Name:	
Class:	

Standardised Competence-Oriented
Written School-Leaving Examination

AHS

9th May 2018

Mathematics

Part 2 Tasks

BMBWF
BUNDESMINISTERIUM
FÜR BILDUNG, WISSENSCHAFT
UND FORSCHUNG

Advice for Completing the Tasks

Dear candidate,

The following booklet for Part 2 contains four tasks, each of which contains between two and four sub-tasks. All sub-tasks can be completed independently of one another. You have *150 minutes* available in which to work on these tasks.

Please use a blue or black pen that cannot be rubbed out. You may use a pencil for tasks that require you to draw a graph, vectors or a geometric construction.

When completing these tasks please use this booklet and the paper provided. Write your name on each piece of paper you use as well as on the first page of this task booklet in the space provided. Please show clearly which sub-task each answer relates to.

In the assessment of your work, everything that is not crossed out will be considered. Your solutions must be clearly marked. If a solution is not clearly marked or if more than one solution is given, the task will be considered to be unsolved. Draw a line through any notes you make.

You may use the official formula booklet for this examination session as well as approved electronic device(s).

Please hand in both the task booklet and the separate sheets you have used at the end of the examination.

Assessment

Every task in Part 1 will be awarded either 0 points or 1 point. Every sub-task in Part 2 will be awarded 0, 1 or 2 points. The tasks marked with an $\boxed{A}$ will be awarded either 0 points or 1 point.

– If at least 16 of the 24 tasks in Part 1 are solved correctly, you will pass the examination.
– If fewer than 16 of the 24 tasks in Part 1 are solved correctly, then the tasks marked with an $\boxed{A}$ from Part 2 may compensate for the shortfall (as part of the “range of essential skills” outlined by the LVBO).
  If, including the tasks marked with an $\boxed{A}$ from Part 2, at least 16 tasks are solved correctly, you will pass the examination.
  If, including the tasks marked with an $\boxed{A}$ from Part 2, fewer than 16 tasks are solved correctly, you will not be awarded enough points to pass the examination.
– If at least 16 tasks are solved correctly (including the compensation tasks marked with an $\boxed{A}$ from Part 2), a grade will be awarded as follows:

Pass	16–23 points
Satisfactory	24–32 points
Good	33–40 points
Very Good	41–48 points

Explanation of the Task Types

Some tasks require a *free answer*. For these tasks, you should write your answer directly underneath each task in the task booklet or on the paper provided. Other task types used in the examination are as follows:

Matching tasks: For this task type you will be given a number of statements, tables or diagrams, which will appear alongside a selection of possible answers. To correctly answer these tasks, you will need to match each statement, table or diagram to its corresponding answer. You should write the letter of the correct answer next to the statement, table or diagram in the space provided.

Example:
You are given two equations.

Task:
Match the two equations to their corresponding description (from A to D).

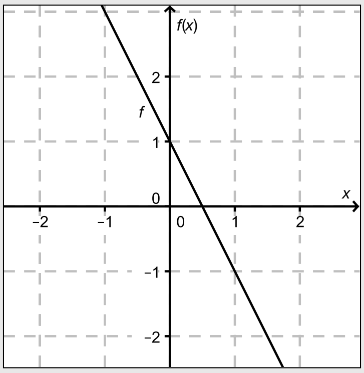
$1 + 1 = 2$	A
$2 \cdot 2 = 4$	C

A	Addition
B	Division
C	Multiplication
D	Subtraction

Construction tasks: This task type requires you to draw points, lines and/or curves in the task booklet.

Example:
Below you will see a linear function f where $f(x) = k \cdot x + d$.

Task:
On the axes provided below, draw the graph of a linear function for which $k = -2$ and $d > 0$.



Multiple-choice tasks of the form "1 out of 6": This task type consists of a question and six possible answers. Only one answer should be selected. You should put a cross next to the only correct answer in the space provided.

Example:
Which equation is correct?

Task:
Put a cross next to the correct equation.

1 + 1 = 1	☐
2 + 2 = 2	☐
3 + 3 = 3	☐
4 + 4 = 8	☒
5 + 5 = 5	☐
6 + 6 = 6	☐

Multiple-choice tasks of the form "2 out of 5": This task type consists of a question and five possible answers, of which two answers should be selected. You should put a cross next to each of the two correct answers in the space provided.

Example:
Which equations are correct?

Task:
Put a cross next to each of the two correct equations.

1 + 1 = 1	☐
2 + 2 = 4	☒
3 + 3 = 3	☐
4 + 4 = 8	☒
5 + 5 = 5	☐

Multiple-choice tasks of the form "x out of 5": This task type consists of a question and five possible answers, of which one, two, three, four *or* five answers may be selected. The task will require you to: "Put a cross next to each correct statement/equation …". You should put a cross next to each correct answer in the space provided.

Example:
Which of the equations given are correct?

Task:
Put a cross next to each correct equation.

1 + 1 = 2	☒
2 + 2 = 4	☒
3 + 3 = 6	☒
4 + 4 = 4	☐
5 + 5 = 10	☒

Gap-fill: This task type consists of a sentence with two gaps, i. e. two sections of the sentence are missing and must be completed. For each gap you will be given the choice of three possible answers. You should put a cross next to **each of the two answers** that are necessary to complete the sentence correctly.

Example:
Below you will see 3 equations.

Task:
Complete the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

The operation in equation ____①____ is known as summation or ____②____.

①	
1 – 1 = 0	☐
1 + 1 = 2	☒
1 · 1 = 1	☐

②	
Multiplication	☐
Subtraction	☐
Addition	☒

Changing an answer for a task that requires a cross:

1. Fill in the box that contains the cross for your original answer.
2. Put a cross in the box next to your new answer.

1 + 1 = 3	☐
2 + 2 = 4	☒
3 + 3 = 5	☐
4 + 4 = 4	☐
5 + 5 = 9	■

In this instance, the answer “5 + 5 = 9” was originally chosen. The answer was later changed to be “2 + 2 = 4”.

Selecting an answer that has been filled in:

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

1 + 1 = 3	☐
2 + 2 = 4	■
3 + 3 = 5	☐
4 + 4 = 4	■
5 + 5 = 9	☐

In this instance, the answer “2 + 2 = 4” was filled in and then selected again.

If you still have any questions, please ask your teacher.

Good Luck!

Task 1

Properties of a Third Degree Polynomial Function

Let f be a third degree polynomial function with equation $f(x) = a \cdot x^3 + b \cdot x$ with coefficients $a, b \in \mathbb{R}\setminus\{0\}$.

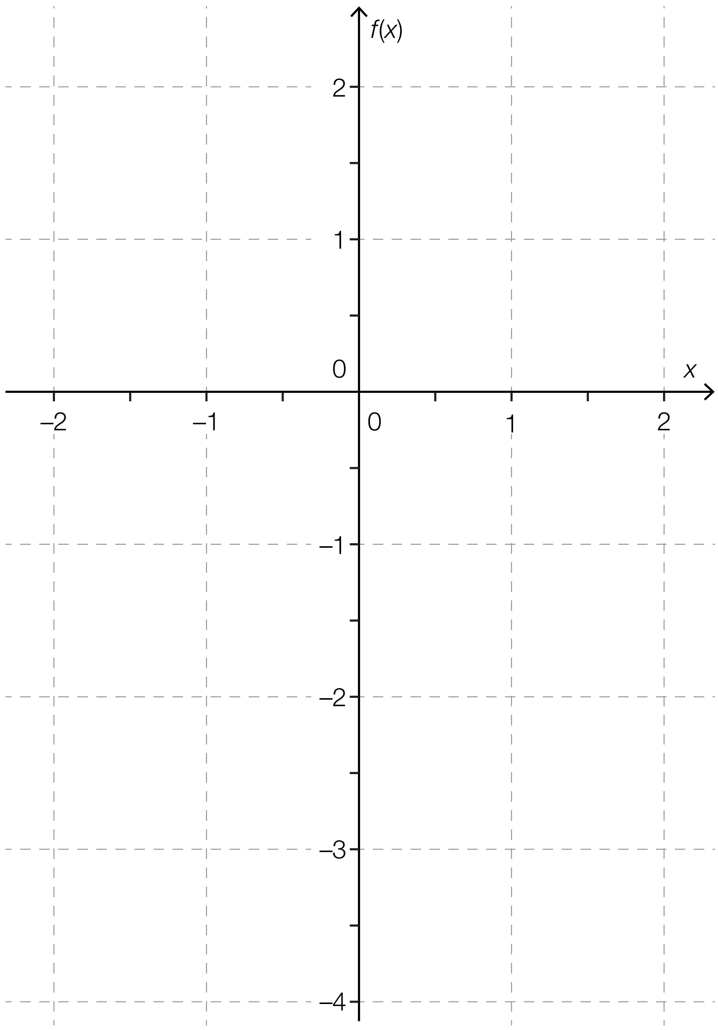
Task:

a) Justify why the function f has exactly three distinct real roots if the coefficients a and b have different signs.

$\boxed{\text{A}}$ The gradient of the tangent to the graph of f at the point where $x = 0$ is equal to the value of the coefficient b. Justify why this statement is true.

b) Determine the relationship between the coefficients a and b such that $\int_0^1 f(x)\,\mathrm{d}x = 0$ holds.

Justify why it follows that f has a root in the interval $(0, 1)$ if it is assumed that $\int_0^1 f(x)\,\mathrm{d}x = 0$ holds. Sketch a possible graph of one such function f in the coordinate system provided below.



Task 2

Hop

Hop is a fast growing climbing plant. The modelling function $h: \mathbb{R}_0^+ \to \mathbb{R}^+$ where $h(t) = \frac{a}{1 + b \cdot e^{k \cdot t}}$ with $a, b \in \mathbb{R}^+$, $k \in \mathbb{R}^-$ can be used to approximate the height of a plant of a particular type of hop at time t, where $h(t)$ is given in metres and t in weeks.

The table below shows the heights of a hop plant measured from the beginning of April ($t = 0$).

Time (in weeks)	0	2	4	6	8	10	12
Height (in m)	0.6	1.2	2.3	4.2	5.9	7.0	7.6

From these values, the values of the parameters $a = 8$, $b = 15$ and $k = -0.46$ for the modelling function h were determined.

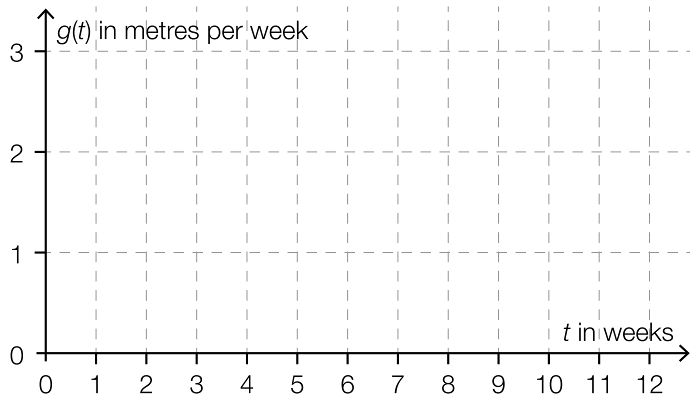
Task:

a) $\boxed{\text{A}}$ Using the modelling function h, write down an expression that can be used to calculate by how many metres the hop plant has grown in the time interval $[0, t_1]$.

Using the modelling function h and your expression, determine by how many metres the plant grew in the first 10 weeks and write down how much this value differs from the actual value measured as a percentage.

b) If the growth is modelled by the function h, there is a time t_2 at which the plant grows the fastest. Write down an equation that can be used to calculate t_2 and determine the value of t_2.

Determine the corresponding maximal rate of growth and sketch the shape of the function g that is based on the modelling function h and describes the rate of growth of the hop plant in terms of t in the coordinate system provided below using the maximum value you have calculated above.



c) Write down a linear function h_1 that gives the correct heights at $t = 0$ and $t = 12$ according to the table and interpret the gradient of this linear function in the given context.

$h_1(t) =$ ____

Using the shape of the graphs of h and h_1, justify why there are at least two points in time at which the rate of growth of the plant has the same value as the gradient of h_1.

d) As the value of t increases, $h(t)$ tends to a value, which is denoted by h_{max}. Using the equation given for the modelling function h, show by calculation that the parameter k (where $k < 0$) has no influence on h_{max} and specify h_{max}.

Favourable weather conditions can mean that the hop plant grows higher and more quickly i. e. that the plant reaches a value larger than h_{max} at an earlier point in time. Write down how a and k would have to be changed to reflect this growth.

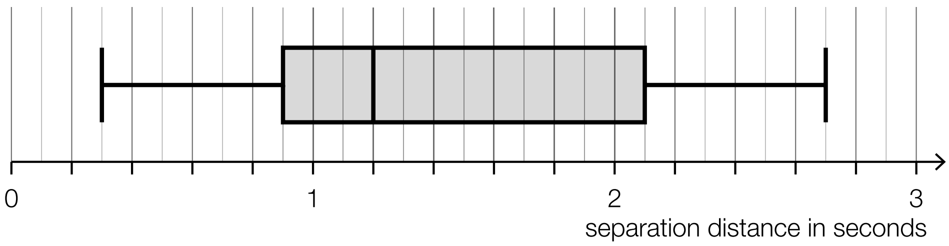
Task 3

Measuring Distances

During public transport checks carried out by the police, distances are measured. In the following description, the term *distance* refers to the length of a line segment and the term *separation distance* refers to a period of time.

If the distance between the back end of the car in front and the front end of the car behind is Δs metres, then the separation distance is the time t in seconds that it would take the car behind to cover the distance Δs.

The box plot below shows the separation distances that were collected during a check of 1 000 vehicles. All of the vehicles that were checked were travelling at a speed of approximately 130 km/h.



Task:

a) $\boxed{A}$ Write down the first quartile q_1 and the third quartile q_3 of the separation distances and state the meaning of the region from q_1 to q_3 in the given context.

According to the values from an Austrian motor club, around three quarters of drivers keep a distance of at least 30 metres from the car in front when driving at speeds of around 130 km/h. Write down whether or not the data displayed in the box plot above can roughly confirm these values and give a reason for your answer.

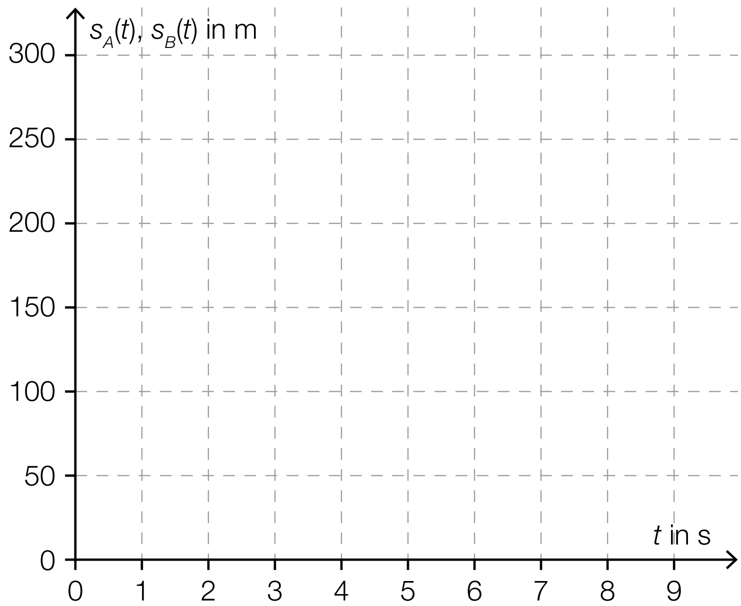
b) A common rule of thumb recommended for the motorway is to keep a separation distance of at least 2 seconds. A person claims that it can be read from the box plot that at least 20 % of the drivers kept to this separation distance. Write down a larger percentage that can be read from the box plot that definitely corresponds to this separation distance and justify your decision.

Assume that the percentage that you have found can be used as the probability that the recommended separation distance is adhered to. Write down the probability that at least six out of a sample of ten independent, randomly chosen measurements from these checks adhere to the recommended separation distance of at least two seconds.

c) During a different check, a vehicle that is to be checked is also filmed in the 300 metres preceding the checkpoint so that the measurement cannot be distorted if the vehicle in front of the vehicle to be checked slows down.

During the measuring process, vehicle *A* travels with constant speed and requires nine seconds to cover the 300 metres that are filmed. Draw the distance covered $s_A(t)$ in terms of the time t in the distance-time diagram provided below ($s_A(t)$ in metres, t in seconds) and write down the speed at which the vehicle is travelling in km/h.

Vehicle *B* also requires nine seconds to cover the 300 metres, but its speed is continually decreasing during this time. Draw a possible graph of the corresponding distance-time function s_B that starts from the origin in the space provided below.



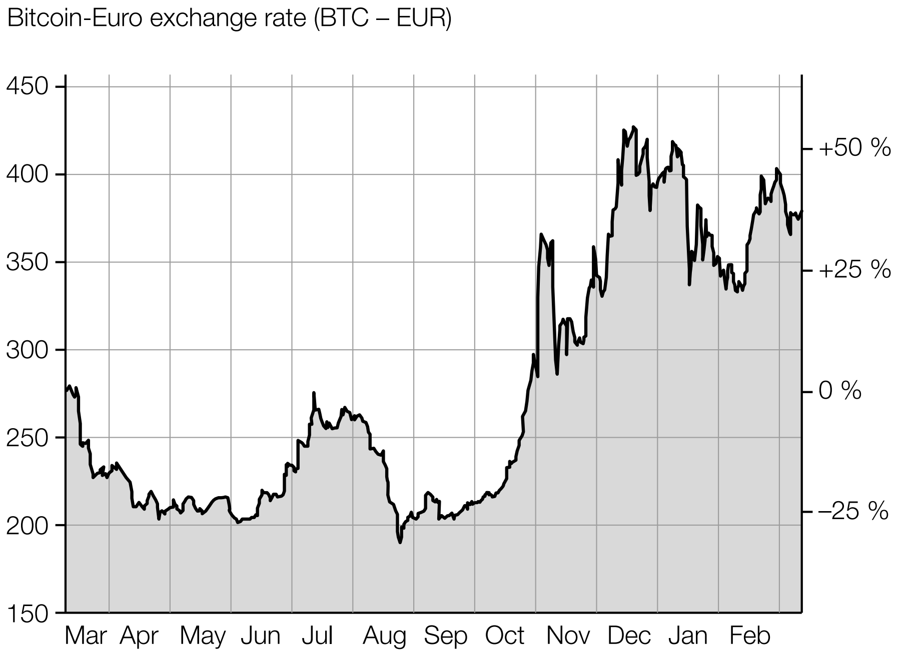
Task 4

Bitcoin

Bitcoin (abbreviation: BTC) is a digital artificial currency. The market value of Bitcoin is determined by supply and demand.

In this task, those people who use Bitcoin are referred to as Bitcoin users.

The diagram below shows the Bitcoin-Euro exchange rate from the 11th March 2015 to the 11th March 2016. The scale on the left-hand side shows the absolute value of a Bitcoin in euros; the scale on the right-hand side shows the percentage change from the 11th March 2015.



Data source: http://www.finanzen.net/devisen/bitcoin-euro-kurs [11.03.2017] (adapted).

Task:

a) Write down in which month from April 2015 to December 2015 the absolute value of the Bitcoin-Euro exchange rate fell the most (from the beginning of the month to the end of the month) and write down this exchange rate loss in euros.

Month: ________________

Exchange rate loss: ________________

Let K_1 be the Bitcoin-Euro exchange rate at the beginning of this particular month, K_2 be the Bitcoin-Euro exchange rate at the end of this particular month, and AT be the number of days in the month considered.

Determine the approximate value of the expression $\frac{K_2 - K_1}{AT}$ and interpret the result in the given context.

b) At the beginning of January 2016, around 15 million Bitcoins were in circulation. The number of Bitcoins in circulation t years after 2009 is approximately $f(t) = 21 \cdot 10^6 - 21 \cdot 10^6 \cdot e^{-0.18 \cdot t}$. At the beginning of January 2009, there were $f(0)$ Bitcoins in circulation.

Determine and interpret the relative (percentage) change in the number of Bitcoins in circulation in the time period [7, 8].

Write down an equation that can be used to calculate the time from which only one million Bitcoins can be put in circulation and determine this time.

c) A study of the demographics of Bitcoin users found that globally 88 % of Bitcoin users are male.
It is to be determined how high this percentage is in Austria. A large number of people were questioned. This survey found that 171 of those asked were Bitcoin users and 138 of these 171 people were male.

$\boxed{\text{A}}$ Using this data, write down a symmetrical 95 % confidence interval for the unknown proportion of male Bitcoin users out of all Bitcoin users in Austria.

Write down the minimum confidence level that would need to be used so that the global proportion of 88 % would be in this interval.