Name:	
Class:	

Standardised Competence-Oriented
Written School-Leaving Examination

AHS

20th September 2018

Mathematics

Part 1 Tasks

Bundesministerium
Bildung, Wissenschaft
und Forschung

Advice for Completing the Tasks

Dear candidate,

The following booklet for Part 1 contains 24 tasks. The tasks can be completed independently of one another. You have *120 minutes* available in which to work through this booklet.

Please use a blue or black pen that cannot be rubbed out. You may use a pencil for tasks that require you to draw a graph, vectors or a geometric construction.

Please do all of your working out solely in this booklet. Write your name on the first page of the booklet in the space provided.

All answers must be written in this booklet. In the assessment of your work, everything that is not crossed out will be considered. Your solutions must be clearly marked. If a solution is not clearly marked or if more than one solution is given, the task will be considered to be unsolved. Draw a line through any notes you make.

You may use the official formula booklet for this examination session as well as approved electronic device(s).

Please hand in the task booklet at the end of the examination.

Assessment

Every task in Part 1 will be awarded either 0 points or 1 point. Every sub-task in Part 2 will be awarded 0, 1 or 2 points. The tasks marked with an $\boxed{\text{A}}$ will be awarded either 0 points or 1 point.

– If at least 16 of the 24 tasks in Part 1 are solved correctly, you will pass the examination.
– If fewer than 16 of the 24 tasks in Part 1 are solved correctly, then the tasks marked with an $\boxed{\text{A}}$ from Part 2 may compensate for the shortfall (as part of the "range of essential skills" outlined by the LVBO).
  If, including the tasks marked with an $\boxed{\text{A}}$ from Part 2, at least 16 tasks are solved correctly, you will pass the examination.
  If, including the tasks marked with an $\boxed{\text{A}}$ from Part 2, fewer than 16 tasks are solved correctly, you will not be awarded enough points to pass the examination.
– If at least 16 tasks are solved correctly (including the compensation tasks marked with an $\boxed{\text{A}}$ from Part 2), a grade will be awarded as follows:

Pass	16–23 points
Satisfactory	24–32 points
Good	33–40 points
Very Good	41–48 points

Explanation of the Task Types

Some tasks require a *free answer*. For these tasks, you should write your answer directly underneath each task in the task booklet. Other task types used in the examination are as follows:

Matching tasks: For this task type you will be given a number of statements, tables or diagrams, which will appear alongside a selection of possible answers. To correctly answer these tasks, you will need to match each statement, table or diagram to its corresponding answer. You should write **the letter of the correct answer** next to the statement, table or diagram in the space provided.

Example:
You are given two equations.

Task:
Match the two equations to their corresponding description (from A to D).

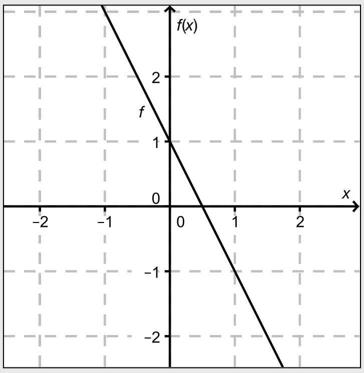
$1 + 1 = 2$	A
$2 \cdot 2 = 4$	C

A	Addition
B	Division
C	Multiplication
D	Subtraction

Construction tasks: This task type requires you to draw points, lines and/or curves in the task booklet.

Example:
Below you will see a linear function f where $f(x) = k \cdot x + d$.

Task:
On the axes provided below, draw the graph of a linear function for which $k = -2$ and $d > 0$.



Multiple-choice tasks of the form "1 out of 6": This task type consists of a question and six possible answers. Only **one answer** should be selected. You should put a cross next to the only correct answer in the space provided.

Example:
Which equation is correct?

Task:
Put a cross next to the correct equation.

1 + 1 = 1	☐
2 + 2 = 2	☐
3 + 3 = 3	☐
4 + 4 = 8	☒
5 + 5 = 5	☐
6 + 6 = 6	☐

Multiple-choice tasks of the form "2 out of 5": This task type consists of a question and five possible answers, of which **two answers** should be selected. You should put a cross next to each of the two correct answers in the space provided.

Example:
Which equations are correct?

Task:
Put a cross next to each of the two correct equations.

1 + 1 = 1	☐
2 + 2 = 4	☒
3 + 3 = 3	☐
4 + 4 = 8	☒
5 + 5 = 5	☐

Multiple-choice tasks of the form "x out of 5": This task type consists of a question and five possible answers, of which **one, two, three, four** *or* **five answers** may be selected. The task will require you to: "Put a cross next to each correct statement/equation …". You should put a cross next to each correct answer in the space provided.

Example:
Which of the equations given are correct?

Task:
Put a cross next to each correct equation.

1 + 1 = 2	☒
2 + 2 = 4	☒
3 + 3 = 6	☒
4 + 4 = 4	☐
5 + 5 = 10	☒

Gap-fill: This task type consists of a sentence with two gaps, i. e. two sections of the sentence are missing and must be completed. For each gap you will be given the choice of three possible answers. You should put a cross next to **each of the two answers** that are necessary to complete the sentence correctly.

Example:
Below you will see 3 equations.

Task:
Complete the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

The operation in equation ____①____ is known as summation or ____②____.

①	
1 – 1 = 0	☐
1 + 1 = 2	☒
1 · 1 = 1	☐

②	
Multiplication	☐
Subtraction	☐
Addition	☒

Changing an answer for a task that requires a cross:

1. Fill in the box that contains the cross for your original answer.
2. Put a cross in the box next to your new answer.

1 + 1 = 3	☐
2 + 2 = 4	☒
3 + 3 = 5	☐
4 + 4 = 4	☐
5 + 5 = 9	■

In this instance, the answer “5 + 5 = 9” was originally chosen. The answer was later changed to be “2 + 2 = 4”.

Selecting an answer that has been filled in:

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

1 + 1 = 3	☐
2 + 2 = 4	■
3 + 3 = 5	☐
4 + 4 = 4	■
5 + 5 = 9	☐

In this instance, the answer “2 + 2 = 4” was filled in and then selected again.

If you still have any questions now, please ask your teacher.

Good Luck!

Task 1

Sets of Numbers

Below you will see statements about numbers belonging to the sets $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$ and $\mathbb{C}$.

Task:

Put a cross next to each of the two true statements.

Irrational numbers can be written in the form $\frac{a}{b}$ with $a, b \in \mathbb{Z}$ and $b \neq 0$.	☐
Every rational number can be written as a terminating or recurring number in its decimal representation.	☐
Every fraction is a complex number.	☐
The set of rational numbers comprises solely positive fractions.	☐
Every real number is also a rational number.	☐

Task 2

The Solution Set of a Quadratic Equation

A quadratic equation of the form $x^2 + a \cdot x = 0$ in x with $a \in \mathbb{R}$ is given.

Task:

Determine the value of a for which the equation given above has the solution set $\left\{0, \frac{6}{7}\right\}$.

$a =$ ____

Task 3

Gas Supplier

A household would like to change its gas provider and is deciding between supplier *A* and supplier *B*.

The energy content of the gas used is measured in kilowatt hours (kWh).

Supplier *A* charges an annual fixed fee of 340 euros and then 2.9 cents per kWh.
Supplier *B* charges an annual fixed fee of 400 euros and then 2.5 cents per kWh.

The inequality $0.025 \cdot x + 400 < 0.029 \cdot x + 340$ can be used to compare the expected costs of each supplier.

Task:

Determine the solution to the inequality given above and interpret the result in the given context.

Task 4

Sales Figures

A specialist sports shop offers n different sports products. The n sports products are arranged in a database according to their product number so that the list of amounts of each product can be written as a vector (with n components).
The vectors B, C and P (with $B, C, P \in \mathbb{R}^n$) are defined as follows:

Vector B: The component $b_i \in \mathbb{N}$ (with $1 \leq i \leq n$) gives the stock level of the i^{th} product on Monday morning of a particular week.
Vector C: The component $c_i \in \mathbb{N}$ (with $1 \leq i \leq n$) gives the stock level of the i^{th} product on Saturday evening of the same week.
Vector P: The component $p_i \in \mathbb{R}$ (with $1 \leq i \leq n$) gives the price per item (in euros) of the i^{th} product in this week.

During the week under consideration, the shop is open from Monday to Saturday, and over the course of the week no stock is delivered nor are the prices of the products changed.

Task:

At the end of the week, the data for the week under consideration (Monday to Saturday) is evaluated. The necessary calculations can be written as expressions. Match each of the four amounts to the corresponding expression (from A to F) that can be used to calculate the amount.

the average sales figures (per product) per day in the week	
the total income resulting from sales of sports products in the week	
the sales figures (per product) in the week	
the value of the remaining stock of sports products at the end of the week	

A	$6 \cdot (B - C)$
B	$B - C$
C	$\frac{1}{6} \cdot (B - C)$
D	$P \cdot C$
E	$P \cdot (B - C)$
F	$6 \cdot P \cdot (B - C)$

Task 5

Line Parallel to the x-Axis

Let g be a line with vector equation $g\colon X = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + t \cdot \vec{a}$ with $t \in \mathbb{R}$.

Task:

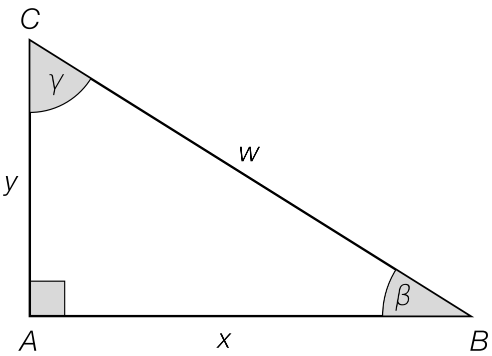
Write down a vector $\vec{a} \in \mathbb{R}^2$ with $\vec{a} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ such that the line g is parallel to the x-axis.

$\vec{a} =$ ____

Task 6

Right-Angled Triangle

The diagram below shows a right-angled triangle.



Task:

Write down an expression that can be used to determine the length of side w in terms of x and β.

$w =$ ____

Task 7

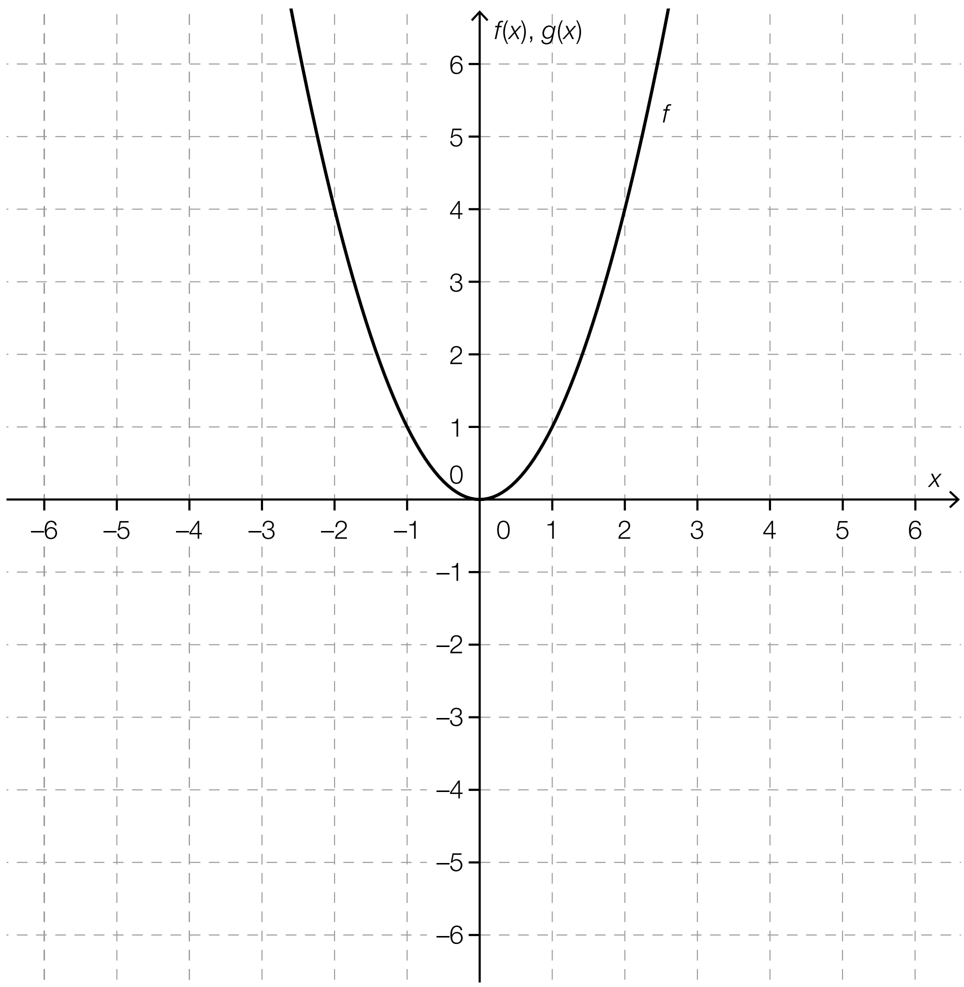
Solving a Quadratic Equation Graphically

The quadratic equation $x^2 + x - 2 = 0$ is given.

The equation shown above can be solved graphically using two functions f and g by considering the equation $f(x) = g(x)$.

Task:

The diagram below shows the graph of the quadratic function f where $f(x) \in \mathbb{Z}$ for each $x \in \mathbb{Z}$.
Draw the graph of the function g in the diagram below.



Task 8

Volume of a Cylinder

The volume of a cylinder can be given by a function V of the two quantities h and r. The height of the cylinder is given by h and the radius of the (circular) base is given by r.

Task:

Doubling the radius r and the height h of a cylinder, we obtain a cylinder whose volume will be x times as big as the volume of the original cylinder.
Determine x.

x = ____

Task 9

Linear Relationships

In certain cases, relationships expressed verbally can be represented by linear functions.

Task:

Which of the following relationships can be described by a linear function? Put a cross next to each of the two correct relationships.

The cost of apartments increases annually by 10 % of the current value.	☐
The area of a square piece of land increases as its side length increases.	☐
The circumference of a circle increases as its radius increases.	☐
The height of a 17 cm candle decreases after it has been lit by 8 mm per minute.	☐
In a culture of bacteria, the number of bacteria doubles per hour.	☐

Task 10

Properties of a Polynomial Function

Let $f\colon \mathbb{R} \to \mathbb{R}$ be a polynomial function with equation $f(x) = a \cdot x^3 + b \cdot x^2 + c \cdot x + d$ ($a, b, c, d \in \mathbb{R}$, $a \neq 0$).

Task:

Statements about the function f are given below.
Which of these statements is/are always true for arbitrary values of $a \neq 0$, b, c and d? Put a cross next to each correct statement.

The function f crosses the x-axis at least once.	☐
The function f has at most two local maxima or minima.	☐
The function f has at most two points in common with the x-axis.	☐
The function f has exactly one point of inflexion.	☐
The function f has at least one local maximum or minimum.	☐

Task 11

Exponential Function

For an exponential function f with $f(x) = 5 \cdot e^{\lambda \cdot x}$ the following equation holds: $f(x + 1) = 2 \cdot f(x)$.

Task:

Write down the value of λ.

λ = ____

Task 12

Half Life

The mass $m(t)$ of a radioactive substance can be written as an exponential function m in terms of the time t.
At the beginning of a measurement, there is 100 mg of the substance. After four hours only 75 mg of the substance remains.

Task:

Determine the half life t_H of this radioactive substance in hours.

Task 13

Water Level of a River

The function $W\colon [0, 24] \to \mathbb{R}_0^+$ assigns each time t to the water level $W(t)$ of a particular river at a certain measuring site. Here, t is measured in hours and $W(t)$ in metres.

Task:

Interpret the expression given below in the context of the water level $W(t)$ of the river.

$$\lim_{\Delta t \to 0} \frac{W(6 + \Delta t) - W(6)}{\Delta t}$$

Task 14

Average Rate of Change

The following table of values for a function *f* is given below:

x	$f(x)$
–3	42
–2	24
–1	10
0	0
1	–6
2	–8
3	–6
4	0
5	10
6	24

Task:

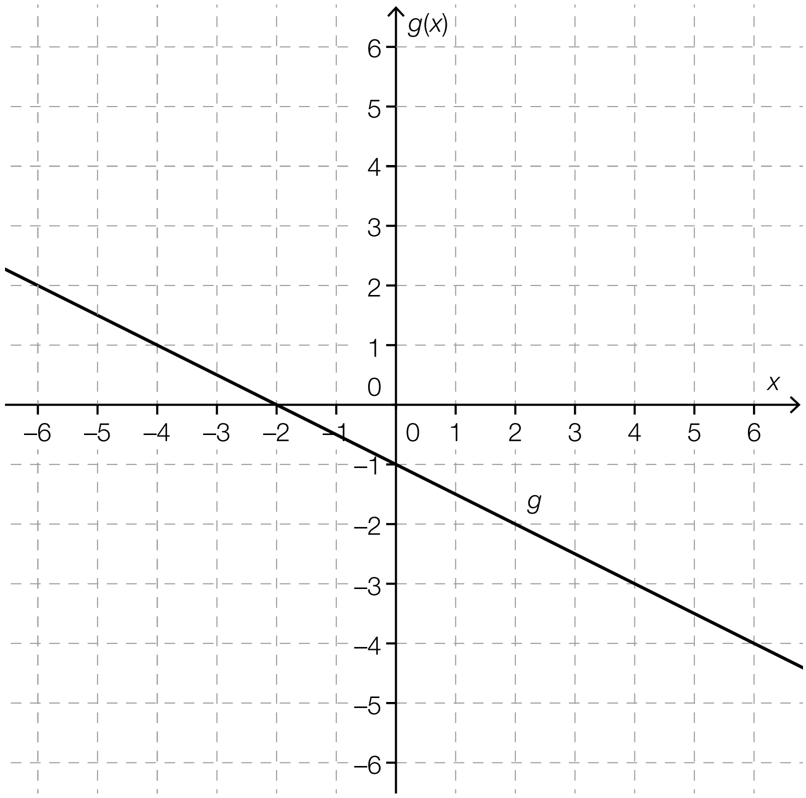
The average rate of change of the function *f* is zero in the interval $[-1, b]$ for precisely one $b \in \{0, 1, 2, 3, 4, 5, 6\}$.
Determine *b*.

b = ____

Task 15

Properties of an Antiderivative

The graph of a linear function g is shown in the diagram below.



Task:

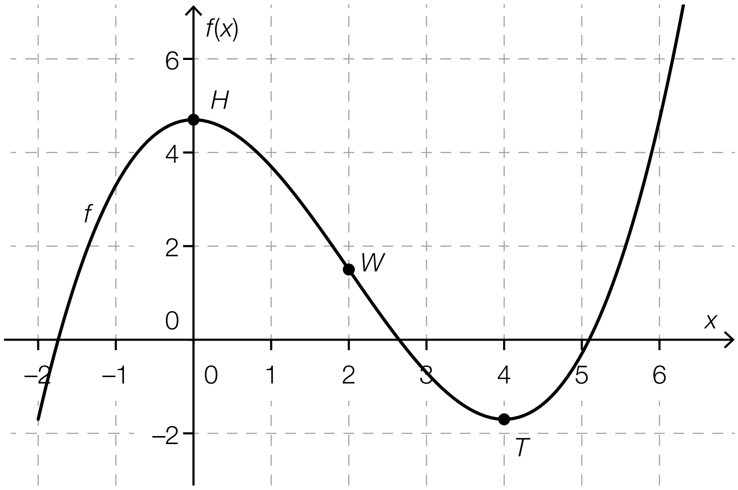
Put a cross next to each of the two statements that are true for the function g.

Every antiderivative of g is a second degree polynomial function.	☐
Every antiderivative of g has a local minimum when $x = -2$.	☐
Every antiderivative of g is strictly monotonically decreasing in the interval (0, 2).	☐
The function G with $G(x) = -0.5$ is an antiderivative of g.	☐
Every antiderivative of g has at least one zero.	☐

Task 16

Second Derivative

The graph of a third degree polynomial function f is shown below.



The points shown in bold are the maximum $H = (0, f(0))$, the point of inflexion $W = (2, f(2))$ and the minimum $T = (4, f(4))$ of the graph.

Task:

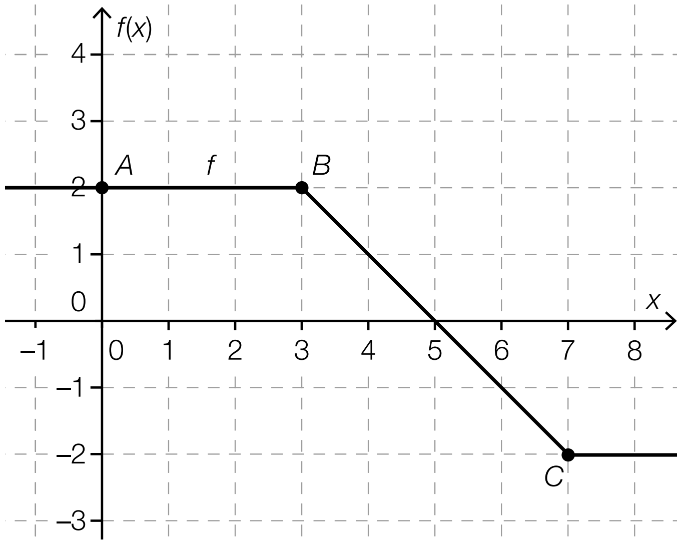
Five statements about the second derivative of f are given below.
Put a cross next to each of the two true statements.

For all x in the interval $[-1, 1]$, $f''(x) < 0$ holds.	☐
For all x in the interval $[1, 3]$, $f''(x) < 0$ holds.	☐
For all x in the interval $[3, 5]$, $f''(x) < 0$ holds.	☐
$f''(0) = f''(4)$	☐
$f''(2) = 0$	☐

Task 17

Definite Integral

The diagram below shows the graph of a piecewise linear function f. The points A, B and C of the graph of the function have integer coordinates.



Task:

Determine the value of the definite integral $\int_0^7 f(x)\,\mathrm{d}x$.

$\int_0^7 f(x)\,\mathrm{d}x =$ ______________________________

Task 18

Acceleration

The function a gives the acceleration of a moving object in terms of the time t in the time interval $[t_1, t_1 + 4]$. The acceleration $a(t)$ is given in m/s^2, and the time t is given in s.

It is known that:

$$\int_{t_1}^{t_1+4} a(t)\,\mathrm{d}t = 2$$

Task:

One of the statements shown below interprets the given definite integral correctly. Put a cross next to the correct statement.

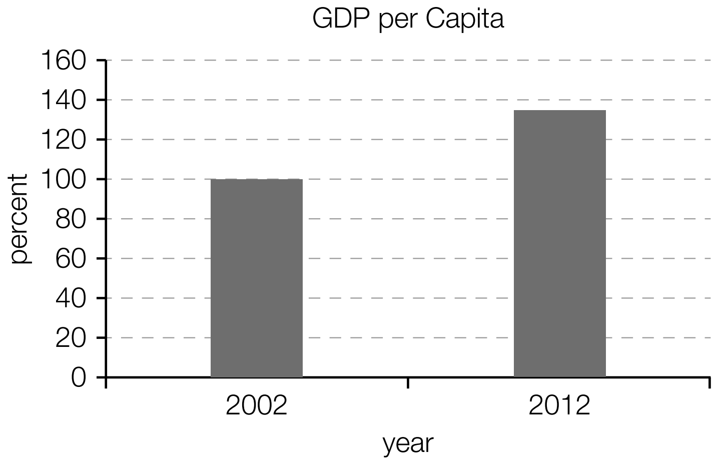
The object covers a distance of 2 m in the given time interval.	☐
The velocity of the object at the end of the given time interval is 2 m/s.	☐
The acceleration of the object at the end of the given time interval is 2 m/s^2 greater than it was at the beginning of the interval.	☐
The velocity of the object increased by 2 m/s in this time interval.	☐
On average, the velocity of the object increases by 2 m/s per second in the given time interval.	☐
In the given time interval, the acceleration of the object increases by $\frac{2}{4}$ m/s^2 per second.	☐

Task 19

Gross Domestic Product

The *nominal gross domestic product* (GDP) gives the total value of all goods produced within the borders of a country over a year at current market prices.
The *GDP per capita* is calculated by dividing the nominal gross domestic product of a country by the number of inhabitants.

The diagram below shows the relative change of the GDP per capita in Austria from 2012 in relation to 2002.



Task:

Write down whether the value of the relative change of the nominal gross domestic product in Austria from 2012 in relation to 2002 can be determined using solely the data given in the diagram shown above and justify your answer.

Task 20

Change in a List of Data

A list of data $x_1, x_2, \dots, x_n$ with n values has a mean of a. Two extra values, x_{n+1} and x_{n+2} are added to the list. The mean of the new list of data, $x_1, x_2, \dots, x_n, x_{n+1}, x_{n+2}$ is also a.

Task:

Write down a relationship between x_{n+1}, x_{n+2} and a as a formula that holds in this case.

Task 21

Red-Green Colour-Blindness

One of the most well-known vision defects is red-green colour-blindness. If a person is affected by this condition, then this vision defect is always present from birth and does not become more or less pronounced over time. Worldwide, around 9 % of all men and 0.8 % of all women have this condition. The proportion of women in the world population is 50.5 %.

Task:

Determine the probability that a randomly selected person is red-green colour-blind.

Task 22

Number of Possibilities

A team has n players. The trainer chooses six players from the team on one day and eight players from the team on another day. The order in which the players are chosen is not relevant. In both cases the number of possibilities for the choice is the same.

Task:

Determine n (the number of players in this team).

n = ____

Task 23

Binomial Distribution

The relative proportion of the Austrian population with the blood type "AB Rhesus Negative" (AB–) is known and is represented by p.
In a random sample of 100 people it is to be determined how many of these randomly selected people have this blood type.

Task:

Match each of the four events described below with the corresponding expression (from A to F) that gives the probability of this event occurring.

Exactly one person has the blood type AB–.	
At least one person has the blood type AB–.	
At most one person has the blood type AB–.	
None of the people has the blood type AB–.	

A	$1 - p^{100}$
B	$p \cdot (1 - p)^{99}$
C	$1 - (1 - p)^{100}$
D	$(1 - p)^{100}$
E	$p \cdot (1 - p)^{99} \cdot 100$
F	$(1 - p)^{100} + p \cdot (1 - p)^{99} \cdot 100$

Task 24

Reducing a Confidence Interval

A company that produces toys conducts a survey of 500 randomly selected households in a town and determines a 95 % confidence interval for the unknown proportion of all households in this town that have heard of the toys made by this company.

A second survey of n randomly selected households resulted in the same value for the relative frequency. Using the same method of calculation, the 95 % confidence interval determined from this survey was less wide than the interval from the first survey.

Task:

Write down all $n \in \mathbb{N}$ for which this case would occur under the specified condition.