consists of a question and five possible answers, of which **one, two, three, four or five** answers may be selected. The task will require you to: “Put a cross next to each correct statement/equation ...”. You should put a cross next to each correct answer in the space provided.

**Example:**

Which of the equations given are correct?

**Task:**

Put a cross next to each correct equation.

$1 + 1 = 2$	<input checked="" type="checkbox"/>
$2 + 2 = 4$	<input checked="" type="checkbox"/>
$3 + 3 = 6$	<input checked="" type="checkbox"/>
$4 + 4 = 4$	<input type="checkbox"/>
$5 + 5 = 10$	<input checked="" type="checkbox"/>

**Gap-fill:** This task type consists of a sentence with two gaps, i.e. two sections of the sentence are missing and must be completed. For each gap you will be given the choice of three possible answers. You should put a cross next to **each of the two answers** that are necessary to complete the sentence correctly.

**Example:**

Below you will see 3 equations.

**Task:**

Complete the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence is a correct statement.

The operation in equation \_\_\_\_\_<sup>①</sup>\_\_\_\_\_ is known as summation or \_\_\_\_\_<sup>②</sup>\_\_\_\_\_.

①		②	
$1 - 1 = 0$	<input type="checkbox"/>	Multiplication	<input type="checkbox"/>
$1 + 1 = 2$	<input checked="" type="checkbox"/>	Subtraction	<input type="checkbox"/>
$1 \cdot 1 = 1$	<input type="checkbox"/>	Addition	<input checked="" type="checkbox"/>

**Changing an answer for a task that requires a cross:**

1. Fill in the box that contains the cross for your original answer.
2. Put a cross in the box next to your new answer.

$1 + 1 = 3$	<input type="checkbox"/>
$2 + 2 = 4$	<input checked="" type="checkbox"/>
$3 + 3 = 5$	<input type="checkbox"/>
$4 + 4 = 4$	<input type="checkbox"/>
$5 + 5 = 9$	<input checked="" type="checkbox"/>

In this instance, the answer “ $5 + 5 = 9$ ” was originally chosen. The answer was later changed to be “ $2 + 2 = 4$ ”.

**Selecting an answer that has been filled in:**

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

$1 + 1 = 3$	<input type="checkbox"/>
$2 + 2 = 4$	<input checked="" type="checkbox"/>
$3 + 3 = 5$	<input type="checkbox"/>
$4 + 4 = 4$	<input checked="" type="checkbox"/>
$5 + 5 = 9$	<input type="checkbox"/>

In this instance, the answer “ $2 + 2 = 4$ ” was filled in and then selected again.

If you still have any questions, please ask your teacher.

**Good Luck!**

# Task 1

## Sets of Numbers

Below you will see statements relating to numbers from the sets  $\mathbb{N}$ ,  $\mathbb{Z}$ ,  $\mathbb{Q}$ ,  $\mathbb{R}$  and  $\mathbb{C}$ .

Task:

Put a cross next to each of the true statements.

Every real number is also a rational number.	<input type="checkbox"/>
Every natural number is also a rational number.	<input type="checkbox"/>
Every integer is also a real number.	<input type="checkbox"/>
Every rational number is also a real number.	<input type="checkbox"/>
Every complex number is also a real number.	<input type="checkbox"/>

## Task 2

### Solutions to a Quadratic Equation

Let  $x^2 + p \cdot x - 3 = 0$  where  $p \in \mathbb{R}$  be a quadratic equation.

Task:

Complete the sentence below by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

This equation has \_\_\_\_\_<sup>①</sup>\_\_\_\_\_, when \_\_\_\_\_<sup>②</sup>\_\_\_\_\_ holds.

①	
infinitely many real solutions	<input type="checkbox"/>
exactly one real solution	<input type="checkbox"/>
no real solutions	<input type="checkbox"/>

②	
$\frac{p^2}{4} + 3 > 0$	<input type="checkbox"/>
$\frac{p^2}{4} + 3 < 0$	<input type="checkbox"/>
$\frac{p^2}{4} + 3 > 1$	<input type="checkbox"/>

# Task 3

## Project Week

A total of 25 students are going to participate in a project week. The number of girls is represented by  $x$  and the number of boys by  $y$ . The girls will sleep in rooms with 3 beds and the boys will sleep in rooms with 4 beds. There are 7 bedrooms available in total. The beds in all 7 rooms will be used; there will be no empty beds left over.

### Task:

Using a system of equations comprised of two of the equations shown below, the number of girls and the number of boys can be calculated.

Put a cross next to each of the two correct equations.

$x + y = 7$	<input type="checkbox"/>
$x + y = 25$	<input type="checkbox"/>
$3 \cdot x + 4 \cdot y = 7$	<input type="checkbox"/>
$\frac{x}{3} + \frac{y}{4} = 7$	<input type="checkbox"/>
$\frac{x}{3} + \frac{y}{4} = 25$	<input type="checkbox"/>

## Task 4

### Sausage Stand

The owner of a sausage stand keeps records of the amount of sausages sold in a day. The records for one particular day are shown in the table below.

Type of sausage	Amount of portions sold	Price at which a portion is sold (in euros)	Cost to the owner of each portion (in euros)
Frankfurter	24	2.70	0.90
Debreziner	14	3.00	1.20
Burenwurst	11	2.80	1.00
Käsekrainer	19	3.20	1.40
Bratwurst	18	3.20	1.20

The numbers of each column of the table can be written as vectors. The vector  $A$  gives the number of portions sold, the vector  $B$  gives the prices at which each portion is sold (in euros), and the vector  $C$  gives the cost to the owner of each portion (in euros).

#### Task:

Write down an expression using the vectors  $A$ ,  $B$ , and  $C$  that gives the sausage stand owner's total profit on this day regarding the sale of the sausages.

Total profit = \_\_\_\_\_



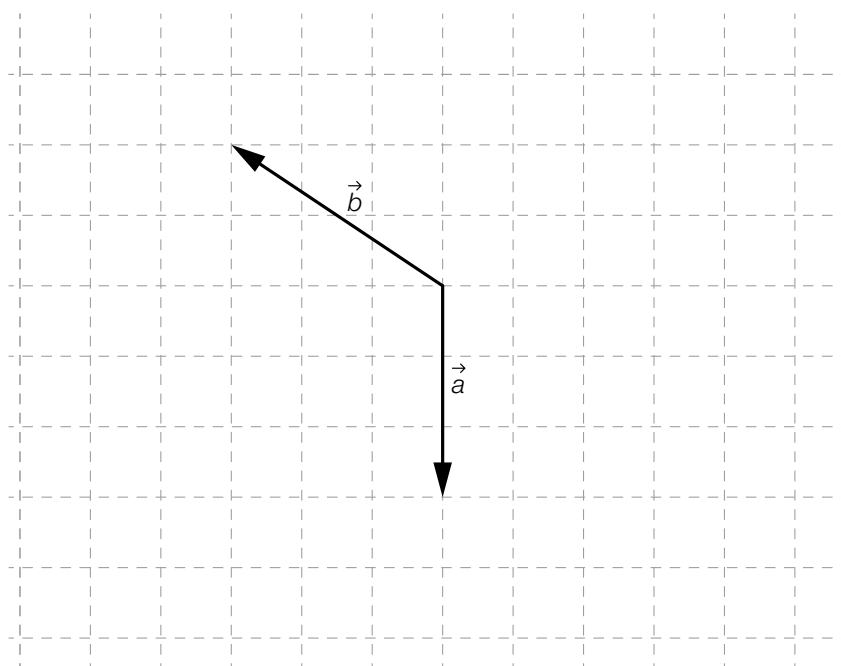
## Task 5

### Two-Dimensional Vectors

The diagram below shows two vectors,  $\vec{a}$  and  $\vec{b}$ .

Task:

In the diagram, draw a vector  $\vec{c}$  such that the sum of the three vectors gives the zero vector, i. e. so that  $\vec{a} + \vec{b} + \vec{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ .



## Task 6

### Rate of Descent

A light aircraft is approaching to land at an angle of  $\alpha$  (in degrees) to the horizontal. The aircraft has an air speed of  $v$  (in m/s).

Task:

Write down a formula for the loss of altitude,  $x$  (in m), that this aircraft experiences in one second at this angle and air speed.

# Task 7

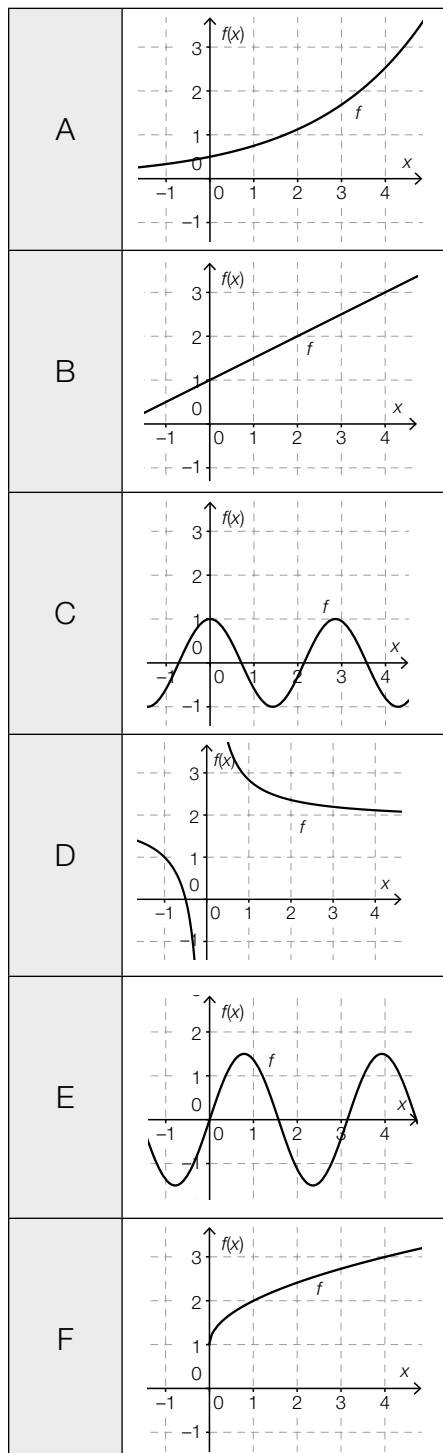
## Types of Functions

Below you will see four equations of functions (where  $a, b \in \mathbb{R}^+$ ) and the graphs of six real functions.

Task:

Match each of the four equations of functions with their corresponding graph (from A to F).

$f(x) = a \cdot \sin(b \cdot x)$	
$f(x) = a \cdot b^x$	
$f(x) = a \cdot \sqrt{x} + b$	
$f(x) = a \cdot x + b$	



## Task 8

### Value of an Asset

The value of a particular asset  $t$  years after its acquisition is represented by  $W(t)$  and can be calculated using the equation  $W(t) = -k \cdot t + d$  ( $k, d \in \mathbb{R}^+$ ) ( $W(t)$  is given in euros).

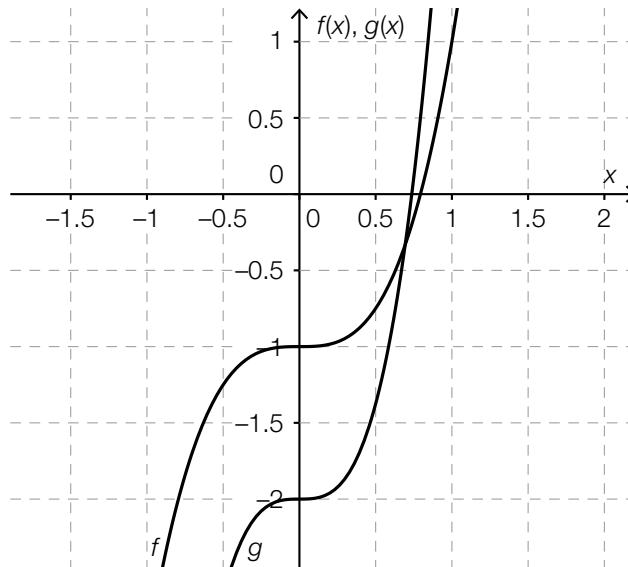
#### Task:

Write down the meaning of the parameters  $k$  and  $d$  with respect to the value of the asset.

# Task 9

## Parameters of a Real Function

The diagram below shows the graphs of two real functions  $f$  and  $g$  with equations  $f(x) = a \cdot x^3 + b$  and  $g(x) = c \cdot x^3 + d$  where  $a, b, c, d \in \mathbb{R}$ .



Task:

Which of the statements below are true for the parameters  $a, b, c,$  and  $d$ ?  
Put a cross next to each of the two correct statements.

$a > c$	<input type="checkbox"/>
$b > d$	<input type="checkbox"/>
$a > 0$	<input type="checkbox"/>
$b > 0$	<input type="checkbox"/>
$c < 1$	<input type="checkbox"/>

## Task 10

### Exponential Function

For an exponential function  $f$ , the following values are known:

$$f(0) = 12 \text{ and } f(4) = 192$$

**Task:**

Write down the equation of the exponential function  $f$ .

$$f(x) = \underline{\hspace{15em}}$$

# Task 11

## Thickness of a Lead Layer

The intensity of electromagnetic radiation decreases exponentially when it penetrates a body.

The half-value thickness of a material is the thickness at which the intensity of radiation is reduced by a half when the material is penetrated. The half-value thickness of lead for the radiation observed is 0.4 cm.

### Task:

Determine the necessary thickness,  $d$ , of a lead layer so that the intensity is reduced to 12.5 % of the original intensity.

$d =$  \_\_\_\_\_ cm

## Task 12

### Periodicity

Let  $f$  be a real function with equation  $f(x) = 3 \cdot \sin(b \cdot x)$  where  $b \in \mathbb{R}$ .

Task:

One of the values below gives the (smallest) period length of the function  $f$ . Put a cross next to the correct value.

$\frac{b}{2}$	<input type="checkbox"/>
$b$	<input type="checkbox"/>
$\frac{b}{3}$	<input type="checkbox"/>
$\frac{\pi}{b}$	<input type="checkbox"/>
$\frac{2\pi}{b}$	<input type="checkbox"/>
$\frac{\pi}{3}$	<input type="checkbox"/>



## Task 13

### Salary of an Employee

The gross salary of a particular employee was € 2,160 per month in the year 2008.

The employee's monthly gross salary increased over the following six years by an average of € 225 per year.

#### Task:

Determine the percentage change of the employee's monthly gross salary across the whole period from 2008 to 2014.

## Task 14

### Swimming Pool

Water is let into a swimming pool from the time  $t = 0$ .

The function  $h$  represents the height of the surface of the water at time  $t$ . The height,  $h(t)$ , is measured in dm and the time,  $t$ , is measured in hours.

**Task:**

Interpret the result of the following calculation in the given context.

$$\frac{h(5) - h(2)}{5 - 2} = 4$$

## Task 15

### Sine Function and Cosine Function

Let  $f$  and  $g$  be functions where  $f(x) = \sin(a \cdot x)$  and  $g(x) = a \cdot \cos(a \cdot x)$  with  $a \in \mathbb{R}$ .

Task:

Which relationship holds between the functions  $f$  and  $g$  and their first derivatives? Put a cross next to the equation that holds for all  $a \in \mathbb{R}$ .

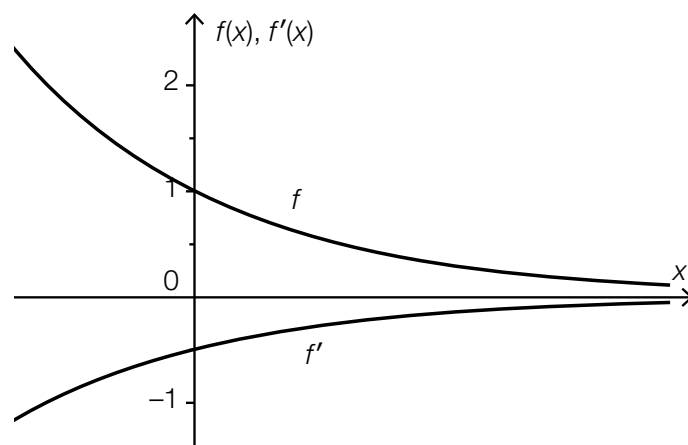
$a \cdot f'(x) = g(x)$	<input type="checkbox"/>
$g'(x) = f(x)$	<input type="checkbox"/>
$a \cdot g(x) = f'(x)$	<input type="checkbox"/>
$f(x) = a \cdot g'(x)$	<input type="checkbox"/>
$f'(x) = g(x)$	<input type="checkbox"/>
$g'(x) = a \cdot f(x)$	<input type="checkbox"/>

## Task 16

### Differentiating an Exponential Function

Let  $f$  be a function where  $f(x) = e^{\lambda \cdot x}$  with  $\lambda \in \mathbb{R}$ .

The diagram below shows the graphs of the function  $f$  and its first derivative  $f'$ .



Task:

Determine the value of the parameter  $\lambda$ .

$\lambda =$  \_\_\_\_\_

# Task 17

## Distance-Time Function

The linear motion of a car is represented by the distance-time function  $s$ . Within a given observation period, the function  $s$  is strictly monotonically increasing and concave down.

Task:

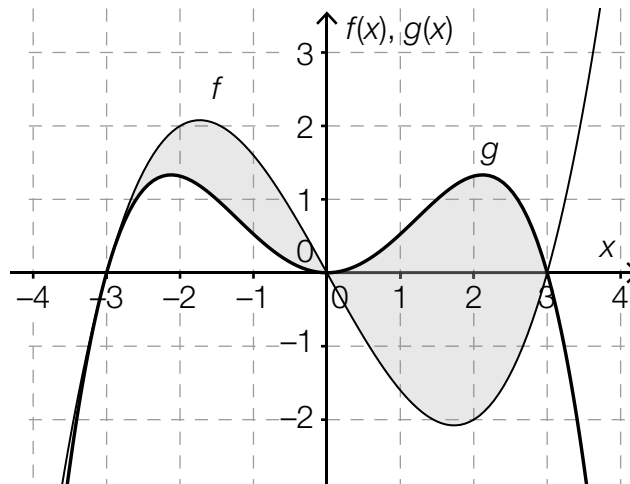
Put a cross next to each of the two true statements for this observation period.

The speed of the car is always increasing.	<input type="checkbox"/>
The values of the function $s'$ are negative.	<input type="checkbox"/>
The values of the function $s''$ are negative.	<input type="checkbox"/>
The value of the difference quotient of $s$ in the given time period is negative.	<input type="checkbox"/>
The value of the differential quotient of $s$ is always decreasing.	<input type="checkbox"/>

# Task 18

## Area Calculation

The graphs of the polynomial functions  $f$  and  $g$  are shown in the diagram below. The graphs cross when  $x = -3$ ,  $x = 0$  and  $x = 3$  and bound the two areas shaded in grey.



Task:

Which of the equations shown below give the area,  $A$ , of the (whole) area shaded in grey? Put a cross next to each of the two correct equations.

$A = \left  \int_{-3}^3 (f(x) - g(x)) dx \right $	<input type="checkbox"/>
$A = 2 \cdot \int_0^3 (g(x) - f(x)) dx$	<input type="checkbox"/>
$A = \int_{-3}^0 (f(x) - g(x)) dx + \int_0^3 (g(x) - f(x)) dx$	<input type="checkbox"/>
$A = \left  \int_{-3}^0 (f(x) - g(x)) dx \right  + \int_0^3 (f(x) - g(x)) dx$	<input type="checkbox"/>
$A = \int_{-3}^0 (f(x) - g(x)) dx + \left  \int_0^3 (f(x) - g(x)) dx \right $	<input type="checkbox"/>

# Task 19

## Stem and Leaf Diagram

The stem and leaf diagrams shown below give the number of cinema goers per showing of the films *A* and *B* over the course of a week. In the diagrams, the unit of the stem is 10 and the unit of the leaf is 1.

Film A	
2	0, 3, 8
3	6, 7
4	1, 1, 5, 6
5	2, 6, 8, 9
6	1, 8

Film B	
2	1
3	1, 4, 5
4	4, 5, 8
5	0, 5, 7, 7
6	1, 2
7	0

### Task:

Put a cross next to each of the statements that are definitely true based on the information shown in the stem and leaf diagrams.

In this week there were more showings of film <i>A</i> than of film <i>B</i> .	<input type="checkbox"/>
The median number of viewers for film <i>A</i> is higher than for film <i>B</i> .	<input type="checkbox"/>
The range of the number of viewers is smaller for film <i>A</i> than for film <i>B</i> .	<input type="checkbox"/>
The total number of viewers for film <i>A</i> was greater than for film <i>B</i> in this week.	<input type="checkbox"/>
In one of the showings for film <i>B</i> , there were more viewers than in any of the individual showings of film <i>A</i> .	<input type="checkbox"/>

## Task 20

### Estimate of a Probability

In a factory, a product is produced by a machine with 100 items being placed in one batch.

Following a recalibration of the machine, three batches of products are produced. These batches are checked to determine the number of rejects they contain. The results of this check are displayed in the table below.

in the first batch	6 rejects
in the second batch	3 rejects
in the third batch	4 rejects

The factory's management requires an estimate of the probability,  $p$ , that a new product produced by the machine is rejected based on the data given above.

#### Task:

Determine an estimate for the probability  $p$  that is as reliable as possible that a new product produced by the machine is rejected.

$p =$  \_\_\_\_\_



## Task 21

### Ludo

In order to place a playing piece on the board at the beginning of a game of *Ludo*, a player has to roll a six using a fair dice (a dice is considered to be “fair” if the probability of the dice showing any of its six faces after being thrown is equal for all six faces).

According to the rules of the game, the number of attempts to roll a six is limited to three attempts before the next player takes their turn.

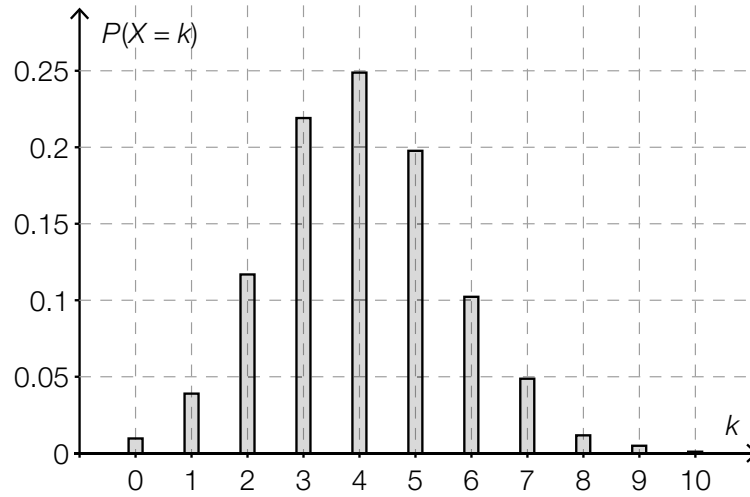
#### Task:

Determine the probability that a playing piece can be placed on the board after a maximum of three attempts to roll a six.

## Task 22

### Determining a Probability

The diagram below shows the probability distribution of a random variable  $X$ .



Task:

Using the diagram, determine an approximate value for the probability  $P(4 \leq X < 7)$ .

$P(4 \leq X < 7) \approx$  \_\_\_\_\_

## Task 23

### Tyres

The probability that a new tyre produced by a particular brand becomes damaged in the first 10,000 km of driving due to a fault in the material is  $p$  %.

A random sample of 80 new tyres from this brand is tested.

#### Task:

Write down an expression that could be used to calculate the probability that at least one of these tyres becomes damaged in the first 10,000 km of driving due to a fault in the material.

## Task 24

### Confidence Interval

For an election forecast, a random sample is selected from all those eligible to vote. Out of 400 people, 80 say they intend to vote for party Y.

#### Task:

Determine a symmetrical 95 % confidence interval for the proportion of voters for party Y out of the whole population of voters.