Name:	
Class:	

Standardised Competence-Oriented
Written School-Leaving Examination

AHS

16th January 2018

Mathematics

Part 2 Tasks

BMB
Bundesministerium
für Bildung

Advice for Completing the Tasks

Dear candidate,

The following booklet for Part 2 contains four tasks, each of which contains between two and four sub-tasks. All sub-tasks can be completed independently of one another. You have *150 minutes* available in which to work on these tasks.

Please use a blue or black pen that cannot be rubbed out. You may use a pencil for tasks that require you to draw a graph, vectors or a geometric construction.

When completing these tasks please use this booklet and the paper provided. Write your name on each piece of paper you use as well as on the first page of this task booklet in the space provided. Please show clearly which sub-task each answer relates to.

In the assessment of your work, everything that is not crossed out will be considered. Your solutions must be clearly marked. If a solution is not clearly marked or if more than one solution is given, the task will be considered to be unsolved. Draw a line through any notes you make.

You may use a pre-approved formula book as well as your usual electronic device(s).

Please hand in both the task booklet and the separate sheets you have used at the end of the examination.

Assessment

Every task in Part 1 will be awarded either 0 points or 1 point. Every sub-task in Part 2 will be awarded 0, 1 or 2 points. The tasks marked with an [A] will be awarded either 0 points or 1 point.

- If at least 16 of the 24 tasks in Part 1 are solved correctly, you will pass the examination.
- If fewer than 16 of the 24 tasks in Part 1 are solved correctly, then the tasks marked with an [A] from Part 2 may compensate for the shortfall (as part of the "range of essential skills" outlined by the LVBO).
  If, including the tasks marked with an [A] from Part 2, at least 16 tasks are solved correctly, you will pass the examination.
  If, including the tasks marked with an [A] from Part 2, fewer than 16 tasks are solved correctly, you will not be awarded enough points to pass the examination.
- If at least 16 tasks are solved correctly (including the compensation tasks marked with an [A] from Part 2), a grade will be awarded as follows:

Pass	16–23 points
Satisfactory	24–32 points
Good	33–40 points
Very Good	41–48 points

Explanation of the Task Types

Some tasks require a *free answer*. For these tasks, you should write your answer directly underneath each task in the task booklet or on the paper provided. Other task types used in the examination are as follows:

Matching tasks: For this task type you will be given a number of statements, tables or diagrams, which will appear alongside a selection of possible answers. To correctly answer these tasks, you will need to match each statement, table or diagram to its corresponding answer. You should write the letter of the correct answer next to the statement, table or diagram in the space provided.

Example:
You are given two equations.

Task:
Match the two equations to their corresponding description (from A to D).

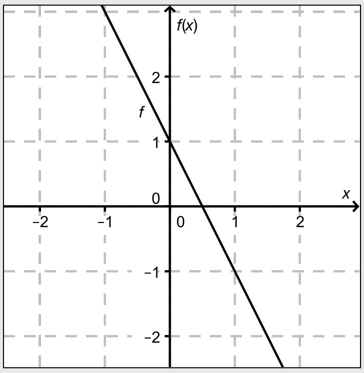
$1 + 1 = 2$	A
$2 \cdot 2 = 4$	C

A	Addition
B	Division
C	Multiplication
D	Subtraction

Construction tasks: This task type requires you to draw points, lines and/or curves in the task booklet.

Example:
Below you will see a linear function *f* where $f(x) = k \cdot x + d$.

Task:
On the axes provided below, draw the graph of a linear function for which $k = -2$ and $d > 0$.



Multiple-choice tasks of the form "1 out of 6": This task type consists of a question and six possible answers. Only one answer should be selected. You should put a cross next to the only correct answer in the space provided.

Example:
Which equation is correct?

Task:
Put a cross next to the correct equation.

1 + 1 = 1	☐
2 + 2 = 2	☐
3 + 3 = 3	☐
4 + 4 = 8	☒
5 + 5 = 5	☐
6 + 6 = 6	☐

Multiple-choice tasks of the form "2 out of 5": This task type consists of a question and five possible answers, of which two answers should be selected. You should put a cross next to each of the two correct answers in the space provided.

Example:
Which equations are correct?

Task:
Put a cross next to each of the two correct equations.

1 + 1 = 1	☐
2 + 2 = 4	☒
3 + 3 = 3	☐
4 + 4 = 8	☒
5 + 5 = 5	☐

Multiple-choice tasks of the form "x out of 5": This task type consists of a question and five possible answers, of which one, two, three, four *or* five answers may be selected. The task will require you to: "Put a cross next to each correct statement/equation …". You should put a cross next to each correct answer in the space provided.

Example:
Which of the equations given are correct?

Task:
Put a cross next to each correct equation.

1 + 1 = 2	☒
2 + 2 = 4	☒
3 + 3 = 6	☒
4 + 4 = 4	☐
5 + 5 = 10	☒

Gap-fill: This task type consists of a sentence with two gaps, i.e. two sections of the sentence are missing and must be completed. For each gap you will be given the choice of three possible answers. You should put a cross next to **each of the two answers** that are necessary to complete the sentence correctly.

Example:
Below you will see 3 equations.

Task:
Complete the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

The operation in equation ___①___ is known as summation or ___②___.

①	
1 – 1 = 0	☐
1 + 1 = 2	☒
1 · 1 = 1	☐

②	
Multiplication	☐
Subtraction	☐
Addition	☒

Changing an answer for a task that requires a cross:

1. Fill in the box that contains the cross for your original answer.
2. Put a cross in the box next to your new answer.

1 + 1 = 3	☐
2 + 2 = 4	☒
3 + 3 = 5	☐
4 + 4 = 4	☐
5 + 5 = 9	■

In this instance, the answer "5 + 5 = 9" was originally chosen. The answer was later changed to be "2 + 2 = 4".

Selecting an answer that has been filled in:

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

1 + 1 = 3	☐
2 + 2 = 4	(■)
3 + 3 = 5	☐
4 + 4 = 4	■
5 + 5 = 9	☐

In this instance, the answer "2 + 2 = 4" was filled in and then selected again.

If you still have any questions, please ask your teacher.

Good Luck!

Task 1

Function

Let f be a quadratic function where $f(x) = a \cdot x^2 + b \cdot x + c$ with coefficients $a, b, c \in \mathbb{R}$.

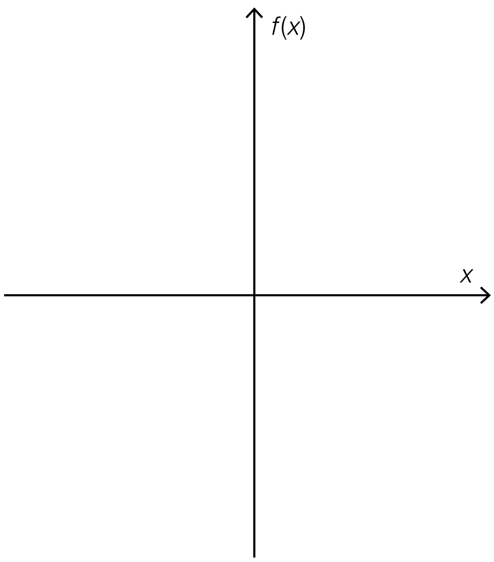
Task:

a) Determine the coordinates of the point P on the graph of one such function f at which the gradient of the tangent to the graph of the function f has the value b and, furthermore, write down a general equation of this tangent t.

The graph of one such function f goes through the point $A = (–1,20)$ and has a tangent t where $t(x) = 9 \cdot x + 4$ at point P. For this function f, write down the values of a, b and c.

b) Write down an expression that gives a in terms of b and c such that the function f has exactly one zero.

In the coordinate system provided below, sketch a possible graph of one such function f with exactly one zero and $a > 0$, $b > 0$, $c > 0$.



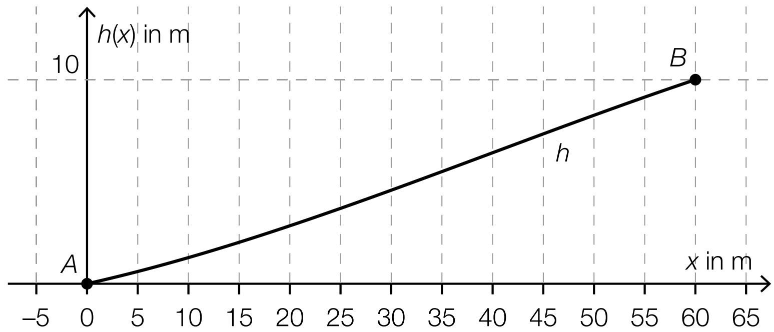
c) [A] For $a = 16$ and $c = 9$, write down both the position of the local maximum or minimum of the function f as well as the corresponding value of the function at this point in terms of b.

Show that this maximum or minimum lies on the graph of the function g where $g(x) = 9 – 16 \cdot x^2$ regardless of the value chosen for b.

Task 2

Road with an Incline

A car travels on a straight section of road that has an incline. Over a certain time period, the car covers the distance between points *A* and *B*. The height of the section of road between *A* and *B* above the height of point *A* can be modelled by a polynomial function *h* in terms of *x*. Here, *x* corresponds to the horizontal distance of the car (which is here modelled as a point) from the point *A* and $h(x)$ gives the height of the position of the car above the height of point *A* ($h(x)$ in m, *x* in m). In this model, the points *A* and *B* have coordinates $A = (0,0)$ and $B = (60,10)$.



An equation of the function *h* is:

$$h(x) = \frac{1}{64\,800} \cdot (-x^3 + 120 \cdot x^2 + 7\,200 \cdot x) \quad \text{for } x \in [0, 60]$$

Task:

a) Write down the value of the difference quotient of the function *h* in the interval [0, 60] and interpret this value in the given context.

A person claims: “If any section of road that has an incline can be modelled by a third degree polynomial function whose point of inflexion lies within this section, then the point of inflexion represents the point at which the incline of the road is the steepest.”

Write down whether this claim is definitely true and justify your decision.

b) There are plans to rebuild the road in such a way that the section between *A* and *B* has a constant gradient.

[A] Determine the equation of the function h_1 that gives the shape of the new road between *A* and *B*. For this function, $h_1(x)$ should give the height (in m) of the position of the car above the height of point *A*.

Determine the size of the angle α that gives the incline of the new road (as measured from the horizontal).

c) When driving in the mountains, there is an uncomfortable pressure on the eardrum that many people describe as an "attack" on the ears. This pressure occurs to a person within a car if the instantaneous rate of change of the height exceeds a value of 4 m/s.

The function g where $g(t) = \frac{1}{5} \cdot t^2 + t$ models the position of the car above the height of A during the journey from $A = (0,0)$ to $B = (60,10)$ as a function of time. The function $g(t)$ gives the height of the car at time t ($g(t)$ in metres, t in seconds measured from the time at which the car is at point A).

Determine how many seconds the car takes to travel from A to B.

Write down whether the instantaneous rate of change of the height during this time period exceeds a value of 4 m/s and justify your answer.

Task 3

Human Development Index

The Human Development Index (*HDI*) is a welfare indicator of countries calculated by the United Nations that should provide a measurement of the development level of a particular country. The *HDI* uses three dimensionless quantities (Life Expectancy Index (*LEI*), Education Index (*EI*) and Income Index (*II*)) and is calculated using the formula $HDI = \sqrt[3]{LEI \cdot EI \cdot II}$. A quantity is dimensionless if it has no units.

Since 2010, the *LEI* and *II* indices have been calculated as follows:

$LEI = \frac{LE - 20}{85 - 20}$ where *LE* is the life expectancy at the time of birth in years

$II = \frac{\ln(B) - \ln(100)}{\ln(75\,000) - \ln(100)}$ where *B* is the gross national income per head in US dollars (always at the beginning of the year)

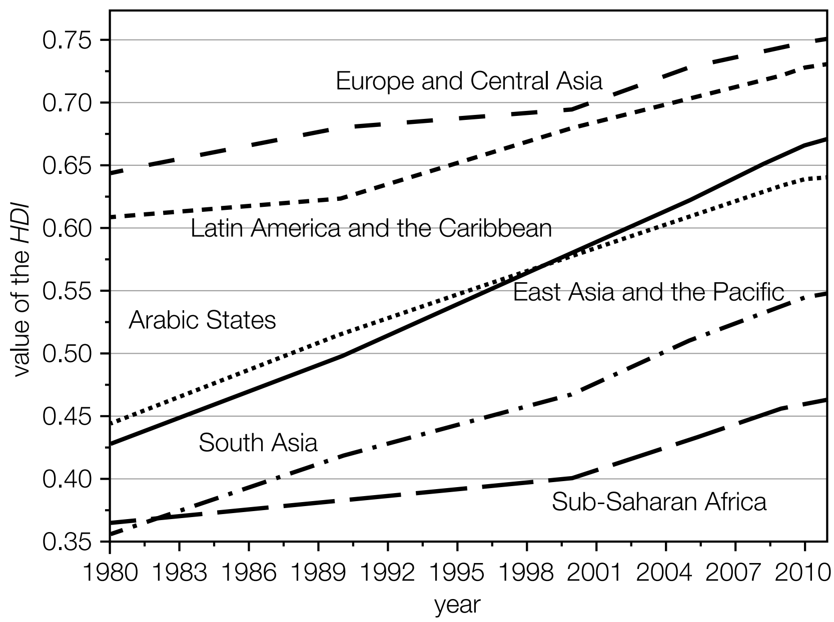
Since 2009, the development programme of the United Nations has organised countries into four development categories according to the value of the *HDI*:

Development category of a country	Value of the *HDI*
E_1	≥ 0.8
E_2	[0.7, 0.8)
E_3	[0.55, 0.7)
E_4	< 0.55

Data source: Deutsche Gesellschaft für die Vereinten Nationen (ed.): *Bericht über die menschliche Entwicklung 2015. Arbeit und menschliche Entwicklung.* Berlin: Berliner Wissenschafts-Verlag 2015, p. 240.

The *HDI* of a region in a particular year is calculated by taking the mean of the *HDIs* of all countries in that region.

The diagram below shows the development of the *HDI* of different regions between 1980 and 2011.

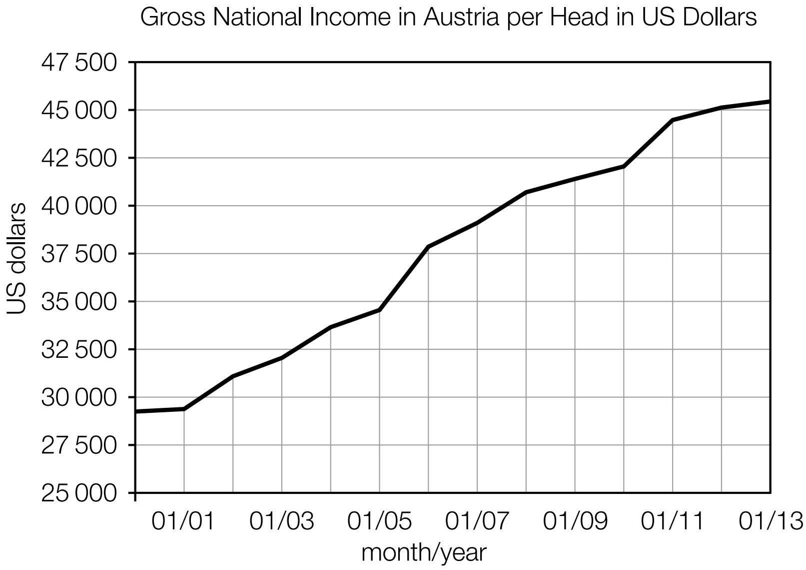


Data source: https://de.wikipedia.org/wiki/Index_der_menschlichen_Entwicklung#/media/File:Human-Development-Index-Trends-2011.svg [08.06.2017].

Task:

a) For Austria, the *Human Development Report* in 2013 found the life expectancy to be $LE = 81.1$ years and the education index to be $EI = 0.819$.

The diagram below shows the gross national income per head for Austria in US dollars from 2000 to 2013 (measurements taken at the beginning of the year).



Data source: http://www.factfish.com/de/statistik/bruttonationaleinkommen [08.06.2017].

For the year 2013, determine the *HDI* for Austria (= HDI_{2013}).

The *HDI* for Austria in the year 2013 (HDI_{2013}) was around 2.5 % greater than the *HDI* for Austria in the year 2008 (HDI_{2008}). Write down an equation that describes this relationship and calculate the HDI_{2008}.

b) The annual trend of the *HDI* for the region of the "Arabic States" from 1980 to 2010 can be approximated by a linear function H with equation $H(t) = k \cdot t + d$ where $k, d \in \mathbb{R}$ and t measured in years. For this function H, $H(0)$ gives the value in the year 1980.

Determine the value of the parameters k and d.

Using the relevant diagram, justify in which region(s) the average annual increase of the *HDI* in the time period from 1980 to 2010 corresponds most closely to the increase seen in the region of the "Arabic States".

c) [A] From the relevant diagram, determine the year at which the region "Latin America and the Caribbean" moved into the development category E_2.

Is it certain that from this point in time around half of the countries in this region were in the category E_2? Justify your answer.

Task 4

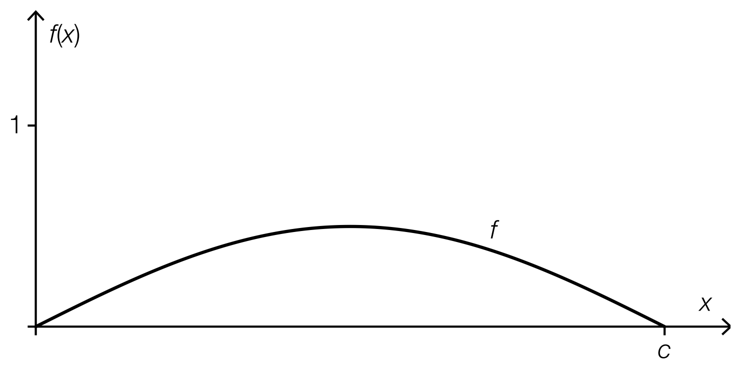
Density Function and Distribution Function

Let X be a random variable for which the probability that X lies in an interval I can be determined using a so-called density function f as follows:

$P(a \leq X \leq b) = \int_a^b f(x)\,\mathrm{d}x$ for all $a, b \in I$ where $a \leq b$

The corresponding distribution function F is given by $F(x) = P(X \leq x)$ for all $x \in \mathbb{R}$.
Therefore $F(b) - F(a) = P(a \leq X \leq b)$ for $a, b \in I$ and $a \leq b$.

The diagram below shows the graph of a density function f with $f(x) = k \cdot \sin(x)$ for $x \in [0, c]$, where $k \in \mathbb{R}, k > 0$ and $f(c) = 0$ holds. For $x \notin [0, c]$ then $f(x) = 0$.



Task:

a) For the given density function f, write down the value of the function $F(0)$ of the corresponding distribution function F and justify why $F(c) = 1$.

$F(0) =$ ____________________

On the diagram above, sketch the graph of the corresponding distribution function F and describe the concavity of F in the interval $[0, c]$.

b) Write down which property of f determines the value of the parameter k and determine the value of k.

Write down an expression of the corresponding distribution function of F in the interval $[0, c]$.

$F(x) =$ ____________________

c) For an event E, $P(E) = 1 - P(X \leq c - a)$ holds for any $a \in [0, c]$.

A Describe this event E in words.

For $a \leq \frac{c}{2}$, draw the probability $P(a \leq X \leq c - a)$ in the diagram below as an area and justify the relationship $P(a \leq X \leq c - a) = 1 - 2 \cdot P(X \leq a)$ using this representation.