Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

January 2018

Mathematics

Supplementary Examination 1
Examiner's Version

BMB
Bundesministerium
für Bildung

Instructions for the supplementary examination

The following supplementary examination is comprised of five tasks that can be completed independently of one another.

Each task contains two parts: The statement of the task requires the candidate to demonstrate core competencies, and the guiding question that follows it requires the candidate to show their ability to communicate their ideas.

In the following document, the examiner will find the tasks as well as the expected solutions and the answer key.

The preparation time is to be at least 30 minutes and the examination time is to be at most 25 minutes.

Assessment

Each task can be awarded zero, one or two points. There is one point available for each demonstration of core competencies as well as for each guiding question. A maximum of 10 points can be achieved.

For the grading of the examination the following scale should be used:

Grade	Minimum number of points
Pass	4 points for the core competencies + 0 points for the guiding questions 3 points for the core competencies + 1 point for the guiding questions
Satisfactory	5 points for the core competencies + 0 points for the guiding questions 4 points for the core competencies + 1 point for the guiding questions 3 points for the core competencies + 2 points for the guiding questions
Good	5 points for the core competencies + 1 point for the guiding questions 4 points for the core competencies + 2 points for the guiding questions 3 points for the core competencies + 3 points for the guiding questions
Very good	5 points for the core competencies + 2 points for the guiding questions 4 points for the core competencies + 3 points for the guiding questions

The examination board will decide on the final grade based on the candidate’s performance in the supplementary examination as well as the result of the written examination.

Evaluation grid for the supplementary examination

This evaluation grid may be used to assist the examiner in assessing the candidate’s performance.

	Point for core competencies reached	Point for the guiding question reached
Task 1		
Task 2		
Task 3		
Task 4		
Task 5		

Task 1

Lines

The vector equation of a line g as well as the equations of three further lines g_1, g_2, g_3 are given below.

g: $X = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + s \cdot \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ where $s \in \mathbb{R}$

g_1: $3 \cdot x + y = 9$

g_2: $y = -3 \cdot x + 10$

g_3: $x - 3 \cdot y = -7$

Task:

Determine which of the lines g_1, g_2, g_3 are perpendicular to the line g and justify your answer.

Guiding question:

Determine which of the four lines are identical and justify your answer.

Determine how the values a_1 and b_2 (where $a_1, b_2 \in \mathbb{R}$) of the line h: $X = \begin{pmatrix} a_1 \\ 3 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ b_2 \end{pmatrix}$ where $t \in \mathbb{R}$ should be chosen so that g and h intersect at exactly one point.
Justify your answer.

Solution of Task 1

Lines

Expected solution of the statement of the task:

The lines g_1 and g_2 are perpendicular to g.

Possible justification:
The direction vector of g, $\vec{v} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$, is also a normal vector of the lines g_1 and g_2.

Answer key:

The point for the core competencies is to be awarded if it has been stated that only g_1 and g_2 are perpendicular to g and a correct justification has been given.

Justifications involving the scalar product or using sketches are also acceptable.

Expected solution of the guiding question:

The lines g and g_3 are identical.

Possible justification:
Both lines have the direction vector $\vec{v} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and go through the point $P = (2,3)$.

For the lines g and h to have exactly one point of intersection, their gradients must be different; therefore, $b_2 \neq \frac{1}{3}$ must hold. As a_1 only determines the position of the point of intersection, not its existence, any real number can be chosen for a_1.

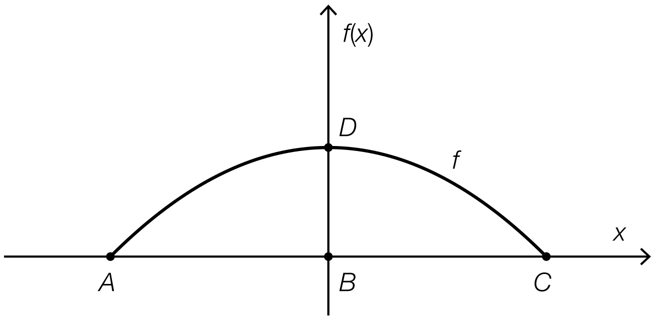
Answer key:

The point for the guiding question is to be awarded if it has been stated that g_3 and g are identical, and this has been justified correctly.
Furthermore, the conditions for the values of a_1 and b_2 must be given correctly with a correct justification.
If specific correct values are given for a_1 and b_2, then the point is to be awarded.

Task 2

Arc of a Bridge

The arc of a bridge is shown in the diagram below. The line segment AC with midpoint B has a length of 40 metres. The maximum height of the arc of the bridge, BD, is 10 metres.



Task:

Find the equation of the function f, where $f(x) = a \cdot x^2 + b$ ($a, b \in \mathbb{R}$), that can be used to model the curve of the arc of the bridge. Explain your approach.

Guiding question:

In order for larger vehicles to also pass under the bridge, the height BD must be increased. Explain whether the parameters a and b of the function f, where $f(x) = a \cdot x^2 + b$ ($a, b \in \mathbb{R}$), should be changed to be greater than, less than or equal to their current values if the distance AC is to remain unchanged.

If the point A is taken to be at the origin, the function g, where $g(x) = c \cdot x^2 + d \cdot x + e$ ($c, d, e \in \mathbb{R}$), should be used to model the situation.

Using the symbols "<", ">" or "=", complete the statements below about c, d and e so that the statements are true for the function g.

c ___ 0; d ___ 0; e ___ 0

Solution of Task 2

Arc of a Bridge

Expected solution of the statement of the task:

Possible approach:

$D = (0,10) \quad \Rightarrow \quad b = 10$

The root of f is at $x = 20$ (and -20).

$f(20) = 0 \quad \Rightarrow \quad 0 = 400 \cdot a + 10 \quad \Rightarrow \quad a = -\frac{1}{40} = -0.025$

$f(x) = -0.025 \cdot x^2 + 10$

Answer key:

The point for the core competencies is to be awarded if a correct equation of the function has been determined and a correct approach has been explained.

Expected solution of the guiding question:

If the height BD is to be increased, then the parameter b needs to be made bigger. So that the zeros remain unchanged, the parameter a needs to be made smaller.

$c < 0; \quad d > 0; \quad e = 0$

Answer key:

The point for the guiding question is to be awarded if the explanation of how both parameters should be changed is correct and the correct symbols have been given in the appropriate locations.

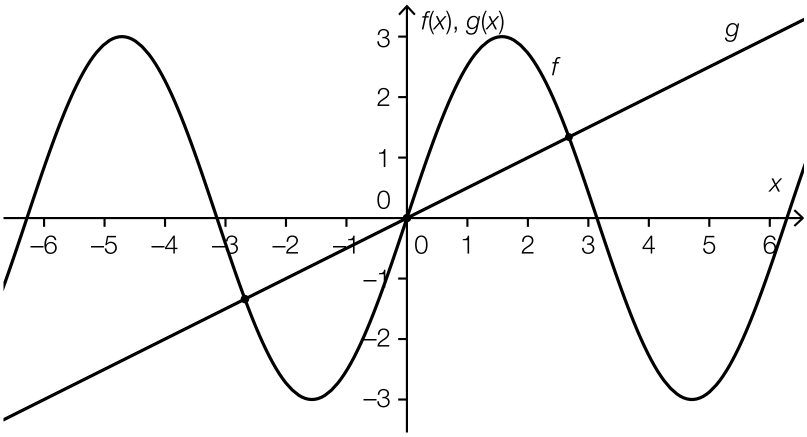
Task 3

Functions

The equations and graphs of the functions f and g are given below:

$f(x) = 3 \cdot \sin(x)$

$g(x) = \frac{x}{2}$



Task:

Determine the value of x_1 in the interval $[0, \pi]$ such that $f'(x_1) = g'(x_1)$ holds and explain how this point can be determined graphically.

Guiding question:

The equation $f(x) = g(x)$ has three solutions for x at a, 0 and c, where $a < 0 < c$.

On the diagram above, represent the value of the expression $\int_0^c (f(x) - g(x))\,\mathrm{d}x$ graphically.

Determine the value of the expression $\int_a^c (f(x) - g(x))\,\mathrm{d}x$.

Solution of Task 3

Functions

Expected solution of the statement of the task:

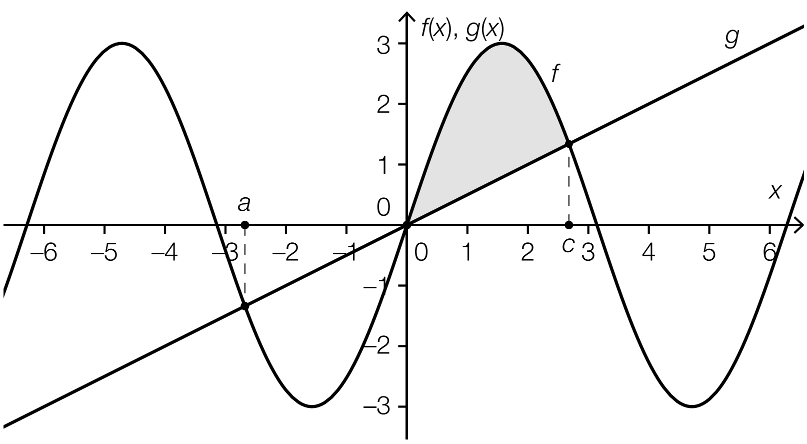
$3 \cdot \cos(x_1) = \frac{1}{2} \quad \Rightarrow \quad x_1 \approx 1.4$

The value of x_1 gives the location of the point (in the interval $[0, \pi]$) at which the gradient of the line is the same as the gradient of the tangent to the graph of f. If the line g is translated so that it becomes a tangent to the graph of f in the interval $[0, \pi]$, then the value is given by the x-coordinate of the point where the two functions meet.

Answer key:

The point for the core competencies is to be awarded if x_1 has been calculated correctly and a method for obtaining the value graphically has been explained correctly.

Expected solution of the guiding question:



The value of the expression $\int_a^c (f(x) - g(x))\,\mathrm{d}x$ is zero.

Answer key:

The point for the guiding question is to be awarded if the correct area has been identified and the value of the integral $\int_a^c (f(x) - g(x))\,\mathrm{d}x$ has been given correctly.

Task 4

Reaction Times

A test subject determines their reaction time (in s) using an online test that they take ten times. The subject obtains the values given below:

0.38 s; 0.27 s; 0.30 s; 0.34 s; 0.25 s; 0.39 s; 0.28 s; 0.24 s; 0.33 s; 0.32 s

Task:

Determine the mean $\bar{t}$ and the standard deviation s of the ten results given above.

Determine what percentage of the reaction times given lie in the interval $[\bar{t} - s, \bar{t} + s]$.

Guiding question:

The test subject carries out the test two more times and obtains the results t_{11} and t_{12} where $t_{11} \neq t_{12}$. The mean that is calculated using all twelve times is referred to as $\bar{t}_{\text{new}}$, and the resulting standard deviation is referred to as s_{new}.

State which conditions the times t_{11} and t_{12} have to fulfil such that $\bar{t}_{\text{new}} = \bar{t}$ and $s_{\text{new}} < s$ hold.

Solution of Task 4

Reaction Times

Expected solution of the statement of the task:

$\bar{t} = 0.31$
$s \approx 0.05$
$[\bar{t} - s, \bar{t} + s] \approx [0.26, 0.36]$

Six of the reaction times lie within the given interval, which corresponds to 60 %.

Answer key:

The point for the core competencies is to be awarded if the mean, the standard deviation and the percentage have been given correctly.

Tolerance intervals:
for the standard deviation: [0.048, 0.052]
for the lower bound of the interval: [0.258, 0.262]
for the upper bound of the interval: [0.358, 0.362]

Expected solution of the guiding question:

The new values must be symmetrical about the mean $\bar{t}$ i.e. $\frac{t_{11} + t_{12}}{2} = \bar{t}$ must hold.

If the new values lie in the interval $(\bar{t} - s, \bar{t} + s)$ then $s_{new} < s$.

Answer key:

The point for the guiding question is to be awarded if the correct conditions for the times t_{11} and t_{12} have been given. Answers that state that the values t_{11} and t_{12} have to be close to the mean can be accepted as a condition for $s_{new} < s$.

Task 5

Raffle

Among 100 raffle tickets, there are 30 winning tickets. Of these 30 tickets, 25 result in winnings of € 10 each and five result in winnings of € 100 each.

Task:

Three tickets are selected at random from the 100 tickets.
Find the probability that no winning tickets are selected and explain your method.

Guiding question:

A person receives a randomly selected ticket from these 100 raffle tickets as a present. Determine the expectation value for this person's winnings.

Another person receives two randomly selected tickets from these 100 raffle tickets as a present. Find an expression that could be used to calculate the probability that this person wins € 110 and explain your approach.

Solution of Task 5

Raffle

Expected solution of the statement of the task:

If no winning ticket is selected, this means that the three tickets have been taken from the 70 losing tickets without replacement.

$$\frac{70}{100} \cdot \frac{69}{99} \cdot \frac{68}{98} \approx 0.3385 = 33.85\ \%$$

Answer key:

The point for the core competencies is to be awarded if the probability has been given correctly and a correct method has been shown.
Tolerance interval for the probability: [33%, 34%]

Expected solution of the guiding question:

$$\frac{25}{100} \cdot 10 + \frac{5}{100} \cdot 100 = 7.5$$

The expectation value for the winnings is € 7.50.

Winnings of € 110 means that out of the two tickets, one ticket wins € 10 and one wins € 100.

A possible expression for calculating the probability: $2 \cdot \frac{25 \cdot 5}{100 \cdot 99}$

Answer key:

The point for the guiding question is to be awarded if the expectation value for the winnings and the correct expression have been given and a correct method has been explained. Equivalent expressions are to be accepted.