Name:	Date:
Class:	

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

January 2018

Mathematics

Supplementary Examination 1
Candidate's Version

BMB
Bundesministerium
für Bildung

Instructions for the supplementary examination

Dear candidate,

The following supplementary examination is comprised of five tasks that can be completed independently of one another.

Each task contains two parts: The statement of the task requires you to demonstrate core competencies, and the guiding question that follows it requires you to demonstrate your ability to communicate your ideas.

You will be given preparation time of at least 30 minutes, and the examination will last at the most 25 minutes.

Assessment

Each task can be awarded zero, one or two points. There is one point available for each demonstration of core competencies as well as for each guiding question. A maximum of 10 points can be achieved.

For the grading of the examination the following scale will be used:

Grade	Minimum number of points
Pass	4 points for the core competencies + 0 points for the guiding questions 3 points for the core competencies + 1 point for the guiding questions
Satisfactory	5 points for the core competencies + 0 points for the guiding questions 4 points for the core competencies + 1 point for the guiding questions 3 points for the core competencies + 2 points for the guiding questions
Good	5 points for the core competencies + 1 point for the guiding questions 4 points for the core competencies + 2 points for the guiding questions 3 points for the core competencies + 3 points for the guiding questions
Very good	5 points for the core competencies + 2 points for the guiding questions 4 points for the core competencies + 3 points for the guiding questions

The examination board will decide on the final grade based on your performance in the supplementary examination as well as the result of the written examination.

Good Luck!

Task 1

Lines

The vector equation of a line g as well as the equations of three further lines g_1, g_2, g_3 are given below.

g: $X = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + s \cdot \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ where $s \in \mathbb{R}$

g_1: $3 \cdot x + y = 9$

g_2: $y = -3 \cdot x + 10$

g_3: $x - 3 \cdot y = -7$

Task:

Determine which of the lines g_1, g_2, g_3 are perpendicular to the line g and justify your answer.

Guiding question:

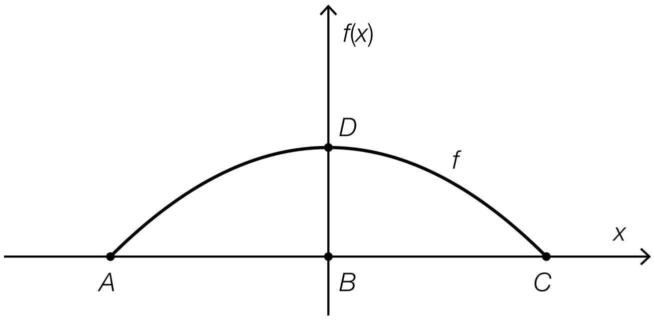
Determine which of the four lines are identical and justify your answer.

Determine how the values a_1 and b_2 (where $a_1, b_2 \in \mathbb{R}$) of the line h: $X = \begin{pmatrix} a_1 \\ 3 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ b_2 \end{pmatrix}$ where $t \in \mathbb{R}$ should be chosen so that g and h intersect at exactly one point.
Justify your answer.

Task 2

Arc of a Bridge

The arc of a bridge is shown in the diagram below. The line segment AC with midpoint B has a length of 40 metres. The maximum height of the arc of the bridge, BD, is 10 metres.



Task:

Find the equation of the function f, where $f(x) = a \cdot x^2 + b$ ($a, b \in \mathbb{R}$), that can be used to model the curve of the arc of the bridge. Explain your approach.

Guiding question:

In order for larger vehicles to also pass under the bridge, the height BD must be increased. Explain whether the parameters a and b of the function f, where $f(x) = a \cdot x^2 + b$ ($a, b \in \mathbb{R}$), should be changed to be greater than, less than or equal to their current values if the distance AC is to remain unchanged.

If the point A is taken to be at the origin, the function g, where $g(x) = c \cdot x^2 + d \cdot x + e$ ($c, d, e \in \mathbb{R}$), should be used to model the situation.

Using the symbols "<", ">" or "=", complete the statements below about c, d and e so that the statements are true for the function g.

c ___ 0; d ___ 0; e ___ 0

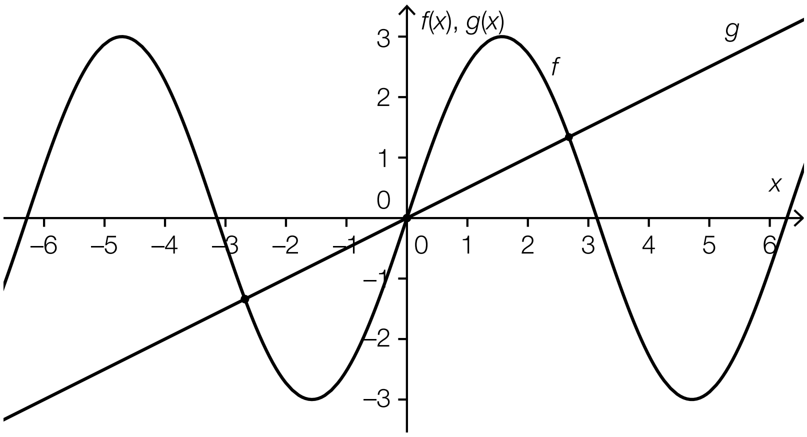
Task 3

Functions

The equations and graphs of the functions f and g are given below:

$f(x) = 3 \cdot \sin(x)$

$g(x) = \frac{x}{2}$



Task:

Determine the value of x_1 in the interval $[0, \pi]$ such that $f'(x_1) = g'(x_1)$ holds and explain how this point can be determined graphically.

Guiding question:

The equation $f(x) = g(x)$ has three solutions for x at a, 0 and c, where $a < 0 < c$.

On the diagram above, represent the value of the expression $\int_0^c (f(x) - g(x))\,dx$ graphically.

Determine the value of the expression $\int_a^c (f(x) - g(x))\,dx$.

Task 4

Reaction Times

A test subject determines their reaction time (in s) using an online test that they take ten times. The subject obtains the values given below:

0.38 s; 0.27 s; 0.30 s; 0.34 s; 0.25 s; 0.39 s; 0.28 s; 0.24 s; 0.33 s; 0.32 s

Task:

Determine the mean $\bar{t}$ and the standard deviation s of the ten results given above.

Determine what percentage of the reaction times given lie in the interval $[\bar{t} - s, \bar{t} + s]$.

Guiding question:

The test subject carries out the test two more times and obtains the results t_{11} and t_{12} where $t_{11} \neq t_{12}$. The mean that is calculated using all twelve times is referred to as $\bar{t}_{\text{new}}$, and the resulting standard deviation is referred to as s_{new}.

State which conditions the times t_{11} and t_{12} have to fulfil such that $\bar{t}_{\text{new}} = \bar{t}$ and $s_{\text{new}} < s$ hold.

Task 5

Raffle

Among 100 raffle tickets, there are 30 winning tickets. Of these 30 tickets, 25 result in winnings of € 10 each and five result in winnings of € 100 each.

Task:

Three tickets are selected at random from the 100 tickets.
Find the probability that no winning tickets are selected and explain your method.

Guiding question:

A person receives a randomly selected ticket from these 100 raffle tickets as a present. Determine the expectation value for this person's winnings.

Another person receives two randomly selected tickets from these 100 raffle tickets as a present. Find an expression that could be used to calculate the probability that this person wins € 110 and explain your approach.