

Name:	
Class:	



Standardised Competence-Oriented
Written School-Leaving Examination

AHS

20th September 2018

Mathematics

Part 2 Tasks

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Advice for Completing the Tasks

Dear candidate,

The following booklet for Part 2 contains four tasks, each of which contains between two and four sub-tasks. All sub-tasks can be completed independently of one another. You have *150 minutes* available in which to work on these tasks.

Please use a blue or black pen that cannot be rubbed out. You may use a pencil for tasks that require you to draw a graph, vectors or a geometric construction.

When completing these tasks please use this booklet and the paper provided. Write your name on each piece of paper you use as well as on the first page of this task booklet in the space provided. Please show clearly which sub-task each answer relates to.

In the assessment of your work, everything that is not crossed out will be considered. Your solutions must be clearly marked. If a solution is not clearly marked or if more than one solution is given, the task will be considered to be unsolved. Draw a line through any notes you make.

You may use the official formula booklet for this examination session as well as approved electronic device(s).

Please hand in both the task booklet and the separate sheets you have used at the end of the examination.

Assessment

Every task in Part 1 will be awarded either 0 points or 1 point. Every sub-task in Part 2 will be awarded 0, 1 or 2 points. The tasks marked with an **A** will be awarded either 0 points or 1 point.

– If at least 16 of the 24 tasks in Part 1 are solved correctly, you will pass the examination.

– If fewer than 16 of the 24 tasks in Part 1 are solved correctly, then the tasks marked with an **A** from Part 2 may compensate for the shortfall (as part of the “range of essential skills” outlined by the LVBO).

If, including the tasks marked with an **A** from Part 2, at least 16 tasks are solved correctly, you will pass the examination.

If, including the tasks marked with an **A** from Part 2, fewer than 16 tasks are solved correctly, you will not be awarded enough points to pass the examination.

– If at least 16 tasks are solved correctly (including the compensation tasks marked with an **A** from Part 2), a grade will be awarded as follows:

Pass	16–23 points
Satisfactory	24–32 points
Good	33–40 points
Very Good	41–48 points

Explanation of the Task Types

Some tasks require a *free answer*. For these tasks, you should write your answer directly underneath each task in the task booklet or on the paper provided. Other task types used in the examination are as follows:

Matching tasks: For this task type you will be given a number of statements, tables or diagrams, which will appear alongside a selection of possible answers. To correctly answer these tasks, you will need to match each statement, table or diagram to its corresponding answer. You should write **the letter of the correct answer** next to the statement, table or diagram in the space provided.

Example:

You are given two equations.

$1 + 1 = 2$	A
$2 \cdot 2 = 4$	C

A	Addition
B	Division
C	Multiplication
D	Subtraction

Task:

Match the two equations to their corresponding description (from A to D).

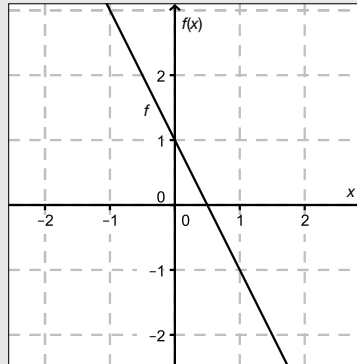
Construction tasks: This task type requires you to draw points, lines and/or curves in the task booklet.

Example:

Below you will see a linear function f where $f(x) = k \cdot x + d$.

Task:

On the axes provided below, draw the graph of a linear function for which $k = -2$ and $d > 0$.



Multiple-choice tasks of the form “1 out of 6”: This task type consists of a question and six possible answers. Only one answer should be selected. You should put a cross next to the only correct answer in the space provided.

Example:

Which equation is correct?

Task:

Put a cross next to the correct equation.

$1 + 1 = 1$	<input type="checkbox"/>
$2 + 2 = 2$	<input type="checkbox"/>
$3 + 3 = 3$	<input type="checkbox"/>
$4 + 4 = 8$	<input checked="" type="checkbox"/>
$5 + 5 = 5$	<input type="checkbox"/>
$6 + 6 = 6$	<input type="checkbox"/>

Multiple-choice tasks of the form “2 out of 5”: This task type consists of a question and five possible answers, of which two answers should be selected. You should put a cross next to each of the two correct answers in the space provided.

Example:

Which equations are correct?

Task:

Put a cross next to each of the two correct equations.

$1 + 1 = 1$	<input type="checkbox"/>
$2 + 2 = 4$	<input checked="" type="checkbox"/>
$3 + 3 = 3$	<input type="checkbox"/>
$4 + 4 = 8$	<input checked="" type="checkbox"/>
$5 + 5 = 5$	<input type="checkbox"/>

Multiple-choice tasks of the form “x out of 5”: This task type consists of a question and five possible answers, of which one, two, three, four or five answers may be selected. The task will require you to: “Put a cross next to each correct statement/equation ...”. You should put a cross next to each correct answer in the space provided.

Example:

Which of the equations given are correct?

Task:

Put a cross next to each correct equation.

$1 + 1 = 2$	<input checked="" type="checkbox"/>
$2 + 2 = 4$	<input checked="" type="checkbox"/>
$3 + 3 = 6$	<input checked="" type="checkbox"/>
$4 + 4 = 4$	<input type="checkbox"/>
$5 + 5 = 10$	<input checked="" type="checkbox"/>

Gap-fill: This task type consists of a sentence with two gaps, i.e. two sections of the sentence are missing and must be completed. For each gap you will be given the choice of three possible answers. You should put a cross next to **each of the two answers** that are necessary to complete the sentence correctly.

Example:

Below you will see 3 equations.

Task:

Complete the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

The operation in equation _____^①_____ is known as summation or _____^②_____.

①	
$1 - 1 = 0$	<input type="checkbox"/>
$1 + 1 = 2$	<input checked="" type="checkbox"/>
$1 \cdot 1 = 1$	<input type="checkbox"/>

②	
Multiplication	<input type="checkbox"/>
Subtraction	<input type="checkbox"/>
Addition	<input checked="" type="checkbox"/>

Changing an answer for a task that requires a cross:

1. Fill in the box that contains the cross for your original answer.
2. Put a cross in the box next to your new answer.

$1 + 1 = 3$	<input type="checkbox"/>
$2 + 2 = 4$	<input checked="" type="checkbox"/>
$3 + 3 = 5$	<input type="checkbox"/>
$4 + 4 = 4$	<input type="checkbox"/>
$5 + 5 = 9$	<input checked="" type="checkbox"/>

In this instance, the answer “ $5 + 5 = 9$ ” was originally chosen. The answer was later changed to be “ $2 + 2 = 4$ ”.

Selecting an answer that has been filled in:

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

$1 + 1 = 3$	<input type="checkbox"/>
$2 + 2 = 4$	<input checked="" type="checkbox"/>
$3 + 3 = 5$	<input type="checkbox"/>
$4 + 4 = 4$	<input checked="" type="checkbox"/>
$5 + 5 = 9$	<input type="checkbox"/>

In this instance, the answer “ $2 + 2 = 4$ ” was filled in and then selected again.

If you still have any questions, please ask your teacher.

Good Luck!

Task 1

Quadratic Function

The graph of a second degree polynomial function f crosses the positive vertical axis at the point $A = (0, y_A)$ and has the point $B = (x_B, 0)$ in common with the positive x -axis. Point B is also a maximum or minimum of f .

The function f has equation $f(x) = \frac{1}{4} \cdot x^2 + b \cdot x + c$ with $b, c \in \mathbb{R}$.

Task:

- a) Write down whether c has to be greater than zero, equal to zero or less than zero and justify your answer.

Write down whether b has to be greater than zero, equal to zero or less than zero and justify your answer.

- b) The function f' is the first derivative of the function f .
Write down whether the following statement is true or false and justify your answer: Point B is a point of intersection of the graphs of f and f' .

For all values of b there is exactly one value of x , x_t , with the following property: At x_t , f and f' have the same gradient. Write down x_t in terms of b .

- c) Write down which relationship has to hold between b and c so that the maximum or minimum x_B of f is also a zero of f .

Write down the coefficients b and c of the function f in terms of x_B .

Task 2

Interaction of Sound Waves

In the simplest case, a sound can be modelled by a sine function s where $s(t) = a \cdot \sin(b \cdot t)$ for $a, b \in \mathbb{R}^+$. The maximum value of a function that represents a sine wave of this kind is known as the amplitude. The number of periods per second is known as the frequency f and is given in Hertz (Hz).

For the frequency, f , the formula $f = \frac{1}{T}$ (with T in seconds) holds; T is the (smallest) period length of the sine wave ($T \in \mathbb{R}^+$).

Three particular sounds are described by the functions h_1 , h_2 and h_3 shown below. The time t ($t \geq 0$) is measured in milliseconds (ms).

$$h_1(t) = \sin(2 \cdot \pi \cdot t)$$

$$h_2(t) = \sin(2.5 \cdot \pi \cdot t)$$

$$h_3(t) = \sin(3 \cdot \pi \cdot t)$$

The interaction of multiple sounds results in a new sound.

The function h with $h(t) = h_1(t) + h_2(t) + h_3(t)$ describes a sound.

The acoustic pressure of a sound depends on time and can be modelled by the function p where $p(t) = \bar{p} \cdot \sin(\omega \cdot t)$. In this function, \bar{p} and ω are constants.

The acoustic pressure is given in pascals (Pa).

Task:

- a) Write down a formula in terms of c for the period length T (in ms) of a sound modelled by the function g with $g(t) = \sin(c \cdot \pi \cdot t)$ where $c \in \mathbb{R}^+$ and t in ms.

The effective value p_{eff} of the acoustic pressure of a sine wave with period length T (in ms) can be calculated using the formula $p_{\text{eff}} = \sqrt{\frac{1}{T} \int_0^T p^2(t) dt}$.

Determine the effective value of the acoustic pressure of a sine wave of a sound for $\bar{p} = 1$ and $\omega = 2 \cdot \pi$.

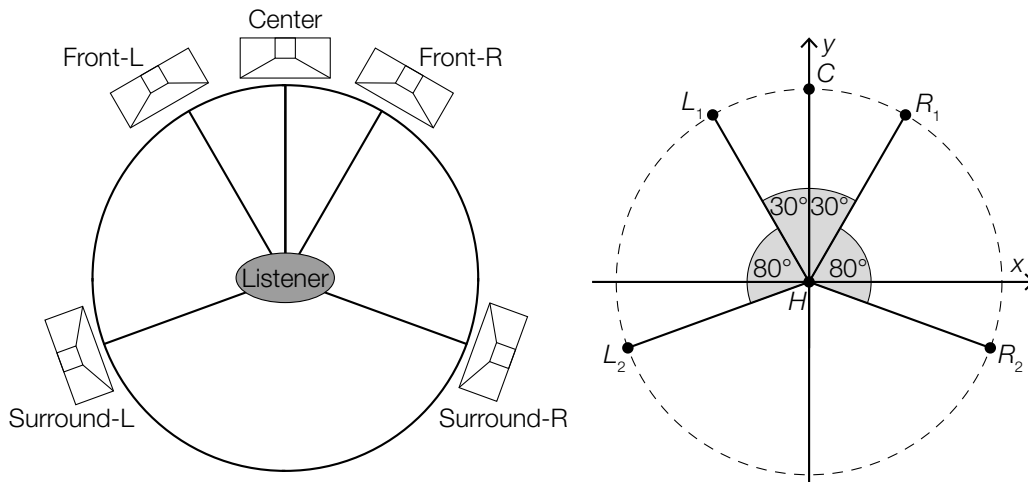
- b) Write down (e.g. with the help of an appropriate graph) the (smallest) period length T (in ms) of the function h .

Write down the frequency, f , of the function h in Hertz.

- c) Write down (e.g. with the help of an appropriate graph) the amplitude of the function h and the time $t \geq 0$ (in ms) at which this amplitude is first reached.

Justify why the amplitude of h is not equal to the sum of the three amplitudes of the functions h_1 , h_2 and h_3 .

- d) For a good surround sound experience (e.g. in a home cinema), it is desirable if the five speakers of a five-channel sound system are arranged as shown in the left-hand diagram below (view from above). This arrangement can be modelled using a Cartesian coordinate system (units in metres) as shown in the right-hand diagram below.



Source: <https://de.wikipedia.org/wiki/5.1> [23.04.2018] (adapted).

In this case, each of the five speakers (C , L_1 , L_2 , R_1 , R_2) is 2 m away from the listener (H). The point H lies at the origin.

A Write down the Cartesian coordinates of R_1 .

Determine the distance between L_2 and R_2 .

Task 3

Salmon Population

The Canadian scientist W. E. Ricker investigated the number of offspring of fish in North American rivers in terms of the number of fish in the parent generation. In 1954, he published Ricker's Model. The expected number of fish $R(n)$ in a successor generation can be approximated from the number of fish n in the respective parental generation using the so-called reproduction function R where $R(n) = a \cdot n \cdot e^{-b \cdot n}$ with $a, b \in \mathbb{R}^+$. After four years at most of living in the sea, salmon return to their place of birth to spawn, i.e. to lay their eggs. After spawning, most salmon die.

As part of his study, Ricker investigated the sockeye salmon population in the Skeena River in Canada. The table shown below gives the salmon population in the years from 1908 to 1923. The numbers given are averages of the observed population size in four consecutive years.

Time period	Observed salmon population (in thousands of fish)
01.01.1908–31.12.1911	1 098
01.01.1912–31.12.1915	740
01.01.1916–31.12.1919	714
01.01.1920–31.12.1923	615

Data source: http://jmahaffy.sdsu.edu/courses/s00/math121/lectures/product_rule/product.html [01.02.2018] (adapted).

Using this data for the salmon population in the Skeena River, the values for the parameters of the reproduction function R $a = 1.535$ and $b = 0.000783$ have been determined ($R(n)$ and n in thousands of fish).

Task:

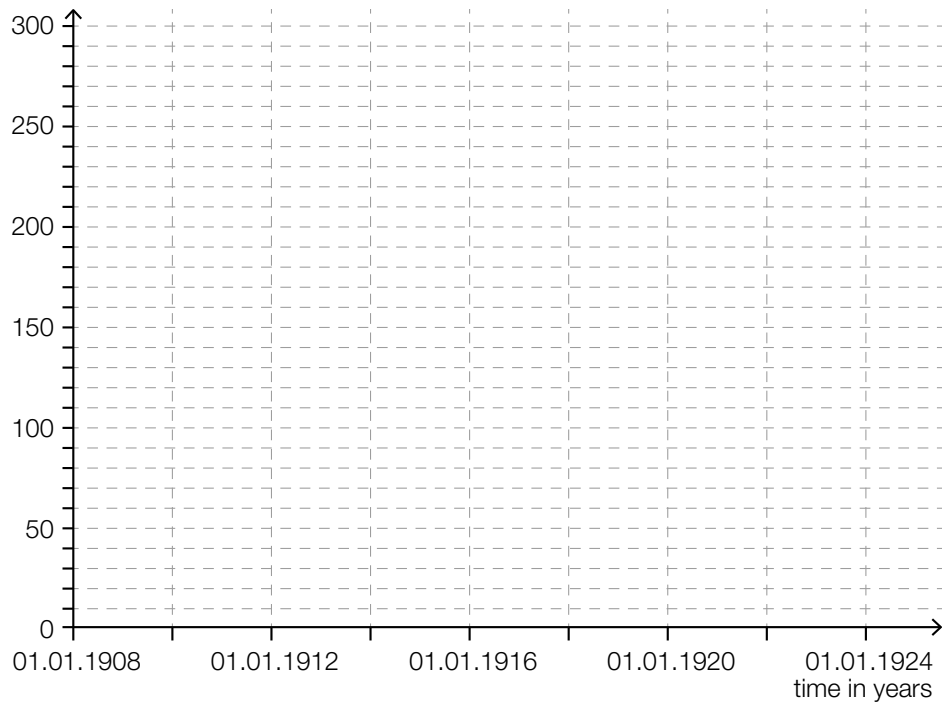
- a) For the salmon population in the Skeena River for $n > 0$, determine the solution n_0 to the equation $R(n) = n$ using the reproduction function in thousands of fish.

A Interpret n_0 in the given context.

- b) Determine the coordinates of the maximum or minimum $E = (n_E, R(n_E))$ of the reproduction function R in terms of a and b and show that n_E is a local maximum for all $a, b \in \mathbb{R}^+$.

Write down for which values of the parameter a the population $R(n_E)$ of the following generation is always greater than the previous population n_E .

c) Represent the data for the salmon population (in thousands of fish) given in the table above in a histogram. The absolute frequencies should be represented by the areas of rectangles.



Ricker's model is one of the standard models for describing population developments. Nevertheless, the values calculated using the reproduction function may deviate from the observed values.

Taking the observed average salmon population of 1098 (in the period between 1908 and 1911) as a starting point, calculate the expected average salmon population in the Skeena River for each of the four-year-periods between 1912 and 1923 according to the expectations based on the reproduction function.

Time period	Calculated salmon population (in thousands of fish)
01.01.1912–31.12.1915	
01.01.1916–31.12.1919	
01.01.1920–31.12.1923	

Task 4

Roulette

Roulette is a game of chance in which a natural number between 0 and 36 is selected at random by a rolling ball. Each of the 37 numbers has an equal probability of being selected, and the result of each turn is independent from previous turns. The region that corresponds to the number zero is green, half of the rest of the numbers are coloured red, and the other half are coloured black. The table below shows a selection of bets that can be placed and their corresponding pay-outs. A “profit of 35-times the initial stake” means, for example, that if the bet is won then the player receives their initial stake plus 35 times that amount (so, in total, 36 times their original stake).

single number (from 0 to 36)	profit of 35-times the initial stake
red/black	profit of the initial stake
even/odd (without zero)	profit of the initial stake

One of the most famous strategies for playing roulette is the Martingale System. In this system, a person always places the same “simple” bet (e. g. on “red” or “even”). If the bet is lost, the player doubles their stake in the next game. If this game is also lost, the player doubles their stake again and continues this pattern game after game. Using this strategy, as soon as a game is won, the game series ends and the player has won a profit amounting to their initial stake in the game series.

Task:

- a) The random variable X describes how often the ball lands on a particular number in 80 games. Determine the probability that the ball lands on a specific number at least four times during the 80 games.

A player would like to increase their chances of winning and proceeds as follows: During a string of e. g. 37 games, he records the number the ball lands on. He assumes, in the subsequent games, that the ball will land on the numbers that he has not recorded and thus places bets on these numbers.

Write down whether the player can increase his chances of winning using this strategy and justify your answer.

- b) A player uses the Martingale System and always places a bet on “red”. The player stops playing either when she wins or once the casino’s highest stake of € 10,000 has been reached and allows no further doubling of the stake.

The table below shows how quickly the stakes can increase from an initial stake of € 10 in the Martingale System if a player has a “losing streak”.

Round	Stake in €
1	10
2	20
3	40
4	80
5	160
6	320
7	640
8	1 280
9	2 560
10	5 120

A Determine the probability that the player loses all ten rounds in this Martingale series.

By calculating the expectation value of the profit, show that the Martingale System is not desirable for the player despite the very low probability of losing ten consecutive games.