Standardised Competence-Oriented
Written School-Leaving Examination

AHS

9th May 2018

Mathematics

Part 1 Tasks

Advice for Completing the Tasks

Dear candidate,

The following booklet for Part 1 contains 24 tasks. The tasks can be completed independently of one another. You have *120 minutes* available in which to work through this booklet.

Please use a blue or black pen that cannot be rubbed out. You may use a pencil for tasks that require you to draw a graph, vectors or a geometric construction.

Please do all of your working out solely in this booklet. Write your name on the first page of the booklet in the space provided.

All answers must be written in this booklet. In the assessment of your work, everything that is not crossed out will be considered. Your solutions must be clearly marked. If a solution is not clearly marked or if more than one solution is given, the task will be considered to be unsolved. Draw a line through any notes you make.

You may use the official formula booklet for this examination session as well as approved electronic device(s).

Please hand in the task booklet at the end of the examination.

Assessment

Every task in Part 1 will be awarded either 0 points or 1 point. Every sub-task in Part 2 will be awarded 0, 1 or 2 points. The tasks marked with an $\boxed{A}$ will be awarded either 0 points or 1 point.

– If at least 16 of the 24 tasks in Part 1 are solved correctly, you will pass the examination.
– If fewer than 16 of the 24 tasks in Part 1 are solved correctly, then the tasks marked with an $\boxed{A}$ from Part 2 may compensate for the shortfall (as part of the "range of essential skills" outlined by the LVBO).
  If, including the tasks marked with an $\boxed{A}$ from Part 2, at least 16 tasks are solved correctly, you will pass the examination.
  If, including the tasks marked with an $\boxed{A}$ from Part 2, fewer than 16 tasks are solved correctly, you will not be awarded enough points to pass the examination.
– If at least 16 tasks are solved correctly (including the compensation tasks marked with an $\boxed{A}$ from Part 2), a grade will be awarded as follows:

Pass	16–23 points
Satisfactory	24–32 points
Good	33–40 points
Very Good	41–48 points

Explanation of the Task Types

Some tasks require a *free answer*. For these tasks, you should write your answer directly underneath each task in the task booklet. Other task types used in the examination are as follows:

Matching tasks: For this task type you will be given a number of statements, tables or diagrams, which will appear alongside a selection of possible answers. To correctly answer these tasks, you will need to match each statement, table or diagram to its corresponding answer. You should write **the letter of the correct answer** next to the statement, table or diagram in the space provided.

Example:
You are given two equations.

Task:
Match the two equations to their corresponding description (from A to D).

$1 + 1 = 2$	A
$2 \cdot 2 = 4$	C

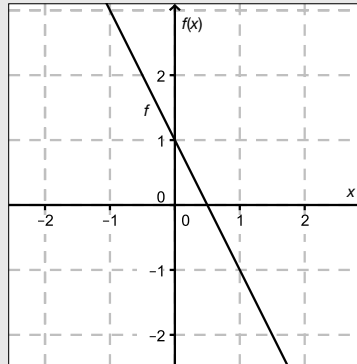| A	Addition
B	Division
C	Multiplication
D	Subtraction

Construction tasks: This task type requires you to draw points, lines and/or curves in the task booklet.

Example:
Below you will see a linear function f where $f(x) = k \cdot x + d$.

Task:
On the axes provided below, draw the graph of a linear function for which $k = -2$ and $d > 0$.

Multiple-choice tasks of the form "1 out of 6": This task type consists of a question and six possible answers. Only **one answer** should be selected. You should put a cross next to the only correct answer in the space provided.

Example:
Which equation is correct?

Task:
Put a cross next to the correct equation.

$1 + 1 = 1$	☐
$2 + 2 = 2$	☐
$3 + 3 = 3$	☐
$4 + 4 = 8$	☒
$5 + 5 = 5$	☐
$6 + 6 = 6$	☐

Multiple-choice tasks of the form "2 out of 5": This task type consists of a question and five possible answers, of which **two answers** should be selected. You should put a cross next to each of the two correct answers in the space provided.

Example:
Which equations are correct?

Task:
Put a cross next to each of the two correct equations.

$1 + 1 = 1$	☐
$2 + 2 = 4$	☒
$3 + 3 = 3$	☐
$4 + 4 = 8$	☒
$5 + 5 = 5$	☐

Multiple-choice tasks of the form "x out of 5": This task type consists of a question and five possible answers, of which **one, two, three, four** *or* **five answers** may be selected. The task will require you to: "Put a cross next to each correct statement/equation …". You should put a cross next to each correct answer in the space provided.

Example:
Which of the equations given are correct?

Task:
Put a cross next to each correct equation.

$1 + 1 = 2$	☒
$2 + 2 = 4$	☒
$3 + 3 = 6$	☒
$4 + 4 = 4$	☐
$5 + 5 = 10$	☒

Gap-fill: This task type consists of a sentence with two gaps, i.e. two sections of the sentence are missing and must be completed. For each gap you will be given the choice of three possible answers. You should put a cross next to **each of the two answers** that are necessary to complete the sentence correctly.

Example:
Below you will see 3 equations.

Task:
Complete the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

The operation in equation _____①_____ is known as summation or _____②_____ .

①	
$1 - 1 = 0$	☐
$1 + 1 = 2$	☒
$1 \cdot 1 = 1$	☐

②	
Multiplication	☐
Subtraction	☐
Addition	☒

Changing an answer for a task that requires a cross:

1. Fill in the box that contains the cross for your original answer.
2. Put a cross in the box next to your new answer.

$1 + 1 = 3$	☐
$2 + 2 = 4$	☒
$3 + 3 = 5$	☐
$4 + 4 = 4$	☐
$5 + 5 = 9$	■

In this instance, the answer "5 + 5 = 9" was originally chosen. The answer was later changed to be "2 + 2 = 4".

Selecting an answer that has been filled in:

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

$1 + 1 = 3$	☐
$2 + 2 = 4$	⬤
$3 + 3 = 5$	☐
$4 + 4 = 4$	■
$5 + 5 = 9$	☐

In this instance, the answer "2 + 2 = 4" was filled in and then selected again.

If you still have any questions now, please ask your teacher.

Good Luck!

Task 1

Relationship between Two Variables

For $a, b \in \mathbb{R}$ the relationship $a \cdot b = 1$ holds.

Task:

Two of the five following statements hold true in any case.
Put a cross next to each of the two correct statements.

If a is less than zero, then b is also less than zero.	☐
The signs of a and b can be different.	☐
For every $n \in \mathbb{N}$, $(a - n) \cdot (b + n) = 1$ holds.	☐
For every $n \in \mathbb{N}\backslash\{0\}$, $(a \cdot n) \cdot \left(\dfrac{b}{n}\right) = 1$ holds.	☐
$a \neq b$ holds.	☐

Task 2

Solar Panel System

A town supports the installation of systems of solar panels in *h* households by providing *p* % of the purchase costs to each household. The mean value for the purchase costs for a system of solar panels in this town is *e* euros.

Task:

Interpret the expression $h \cdot e \cdot \dfrac{p}{100}$ in the given context.

Task 3

Number of Solutions to a Quadratic Equation

Let $r \cdot x^2 + s \cdot x + t = 0$ be a quadratic equation in the variable x with coefficients $r, s, t \in \mathbb{R}\backslash\{0\}$.

The number of real solutions to the equation depends on r, s and t.

Task:

Find the number of real solutions to the given equation if r and t have different signs and provide a general justification for your answer.
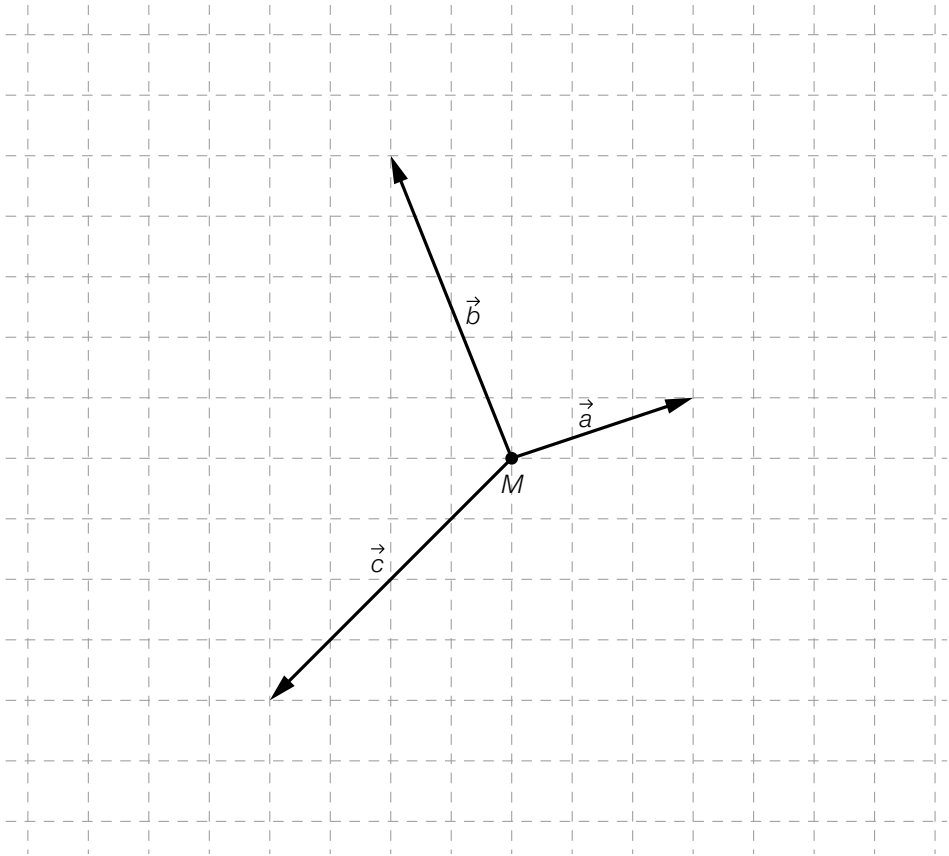
Task 4

Forces

Three forces act on a point mass M. These forces are represented by the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$.

Task:

In the diagram below, draw a force vector $\vec{d}$ such that the sum of all four forces is zero (for all components).

Task 5

Right Angle

Let AB be a line segment in $\mathbb{R}^2$ where $A = (3,4)$ and $B = (-2,1)$.

Task:

Write down a possible vector $\vec{n} \in \mathbb{R}^2$ where $\vec{n} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ that is perpendicular to the line segment AB.

Task 6

Sine and Cosine

The diagram below shows a circle with centre O and radius 1. The points $A = (1,0)$ and P lie on the circumference of the circle. The angle α shown below is measured from the leg OA to the leg OP in a counterclockwise direction.
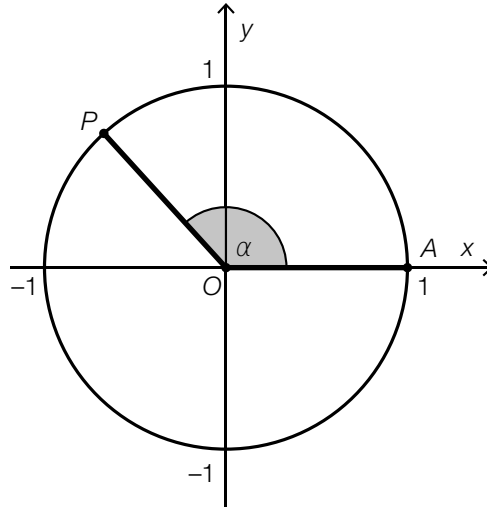
A point Q on the circumference of the circle should give rise to an angle β such that the following conditions hold:

$\sin(\beta) = -\sin(\alpha)$ and $\cos(\beta) = \cos(\alpha)$

Task:

Draw the point Q on the diagram above.

Task 7

Square Pyramid

The surface area of a regular square pyramid can be written as a function O of the length of the edge of the base a and the height of the lateral face h_1.

The equation of the function is: $O(a, h_1) = a^2 + 2 \cdot a \cdot h_1$, where $a \in \mathbb{R}^+$ and $h_1 > \dfrac{a}{2}$.

Task:

Six statements about the surface area of regular square pyramids are given below. Put a cross next to the correct statement.

If h_1 is constant, then the surface area is directly proportional to a.	☐
If a is constant, then the surface area is directly proportional to h_1.	☐
For $a = 1$ cm, the surface area is definitely greater than 2 cm².	☐
For $a = 1$ cm, the surface area is definitely less than 10 cm².	☐
If both a and h_1 are doubled, then the surface area is doubled.	☐
If $h_1 = a^2$, then the surface area can be written as an exponential function in terms of a.	☐

Task 8

Cyclists

Two cyclists *A* and *B* both start from the same starting point and cycle using electronic bicycles along a straight road in the same direction with constant speed.

The diagram below shows the graphs of the functions s_A and s_B, which show the distance covered by the cyclists with respect to the time spent cycling. The points shown in bold have coordinates (0,0), (2,0) and (8,2 400).
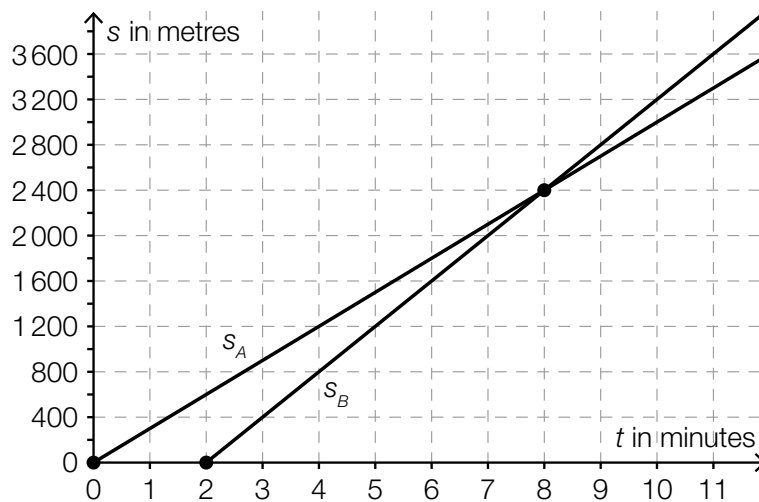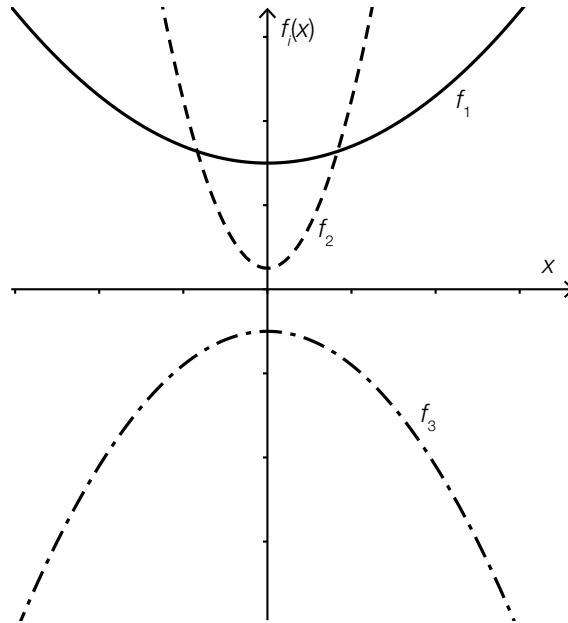

Task:

Put a cross next to each of the two statements that can be obtained from the diagram above.

Cyclist *B* starts two minutes later than cyclist *A*.	☐
The speed of cyclist *A* is 200 metres per minute.	☐
Cyclist *B* overtakes cyclist *A* after a distance of 2.4 kilometres.	☐
Both cyclists are the same distance away from the starting point eight minutes after cyclist *B* has started.	☐
Four minutes after cyclist *A* sets off, the two cyclists are 200 metres away from each other.	☐

Task 9

Graphs of Quadratic Functions

The diagram below shows the graphs of the quadratic functions f_1, f_2 and f_3 with equations $f_i(x) = a_i \cdot x^2 + b_i$ where $a_i, b_i \in \mathbb{R}, i \in \{1, 2, 3\}$.

Task:

Write down the values of the parameters a_i and b_i in order from smallest to largest in the spaces provided below.
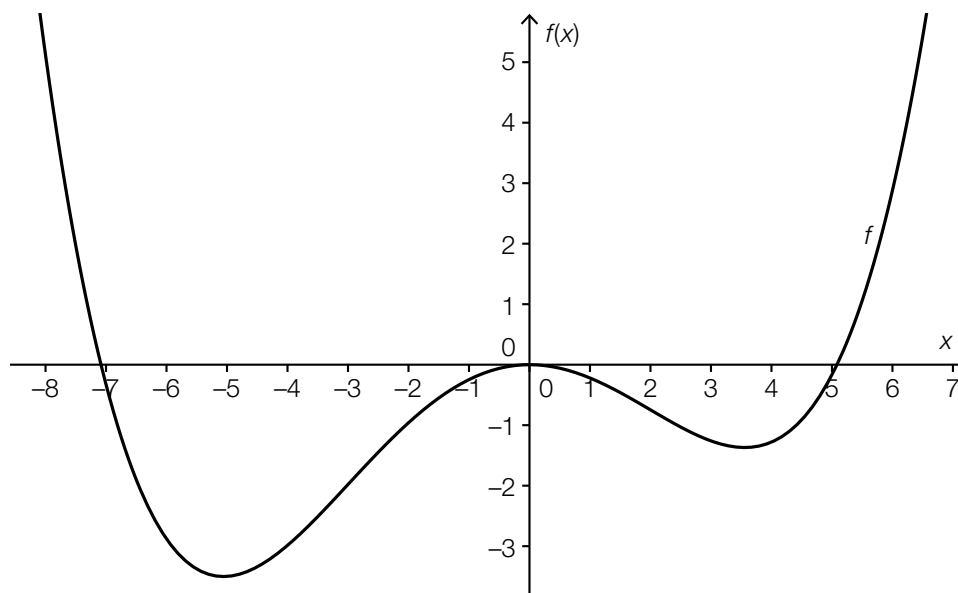
Parameter a_i: _____ < _____ < _____

Parameter b_i: _____ < _____ < _____

Task 10

Polynomial Function

The diagram below shows the graph of a polynomial function *f*.

Task:

Justify why the function shown above cannot be a third degree polynomial function.

Task 11

Cell Cultures

In the context of a biological experiment, six cell cultures are placed under favourable or unfavourable external conditions such that the number of cells either increases exponentially or decreases exponentially.

The number of cells in each cell culture t days after the start of the experiment is given by $N_i(t)$ ($i = 1, 2, 3, 4, 5, 6$).

Task:

Match each of the four changes described below to the corresponding equation of a function (from A to F).

The number of cells doubles each day.	
The number of cells increases by 85 % each day.	
The number of cells decreases by 85 % each day.	
The number of cells decreases by half each day.	

A	$N_1(t) = N_1(0) \cdot 0.15^t$
B	$N_2(t) = N_2(0) \cdot 0.5^t$
C	$N_3(t) = N_3(0) \cdot 0.85^t$
D	$N_4(t) = N_4(0) \cdot 1.5^t$
E	$N_5(t) = N_5(0) \cdot 1.85^t$
F	$N_6(t) = N_6(0) \cdot 2^t$

Task 12

Sine Function

For $a, b \in \mathbb{R}^+$, let the function $f\colon \mathbb{R} \to \mathbb{R}$ be with $f(x) = a \cdot \sin(b \cdot x)$ for $x \in \mathbb{R}$.

The function f is known to have both of the properties given below:

- The (smallest) period of the function f is π.
- The difference between the largest and smallest value of the function f is 6.
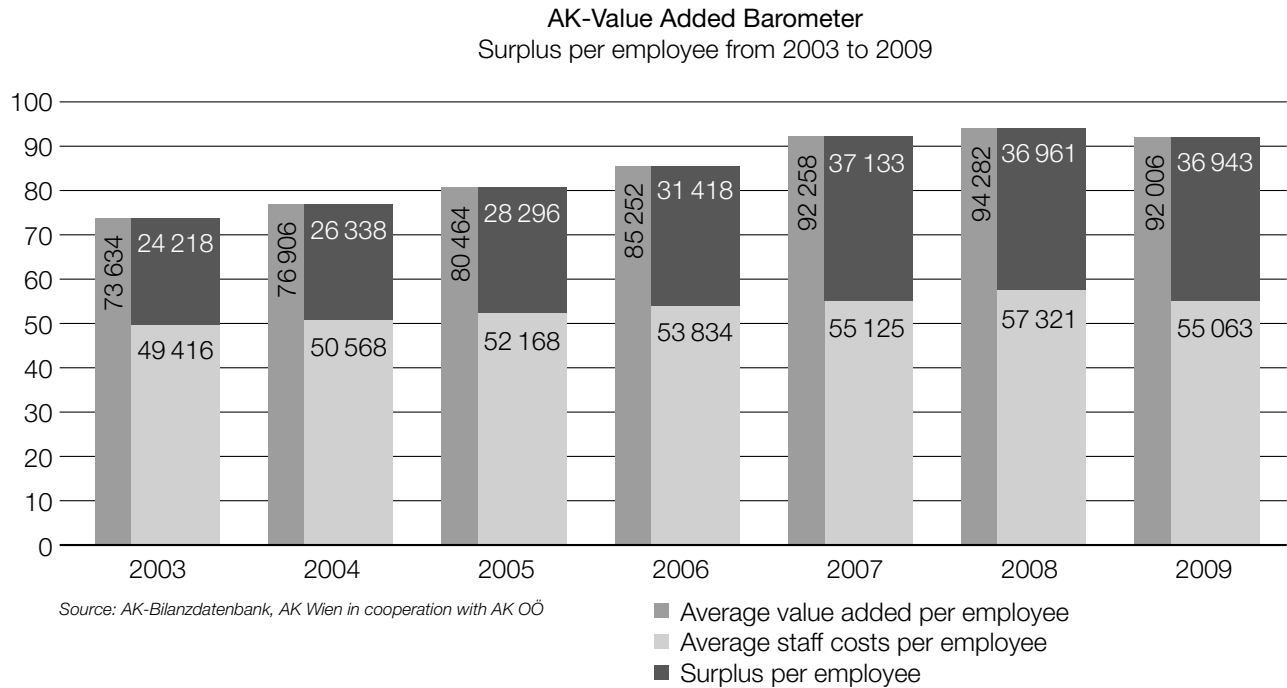
Task:

Write down a and b.

$a =$ _____

$b =$ _____

Task 13

Value Added

AK-Value Added Barometer
Surplus per employee from 2003 to 2009

Year	Average value added per employee	Average staff costs per employee	Surplus per employee
2003	73 634	49 416	24 218
2004	76 906	50 568	26 338
2005	80 464	52 168	28 296
2006	85 252	53 834	31 418
2007	92 258	55 125	37 133
2008	94 282	57 321	36 961
2009	92 006	55 063	36 943

Source: AK-Bilanzdatenbank, AK Wien in cooperation with AK OÖ

■ Average value added per employee
■ Average staff costs per employee
■ Surplus per employee

Data source: Arbeiterkammer Oberösterreich (ed.): *AK Wertschöpfungsbarometer: Trotz Krise: Eigentümer profitierten*, April 2011, p. 3.
https://media.arbeiterkammer.at/ooe/betriebsraete/PKU_2011_Wertschoepfungsbarometer.pdf [12.09.2017].

The AK-Value Added Barometer shows the trend of the amount that Austrian medium-sized and large businesses earn on average for each employee per year.

The surplus per employee, i.e. the difference between the average value added per employee and the average staff costs per employee, is determined.

Task:

For the year 2007, determine the proportion of this surplus to the value added per head as a percentage.

Task 14

Cooling Process

A liquid is cooled. The function T can be used to approximate the temperature of the liquid, where $T(t)$ gives the temperature of the liquid at time $t \geq 0$ ($T(t)$ in °C, t in minutes). The cooling process starts at time $t = 0$.

Task:

Interpret the equation $T'(20) = -0.97$ in the given context using the correct units.

Task 15

Debt Repayment

A person has taken out a bank loan to finance the purchase of an apartment. At the end of each month, the level of debt increases by 0.4 % due to the interest rate and a monthly instalment of € 450 is paid off the loan.

The level of debt at the end of t months is given by $S(t)$.

Task:

Write down a difference equation to help calculate, knowing the level of debt at the end of a month, the level of debt of the following month.

Task 16

Relationship between a Function, the Derivative and the Antiderivative

Let f be a third degree polynomial function with derivate f' and antiderivative F.

Task:

Complete the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence is a correct statement.

The second derivative of the function _____①_____ is the function _____②_____ .

①	
f	☐
f'	☐
F	☐

②	
f	☐
f'	☐
F	☐

Task 17

Graph of a Function

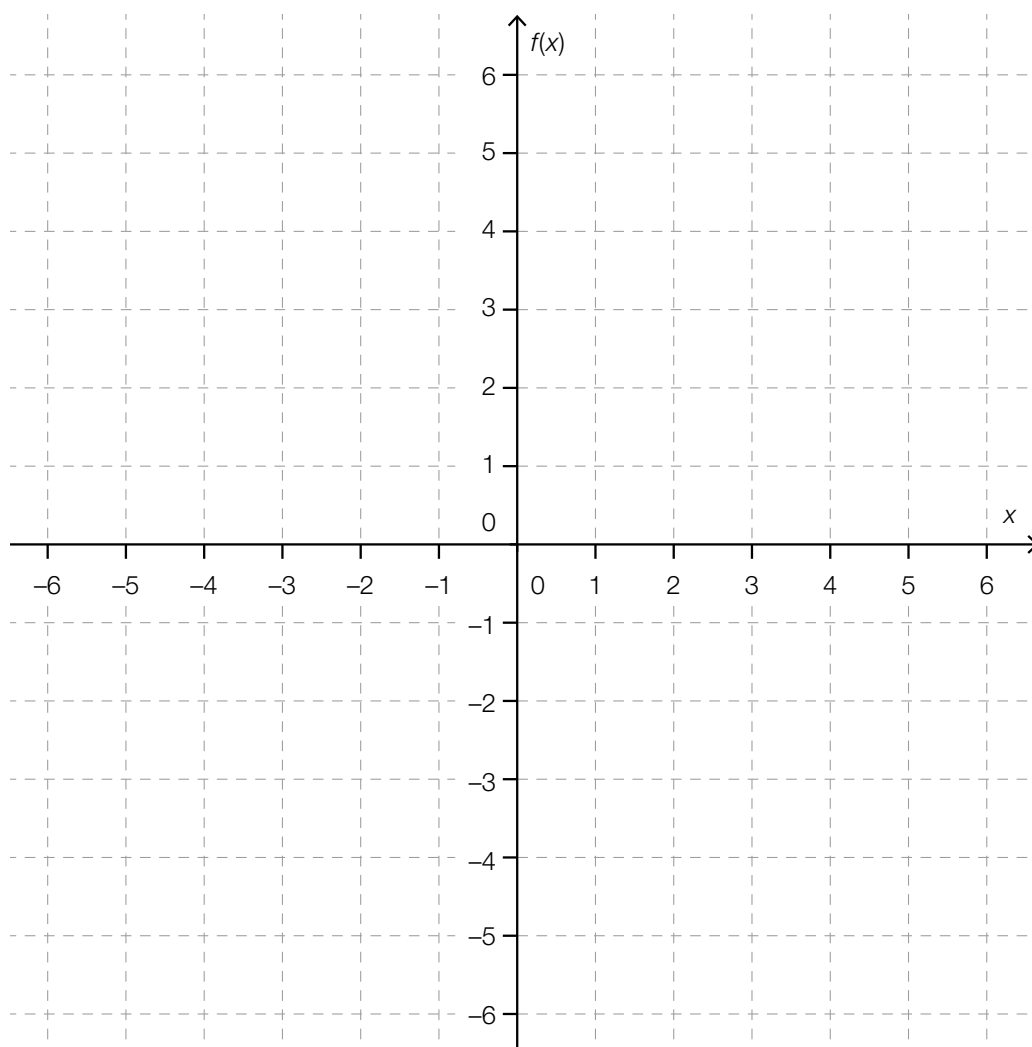A function $f: \mathbb{R} \to \mathbb{R}$ that is not constant has the following properties:

$f(4) = 2$
$f'(4) = 0$
$f''(4) = 0$
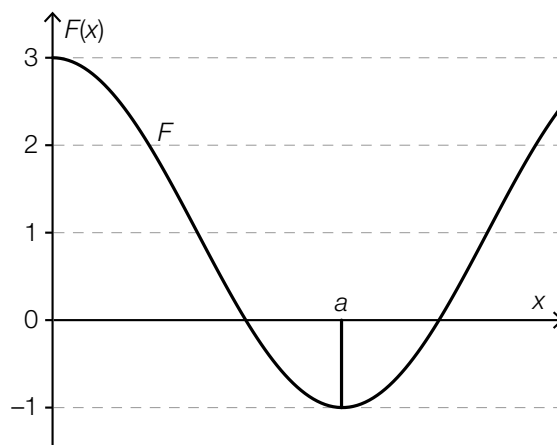$f'(x) \leq 0$ for all $x \in \mathbb{R}$

Task:

In the space provided below, sketch a possible graph of one such function f.

Task 18

Value of a Definite Integral

For a real function f the graph of an antiderivative F is shown below.

Task:

Write down the value of the definite integral $I = \int_0^a f(x)\,dx$.

$I = $ _____

Task 19

Homework and Examination

In a class of only girls, 15 pieces of homework were to be handed in before an examination. In the examination, a maximum of 48 points could be achieved.

In the scatter diagram shown below, the number of pieces of homework that were handed in and the number of points obtained in the examination are shown for each of the 20 pupils in this class.
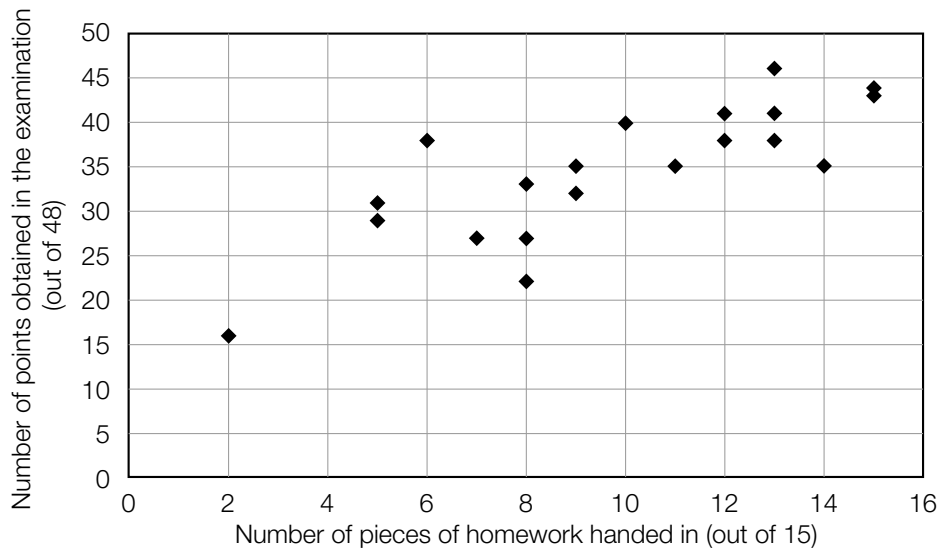
Task:

Two of the five statements shown below interpret the scatter diagram correctly. Put a cross next to each of the two correct statements.

Only pupils that handed in more than 10 pieces of homework were awarded more than 35 points on the examination.	☐
The pupil with the lowest number of points on the examination had handed in the fewest pieces of homework.	☐
The pupil with the highest number of points on the examination had handed in all of the pieces of homework.	☐
Pupils who had handed in at least 10 pieces of homework achieved on average more points in the examination than those who had handed in fewer than 10 pieces of homework.	☐
From the number of points awarded in the examination, it is possible to determine unequivocally the number of pieces of homework that had been handed in.	☐

Task 20

Donations

For a good cause, 20 people have donated money. Each person has donated a different amount. These 20 amounts of money (in euros) comprise the data set $x_1, x_2, \ldots, x_{20}$. From this data set, the minimum, the maximum, the mean, the median as well as the lower (first) and upper (third) quartiles can be determined.

Mrs. Müller is one of these 20 people and has donated 50 euros.

Task:

Each of the four questions in the table on the left-hand side can be answered correctly using one of the statistical values from the table on the right-hand side.

Match each of the four questions to the corresponding statistical value (from A to F).

Is Mrs. Müller's donation one of the five largest donations?	
Is Mrs. Müller's donation one of the ten largest donations?	
Is Mrs. Müller's donation the smallest donation?	
How many euros have the 20 people donated in total?	

A	Minimum
B	Maximum
C	Mean
D	Median
E	Lower quartile
F	Upper quartile

Task 21

Sweets

There are 50 sweets in a packet; 20 are red, 16 are white and 14 are green.
A child places their hand into the packet and takes out three sweets without looking at the colours.

Task:

Assuming that each sweet has an equal probability of being chosen, write down the probability that at least one of the three sweets is red.

Task 22

Comparing Two Probability Distributions

The diagrams below show the probability distributions of two random variables X and Y. The expectation values of the random variables are given by $E(X)$ and $E(Y)$ and the standard deviations are given by $\sigma(X)$ and $\sigma(Y)$.
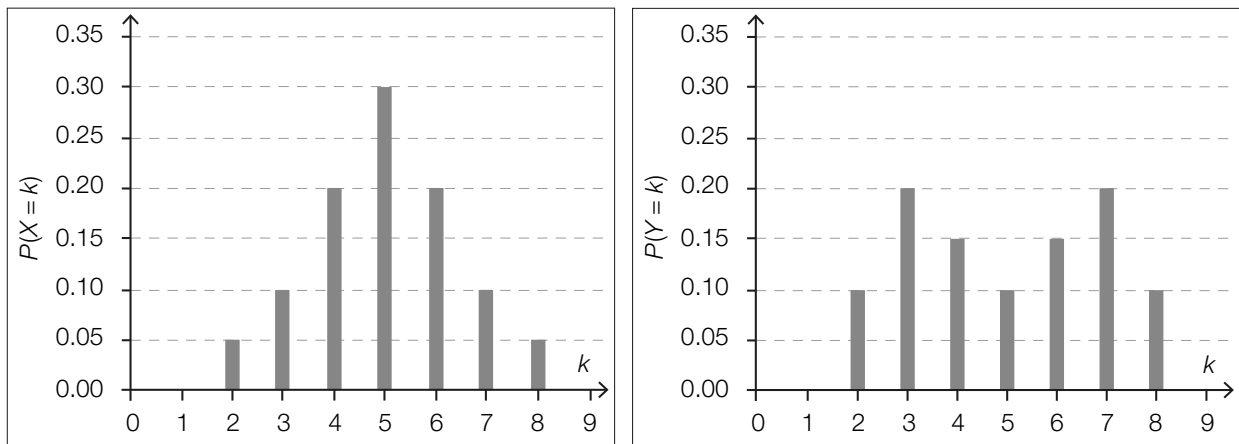
Task:

Put a cross next to each of the two correct statements.

$E(X) = E(Y)$	☐
$\sigma(X) > \sigma(Y)$	☐
$P(X \leq 3) < P(Y \leq 3)$	☐
$P(3 \leq X \leq 7) = P(3 \leq Y \leq 7)$	☐
$P(X \leq 5) = 0.3$	☐

Task 23

Mass Production

A particular product is mass-produced in packs of 100 items. In a pack, the probability that an individual item (independent of the other items in the pack) is faulty is 6 %.

Task:

Determine the probability of the package containing at the most two faulty items.

Task 24

Widths of Confidence Intervals

Four confidence intervals (*A*, *B*, *C* and *D*) are calculated for an unknown proportion using the same method involving the sample size, *n*, the confidence level, γ, and the relative frequency. The relative frequency used for all four confidence intervals is the same. The confidence intervals are symmetrical about the relative frequency.

Confidence Interval	Sample Size *n*	Confidence Level γ
A	500	90 %
B	500	95 %
C	2 000	90 %
D	2 000	95 %

Task:

Compare the widths of these four confidence intervals and write down the confidence intervals with the smallest and the largest widths.

Confidence interval with the smallest width: _____

Confidence interval with the largest width: _____