| Name: | |
|---|---|
| Class: | |

Standardised Competence-Oriented
Written School-Leaving Examination

AHS

16th January 2018

# Mathematics

## Part 1 Tasks

BMB
Bundesministerium
für Bildung

# Advice for Completing the Tasks

Dear candidate,

The following booklet for Part 1 contains 24 tasks. The tasks can be completed independently of one another. You have *120 minutes* available in which to work through this booklet.

Please use a blue or black pen that cannot be rubbed out. You may use a pencil for tasks that require you to draw a graph, vectors or a geometric construction.

Please do all of your working out solely in this booklet. Write your name on the first page of the booklet in the space provided.

All answers must be written in this booklet. In the assessment of your work, everything that is not crossed out will be considered. Your solutions must be clearly marked. If a solution is not clearly marked or if more than one solution is given, the task will be considered to be unsolved. Draw a line through any notes you make.

You may use a pre-approved formula book as well as your usual electronic device(s).

Please hand in the task booklet at the end of the examination.

## Assessment

Every task in Part 1 will be awarded either 0 points or 1 point. Every sub-task in Part 2 will be awarded 0, 1 or 2 points. The tasks marked with an A will be awarded either 0 points or 1 point.

– If at least 16 of the 24 tasks in Part 1 are solved correctly, you will pass the examination.
– If fewer than 16 of the 24 tasks in Part 1 are solved correctly, then the tasks marked with an A from Part 2 may compensate for the shortfall (as part of the "range of essential skills" outlined by the LVBO).
  If, including the tasks marked with an A from Part 2, at least 16 tasks are solved correctly, you will pass the examination.
  If, including the tasks marked with an A from Part 2, fewer than 16 tasks are solved correctly, you will not be awarded enough points to pass the examination.
– If at least 16 tasks are solved correctly (including the compensation tasks marked with an A from Part 2), a grade will be awarded as follows:

| | |
|---|---|
| Pass | 16–23 points |
| Satisfactory | 24–32 points |
| Good | 33–40 points |
| Very Good | 41–48 points |

## Explanation of the Task Types

Some tasks require a ***free answer***. For these tasks, you should write your answer directly underneath each task in the task booklet. Other task types used in the examination are as follows:

*Matching tasks:* For this task type you will be given a number of statements, tables or diagrams, which will appear alongside a selection of possible answers. To correctly answer these tasks, you will need to match each statement, table or diagram to its corresponding answer. You should write **the letter of the correct answer** next to the statement, table or diagram in the space provided.

Example:
You are given two equations.

Task:
Match the two equations to their corresponding description (from A to D).

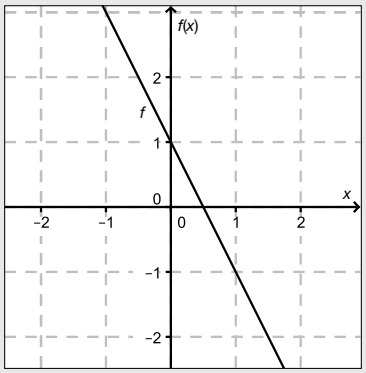
| | |
|---|---|
| $1 + 1 = 2$ | $A$ |
| $2 \cdot 2 = 4$ | $C$ |

| | |
|---|---|
| A | Addition |
| B | Division |
| C | Multiplication |
| D | Subtraction |

*Construction tasks:* This task type requires you to draw points, lines and/or curves in the task booklet.

Example:
Below you will see a linear function $f$ where $f(x) = k \cdot x + d$.

Task:
On the axes provided below, draw the graph of a linear function for which $k = -2$ and $d > 0$.




*Multiple-choice tasks of the form "1 out of 6":* This task type consists of a question and six possible answers. Only **one answer** should be selected. You should put a cross next to the only correct answer in the space provided.

Example:
Which equation is correct?

Task:
Put a cross next to the correct equation.

| | |
|---|---|
| 1 + 1 = 1 | ☐ |
| 2 + 2 = 2 | ☐ |
| 3 + 3 = 3 | ☐ |
| 4 + 4 = 8 | ☒ |
| 5 + 5 = 5 | ☐ |
| 6 + 6 = 6 | ☐ |

*Multiple-choice tasks of the form "2 out of 5":* This task type consists of a question and five possible answers, of which **two answers** should be selected. You should put a cross next to each of the two correct answers in the space provided.

Example:
Which equations are correct?

Task:
Put a cross next to each of the two correct equations.

| | |
|---|---|
| 1 + 1 = 1 | ☐ |
| 2 + 2 = 4 | ☒ |
| 3 + 3 = 3 | ☐ |
| 4 + 4 = 8 | ☒ |
| 5 + 5 = 5 | ☐ |

*Multiple-choice tasks of the form "x out of 5":* This task type consists of a question and five possible answers, of which **one, two, three, four *or* five answers** may be selected. The task will require you to: "Put a cross next to each correct statement/equation …". You should put a cross next to each correct answer in the space provided.

Example:
Which of the equations given are correct?

Task:
Put a cross next to each correct equation.

| | |
|---|---|
| 1 + 1 = 2 | ☒ |
| 2 + 2 = 4 | ☒ |
| 3 + 3 = 6 | ☒ |
| 4 + 4 = 4 | ☐ |
| 5 + 5 = 10 | ☒ |

*Gap-fill:* This task type consists of a sentence with two gaps, i. e. two sections of the sentence are missing and must be completed. For each gap you will be given the choice of three possible answers. You should put a cross next to **each of the two answers** that are necessary to complete the sentence correctly.

Example:
Below you will see 3 equations.

Task:
Complete the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

The operation in equation _____ ① _____ is known as summation or _____ ② _____.

| ① | |
|---|---|
| 1 – 1 = 0 | ☐ |
| 1 + 1 = 2 | ☒ |
| 1 · 1 = 1 | ☐ |

| ② | |
|---|---|
| Multiplication | ☐ |
| Subtraction | ☐ |
| Addition | ☒ |

*Changing an answer for a task that requires a cross:*

1. Fill in the box that contains the cross for your original answer.
2. Put a cross in the box next to your new answer.

| | |
|---|---|
| 1 + 1 = 3 | ☐ |
| 2 + 2 = 4 | ☒ |
| 3 + 3 = 5 | ☐ |
| 4 + 4 = 4 | ☐ |
| 5 + 5 = 9 | ■ |

In this instance, the answer "5 + 5 = 9" was originally chosen. The answer was later changed to be "2 + 2 = 4".

*Selecting an answer that has been filled in:*

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

| | |
|---|---|
| 1 + 1 = 3 | ☐ |
| 2 + 2 = 4 | (■) |
| 3 + 3 = 5 | ☐ |
| 4 + 4 = 4 | ■ |
| 5 + 5 = 9 | ☐ |

In this instance, the answer "2 + 2 = 4" was filled in and then selected again.

If you still have any questions now, please ask your teacher.

**Good Luck!**

# Task 1

## Number of People on a Bus

The variable $F$ represents the number of female passengers on a bus. The variable $M$ represents the number of male passengers on the same bus. Including the (male) driver, there are twice as many men as women on the bus. (The driver is not included in the passenger count.)

Task:

Put a cross next to the equation that correctly describes the relationship between the number of women and the number of men on the bus.

| | |
|---|---|
| $2 \cdot (M + 1) = F$ | ☐ |
| $M + 1 = 2 \cdot F$ | ☐ |
| $F = 2 \cdot M + 1$ | ☐ |
| $F + 1 = 2 \cdot M$ | ☐ |
| $M - 1 = 2 \cdot F$ | ☐ |
| $2 \cdot F = M$ | ☐ |

# Task 2

## Train Journeys

A goods train departs from Salzburg in the direction of Linz at 8.00 a. m. The train station at Linz is 124 km away from the train station at Salzburg. Half an hour after the goods train has departed, an express train leaves Linz and travels towards Salzburg. The goods train travels at an average speed of 100 km/h, and the average speed of the express train is 150 km/h.

Task:

The time it takes for the goods train to encounter the train from Linz is represented by $t$ and is measured in hours.
Write down an equation that can be used to calculate the time $t$ and calculate this time.

# Task 3

## Solution to a Quadratic Equation

An equation that can be brought into the form $a \cdot x^2 + b \cdot x + c = 0$ where $a, b, c \in \mathbb{R}$ and $a \neq 0$ is known as a quadratic equation in the variable $x$ with coefficients $a$, $b$, $c$.

Task:

Complete the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence is a correct statement.

A quadratic equation of the form $a \cdot x^2 + b \cdot x + c = 0$ where _____①_____ definitely has _____②_____.

| ① | |
|---|---|
| $a > 0$ and $c > 0$ | ☐ |
| $a > 0$ and $c < 0$ | ☐ |
| $a < 0$ and $c < 0$ | ☐ |

| ② | |
|---|---|
| two distinct real solutions | ☐ |
| exactly one real solution | ☐ |
| no real solutions | ☐ |

# Task 4

## Orthogonal Vectors

Below, you will see some vectors.

$\vec{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$\vec{b} = \begin{pmatrix} x \\ 0 \end{pmatrix}, x \in \mathbb{R}$

$\vec{c} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$\vec{d} = \vec{a} - \vec{b}$

Task:

Determine the value of $x$ such that the vectors $\vec{c}$ and $\vec{d}$ are perpendicular to each other.

# Task 5

## Gradient of a Gutter

A gutter has a particular length $l$ (in metres). So that water can run off well, the gutter has to be positioned at an angle of at least $\alpha$ to the horizontal. Therefore, there is a difference in height of at least $h$ metres between the two end points of the gutter.

Task:

Write down a formula that can be used to calculate $h$ in terms of $l$ and $\alpha$.

$h =$ _____

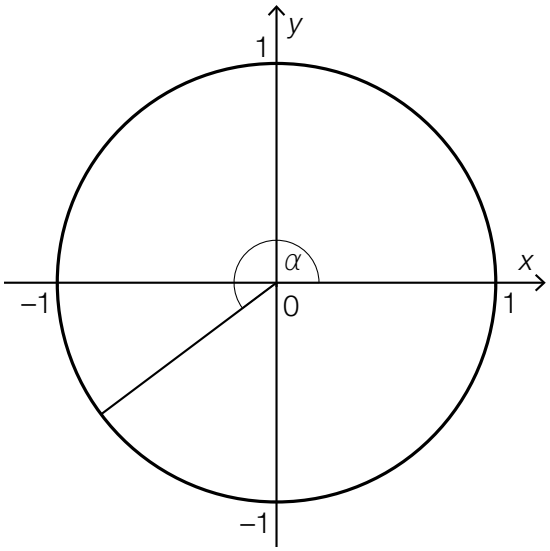
# Task 6

## Angles in the Unit Circle

The diagram below shows an angle $\alpha$ in the unit circle.

Task:

In the diagram, sketch the angle $\beta$ in the interval [0°, 360°] where $\beta \neq \alpha$ for which $\cos(\beta) = \cos(\alpha)$ holds.



# Task 7

## Stefan-Boltzmann Law

The luminosity $L$ of a star is given by the following formula:

$L = 4 \cdot \pi \cdot R^2 \cdot T^4 \cdot \sigma$

In the formula, $R$ is the radius of the star and $T$ is the surface temperature of the star; $\sigma$ is a constant (the so-called Stefan-Boltzmann constant).

Task:

Complete the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence is a correct statement.

For different stars with the same, known radius $R$, the luminosity $L$ is a function of _____①_____, and the function becomes _____②_____.

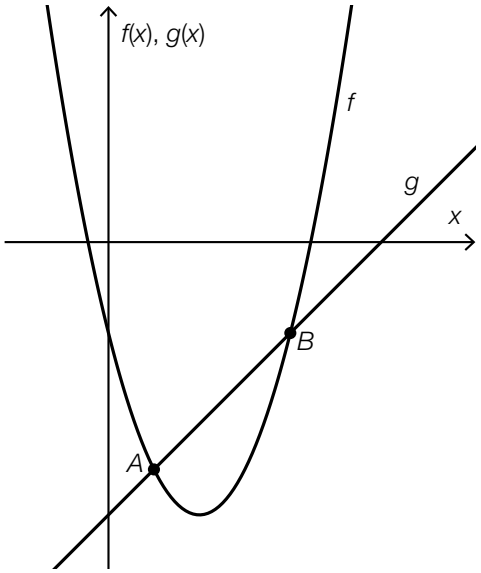
| ① | |
|---|---|
| the radius of the star, $R$ | ☐ |
| the surface temperature, $T$ | ☐ |
| the constant, $\sigma$ | ☐ |

| ② | |
|---|---|
| a linear function | ☐ |
| a power function | ☐ |
| an exponential function | ☐ |

# Task 8

## Points of Intersection

The diagram below shows the graph of the function $f$ where $f(x) = x^2 - 4 \cdot x - 2$ and the graph of the function $g$ where $g(x) = x - 6$ as well as their points of intersection $A$ and $B$.




Task:

Determine the coefficients $a$ and $b$ of the quadratic equation $x^2 + a \cdot x + b = 0$ such that the two solutions to this equation are the $x$-coordinates of the points of intersection $A$ and $B$.

# Task 9

## Gradient of a Linear Function

The graph of a linear function $f$ goes through the points $A = (a\,,b)$ and $B = (5 \cdot a\,,-3 \cdot b)$ where $a, b \in \mathbb{R}\backslash\{0\}$.

Task:

Determine the gradient $m$ of the linear function $f$.

$m =$ _____

# Task 10

## Pattern of Change

The pattern of how a quantity $N$ changes over time $t$ is described by the equation $N(t) = 1.2 \cdot 0.98^t$.

Task:

Which of the patterns of change described below can be represented by the equation given above? Put a cross next to the correct pattern of change.

| | |
|---|---|
| Per unit of time, 0.02 % of the amount of a radioactive substance left on a particular day decays. | ☐ |
| Per unit of time, 0.02 $m^3$ of water flows into a reservoir. | ☐ |
| Per unit of time, 1.2 mg of the active ingredient in a medication is broken down. | ☐ |
| The number of inhabitants in a country increases by 1.2 % per unit of time. | ☐ |
| The value of a property increases by 2 % per unit of time. | ☐ |
| The temperature of a body reduces by 2 % per unit of time. | ☐ |

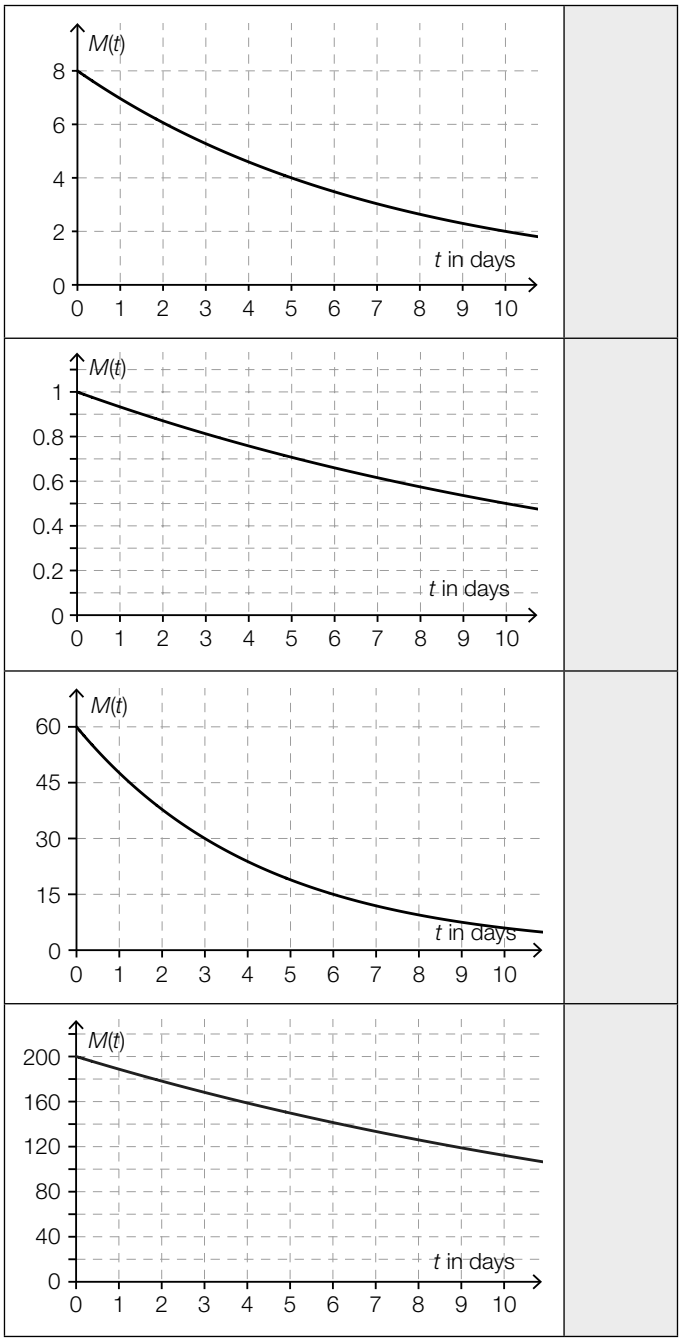
# Task 11

## Half-Life

The diagrams below show the graphs of exponential functions that each represents the amount remaining of a radioactive substance in terms of time.
The amount is given by $M(t)$ (in mg) at time $t$ (in days).

Task:

Match each of the four graphs with the corresponding half-life (from A to F).

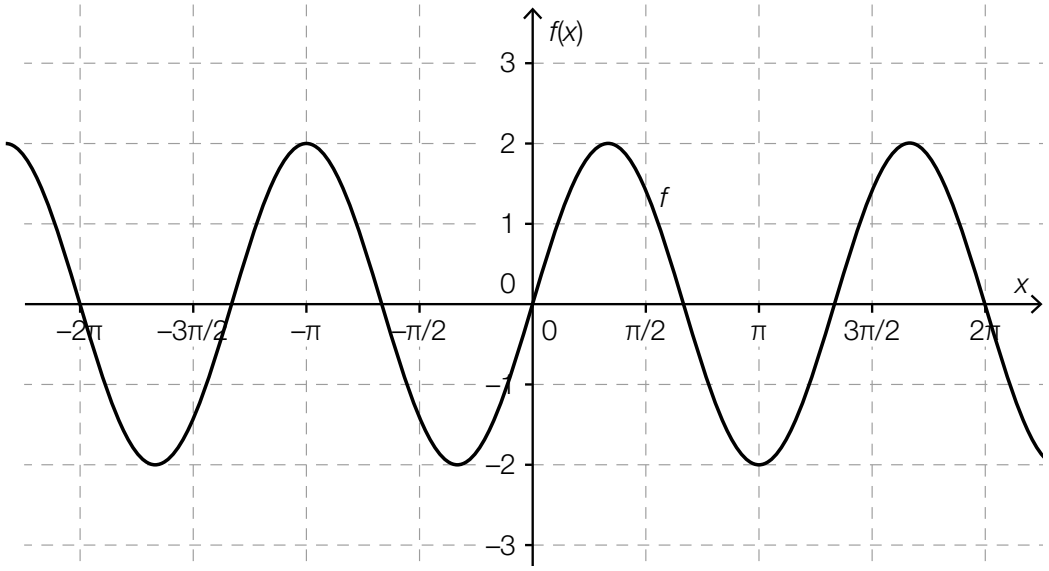



| | |
|---|---|
| A | 1 day |
| B | 2 days |
| C | 3 days |
| D | 5 days |
| E | 10 days |
| F | more than 10 days |

# Task 12

## Parameters of a Sine Function

The graph of a function $f$ where $f(x) = a \cdot \sin(b \cdot x)$ and $a, b \in \mathbb{R}^+$ is shown below.




Task:

For the graph shown above, write down appropriate values for the parameters $a$ and $b$.

$a$ = _____

$b$ = _____

# Task 13

## Radioactive Decay

The value $m(t)$ represents the amount of a radioactive substance that is left after $t$ days.

Task:

One of the expressions shown below describes the relative change in the amount of the radioactive substance over the first three days.
Put a cross next to the correct expression.

| | |
|---|---|
| $m(3) - m(0)$ | ☐ |
| $\frac{m(3) - m(0)}{3}$ | ☐ |
| $\frac{m(0)}{m(3)}$ | ☐ |
| $\frac{m(3) - m(0)}{m(0)}$ | ☐ |
| $\frac{m(3) - m(0)}{m(0) - m(3)}$ | ☐ |
| $m'(3)$ | ☐ |

# Task 14

## Differentiation

Below you will see six equations of functions, each with a parameter $k$, where $k \in \mathbb{Z}$ and $k \neq 0$.

Task:

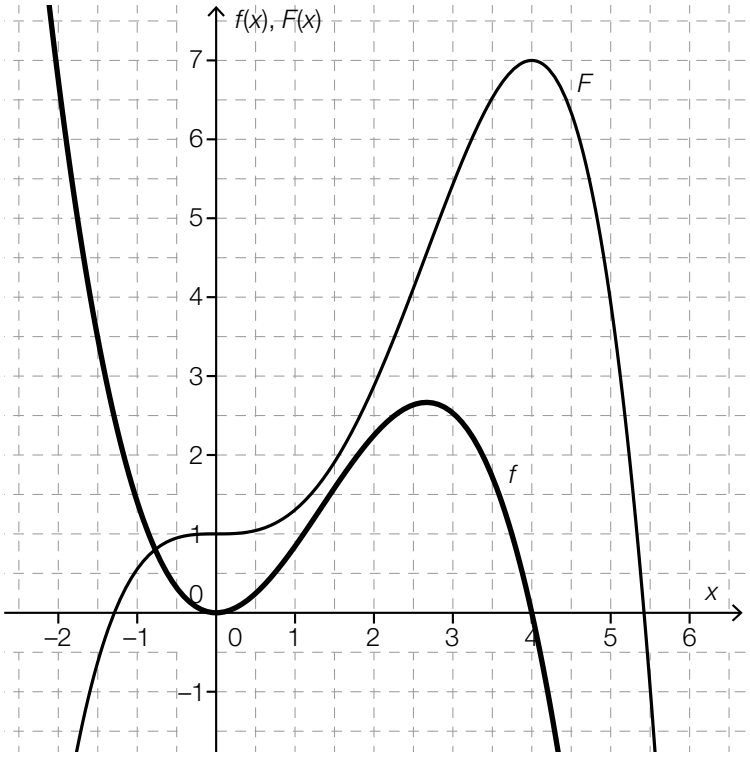
For which of the equations of functions shown below does the relationship $f'(x) = k \cdot f(x)$ hold for all $x \in \mathbb{R}$?
Put a cross next to the correct equation of a function.

| | |
|---|---|
| $f(x) = k$ | ☐ |
| $f(x) = \frac{k}{x}$ | ☐ |
| $f(x) = k \cdot x$ | ☐ |
| $f(x) = x^k$ | ☐ |
| $f(x) = e^{k \cdot x}$ | ☐ |
| $f(x) = \sin(k \cdot x)$ | ☐ |

# Task 15

## Area

The diagram below shows the graph of a third degree polynomial function $f$ and the graph of its antiderivative $F$.




Task:

The graph of $f$ and the positive $x$-axis enclose a finite area in the interval [0, 4]. Determine the area of this finite area.

# Task 16

## Point of Inflexion

A third degree polynomial function $f$ can be differentiated to give $f'$ where $f'(x) = 12 \cdot x^2 - 4 \cdot x - 8$.

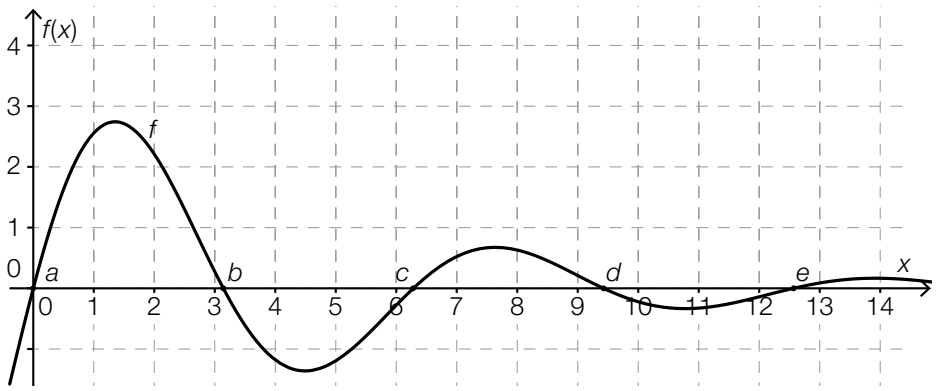
Task:

Write down whether the function $f$ has a point of inflexion at $x = 6$ and justify your answer.

# Task 17

## Definite Integral

The graph of a function $f$ crosses the $x$-axis in a given region at the points $a$, $b$, $c$, $d$ and $e$.




Task:

Which of the following definite integrals have a value that is greater than 0? Put a cross next to each of the two correct definite integrals.

| | |
|---|---|
| $\int_a^c f(x)\,dx$ | ☐ |
| $\int_b^c f(x)\,dx$ | ☐ |
| $\int_b^d f(x)\,dx$ | ☐ |
| $\int_a^b f(x)\,dx$ | ☐ |
| $\int_d^e f(x)\,dx$ | ☐ |

# Task 18

## Emissions

On a winter's day the emissions of a fireplace are measured.
The function $A\colon \mathbb{R}^+ \to \mathbb{R}^+$ represents the instantaneous amount of emissions $A(t)$ as a function of the time $t$ in which $A(t)$ is measured in grams per hour and $t$ in hours ($t = 0$ corresponds to midnight).

Task:

Interpret the expression $\int_7^{15} A(t)\,\mathrm{d}t$ in the given context.

# Task 19

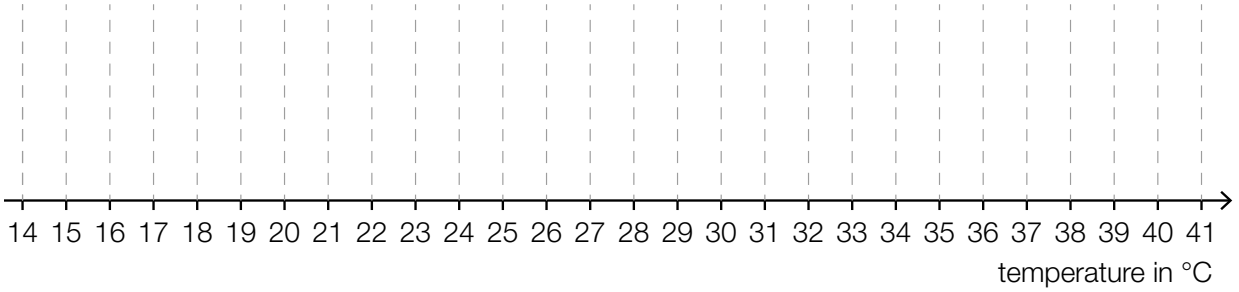
## Statistical Representations

The maximum temperature on each day during a very hot summer is measured by a meteorological station for the time period of one month. The readings in degrees Celsius can be read from the following stem and leaf diagram.

| | |
|---|---|
| 1 | 9 |
| 2 | 2 2 3 3 3 |
| 2 | 5 6 6 6 6 7 7 7 7 7 7 7 |
| 3 | 1 1 1 2 3 3 3 4 4 4 |
| 3 | 8 |
| 4 | 0 0 |

Task:

Draw a boxplot that represents the maximum daily temperatures listed above.



# Task 20

## Mean

There are 25 pupils in a class of which one pupil is an extraordinary pupil.

In a test, the mean score of all 25 pupils is 12.6 points. The mean score among the non-extraordinary pupils is 12.5 points.

Task:

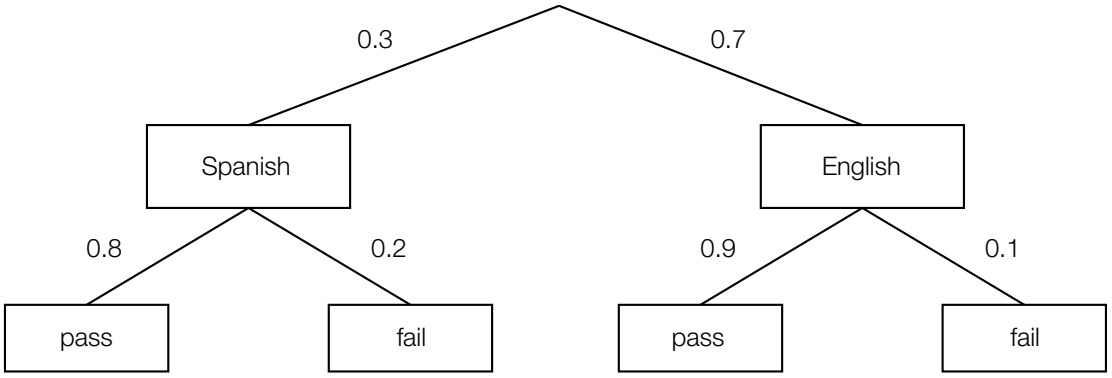
Determine how many points the extraordinary pupil scored on this test.

# Task 21

## Examination

In order to receive funding for a period abroad, students have to complete an examination in either Spanish or English.

The tree diagram shown below shows the proportions of students who have taken the examination in each language. The tree diagram also shows the proportions of students who pass and fail the examination.




Task:

The examination paper of one particular student is selected at random.

Interpret the expression $0.7 \cdot 0.9 + (1 - 0.7) \cdot 0.8$ in the given context.

# Task 22

## Probability

The random variable $X$ can take the values $\{0, 1, \ldots, 9, 10\}$.
The two probabilities $P(X = 0) = 0.35$ and $P(X = 1) = 0.38$ are known.

Task:

Determine the probability $P(X \geq 2)$.

$P(X \geq 2) =$ _____

# Task 23

## Rose Bushes

A particular percentage of bushes of a type of rose bloom with yellow flowers.
A number of rose bushes of this type are planted in a bed. The random variable $X$ is binomially distributed and gives the number of yellow-flowering rose bushes. The expectation value for the number $X$ of yellow-flowering rose bushes is 32 and the standard deviation has the value 4.

The following comparison is made:
“The probability that there are at least 28 and at most 36 yellow-flowering rose bushes in this bed is greater than the probability that there are more than 32 yellow-flowering rose bushes.”

Task:

Write down whether this comparison is true and justify your answer.

# Task 24

## Certainty of a Confidence Interval

The filling system of a company has to be checked at regular intervals and may need to be reconfigured.

After a filling system has been configured, 30 out of 1 000 packs that were checked had not been filled correctly. For an unknown relative proportion $p$ of packs that are not filled correctly, the company has determined the symmetrical confidence interval of [0.02, 0.04].

Task:

Using a normal approximation of the binomial distribution, determine the certainty of this confidence interval.