Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

October 2018

Mathematics

Supplementary Examination 1
Examiner's Version

Bundesministerium
Bildung, Wissenschaft
und Forschung

Instructions for the supplementary examination

The following supplementary examination is comprised of five tasks that can be completed independently of one another.

Each task contains two parts: The statement of the task requires the candidate to demonstrate core competencies, and the guiding question that follows it requires the candidate to show their ability to communicate their ideas.

In the following document, the examiner will find the tasks as well as the expected solutions and the answer key.

The preparation time is to be at least 30 minutes and the examination time is to be at most 25 minutes.

Assessment

Each task can be awarded zero, one or two points. There is one point available for each demonstration of core competencies as well as for each guiding question. A maximum of 10 points can be achieved.

For the grading of the examination the following scale should be used:

Grade	Minimum number of points
Pass	4 points for the core competencies + 0 points for the guiding questions 3 points for the core competencies + 1 point for the guiding questions
Satisfactory	5 points for the core competencies + 0 points for the guiding questions 4 points for the core competencies + 1 point for the guiding questions 3 points for the core competencies + 2 points for the guiding questions
Good	5 points for the core competencies + 1 point for the guiding questions 4 points for the core competencies + 2 points for the guiding questions 3 points for the core competencies + 3 points for the guiding questions
Very good	5 points for the core competencies + 2 points for the guiding questions 4 points for the core competencies + 3 points for the guiding questions

The examination board will decide on the final grade based on the candidate's performance in the supplementary examination as well as the result of the written examination.

Evaluation grid for the supplementary examination

This evaluation grid may be used to assist the examiner in assessing the candidate’s performance.

	Point for core competencies reached	Point for the guiding question reached
Task 1		
Task 2		
Task 3		
Task 4		
Task 5		

Task 1

Price

The net price of a product is N euros. The gross price is the sum of the net price and m % Value Added Tax (calculated using the net price).
The sale price V is obtained by subtracting a discount of r % of the gross price from the gross price.

Task:

Write down a formula for the sale price V in terms of N, m and r.

V = ____

Guiding question:

Write down whether the same sale price V would be obtained if a discount of r % of the net price were subtracted from the net price first, before m % of the resulting price were added on to this result. Justify your decision.

Furthermore, write down the size of the discount that would have to be applied to a product that has 20 % Value Added Tax so that the net price were equal to the sale price.

Solution to Task 1

Price

Expected solution to the statement of the task:

Possible formula:

$$V = N \cdot \left(1 + \frac{m}{100}\right) \cdot \left(1 - \frac{r}{100}\right)$$

Answer key:

The point for the core competencies is to be given if a correct expression has been given. Equivalent terms are also to be considered correct.

Expected solution to the guiding question:

Possible justification:

The same sale price V is obtained if the discount is applied first and then the VAT is added due to the commutative rule of multiplication:

$$N \cdot \left(1 + \frac{m}{100}\right) \cdot \left(1 - \frac{r}{100}\right) = N \cdot \left(1 - \frac{r}{100}\right) \cdot \left(1 + \frac{m}{100}\right)$$

$$V = V \cdot \left(1 + \frac{20}{100}\right) \cdot \left(1 - \frac{r}{100}\right) \quad \Rightarrow \quad r = 16.\dot{6}$$

A discount of around 17 % has to be applied.

Answer key:

The point for the guiding question is to be awarded if the alternative calculation of V has been both given and explained correctly. Furthermore, the percentage has to have been given correctly.
Tolerance interval: [16 %, 17 %]

Task 2

Iodine-131

The isotope iodine-131 is radioactive.
After t days, the remaining amount $N(t)$ of iodine-131 decreases approximately exponentially. The amount of iodine-131 at time $t = 0$ is given by N_0.

Task:

After four days, 30 % of the original amount of iodine-131 has decayed. Determine the percentage of the amount of iodine-131 that decays per day and explain your method.

Guiding question:

Determine the general (without using concrete values) relative change of an exponential function of the form $N(t) = N_0 \cdot a^t$ in the time periods $[0, t_1]$ and $[t_0, t_0 + t_1]$ (where $t_0, t_1 \in \mathbb{R}^+$).

Interpret the results (drawing on a characteristic property of exponential functions).

Solution to Task 2

Iodine-131

Expected solution to the statement of the task:

After four days, 70 % of the original amount is still left.
$N(4) = 0.7 \cdot N_0 = N_0 \cdot a^4 \quad \Rightarrow \quad a \approx 0.915$

Thus, around 91.5 % of the original amount of isotope iodine-131 is left after one day.
Therefore, around 8.5 % of the original amount of iodine-131 decays per day.

Answer key:

The point for the core competencies is to be given if a correct percentage has been calculated and a correct method has been explained.
Tolerance interval: [8 %, 9 %]

Expected solution to the guiding question:

$$\frac{N(t_1) - N(0)}{N(0)} = \frac{N_0 \cdot a^{t_1} - N_0}{N_0} = a^{t_1} - 1$$

$$\frac{N(t_0 + t_1) - N(t_0)}{N(t_0)} = \frac{N_0 \cdot a^{t_0 + t_1} - N_0 \cdot a^{t_0}}{N_0 \cdot a^{t_0}} = a^{t_1} - 1$$

Possible interpretation:

The relative change of the value of the function is constant across time periods of equal length.

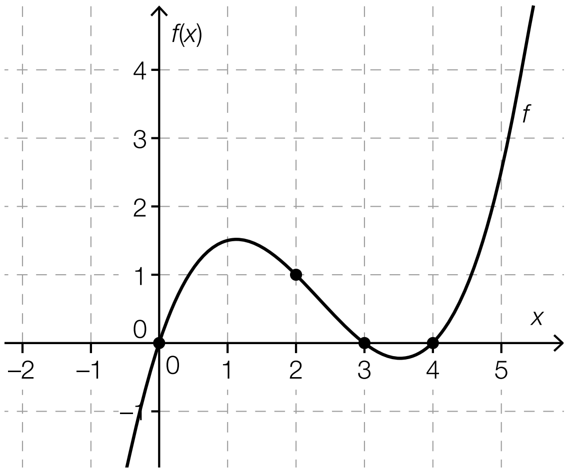
Answer key:

The point for the guiding question is to be awarded if the relative change for both intervals has been determined correctly and a correct interpretation has been given.

Task 3

Function and the Antiderivative

The diagram below shows a section of the graph of a third degree polynomial function, *f*. The points shown in bold have integer coordinates.



Task:

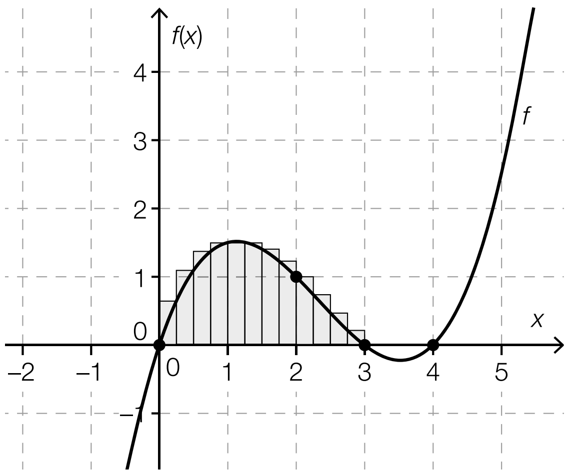
Write down the number and positions of any maxima, minima and points of inflexion of an antiderivative, *F*, of *f*.

Guiding question:

The graph of the function *f* encloses two finite areas bounded by the *x*-axis.

Write down two formulae, one using *f* and one using *F*, that could be used to calculate the total area of these finite areas.

In the diagram shown, the larger area has been divided into 12 rectangles of equal width. The sum of these rectangular areas (the upper sum) can be used to approximate the value of the area of the larger area.
Write down the value of the upper sum and determine by how much this value differs from the actual area of this region by using an adequate model function *f*.



Solution to Task 3

Function and the Antiderivative

Expected solution to the statement of the task:

The antiderivative F has three maxima/minima.
These are at the points where $x = 0$, $x = 3$ and $x = 4$.
The antiderivative F has two points of inflexion.
These are at the points where $x \approx 1.1$ and $x \approx 3.5$.

Answer key:

The point for the core competencies is to be given if the number and positions of the maxima, minima and points of inflexion of F have been given correctly.
Tolerance intervals for the points of inflexion: [1, 1.3] and [3.4, 3.7]

Expected solution to the guiding question:

Possible formulae:
$A = \int_0^3 f(x)\,\mathrm{d}x - \int_3^4 f(x)\,\mathrm{d}x$
$A = [F(3) - F(0)] - [F(4) - F(3)]$

$O_s \approx 3.17$
$f(x) = 0.25 \cdot x^3 - 1.75 \cdot x^2 + 3 \cdot x$
$A \approx 2.81$
$O_s - A \approx 0.36$

Answer key:

The point for the guiding question is to be awarded if correct formulae for both f and F for calculating the value of the area have been given and a correct value of the upper sum has been given, and the difference to the actual value has been given correctly.
Tolerance interval for the deviation: [0.3, 0.4]

Task 4

Vertical Throw

The height of a body thrown at time $t = 0$ can be described by a function h where $h(t) = 50 + 60 \cdot t - 5 \cdot t^2$ ($h(t)$ in metres, t in seconds).

Task:

Determine the equation of the first derivative of h and write down the meaning of this function in the context of the movement of the body using the correct units.

Determine the maximum $E = (t_1, h(t_1))$ of the function h and interpret the meaning of both coordinates, t_1 and $h(t_1)$, in the given context.

Guiding question:

Determine the average rate of change of the function h in the time period $[0, t_1]$ and interpret the result in the context of the movement of the body.

The Mean Value Theorem of differentiation says that for a function f, under certain conditions, in an interval $[a, b]$ there exists at least one point $x_0 \in (a, b)$ such that $f'(x_0) = \frac{f(b) - f(a)}{b - a}$ holds.

Interpret this statement for the function h in the given context for the time period $[0, t_1]$.

Determine the point t_0 as described in the Mean Value Theorem for the function h in the time period $[0, t_1]$.

Solution to Task 4

Vertical Throw

Expected solution to the statement of the task:

$h'(t) = 60 - 10 \cdot t$
Meaning: $h'(t)$ gives the instantaneous velocity of the body (at time t) in m/s.

$h'(t) = 0 \quad \Rightarrow \quad t_1 = 6,\ h(6) = 230 \quad \Rightarrow \quad E = (6{,}230)$
Meaning: After 6 seconds, the body has reached a (maximum) height of 230 metres.

Answer key:

The point for the core competencies is to be given if the derivative and its meaning (using the correct units) have been given correctly, the maximum point has been determined correctly and the meaning of both coordinates has been explained correctly.

Expected solution to the guiding question:

$\frac{h(6) - h(0)}{6 - 0} = \frac{230 - 50}{6} = 30$
Meaning: The average velocity (or speed) of the body in the time period [0 s, 6 s] is 30 m/s.

Possible interpretation:
In the time period [0, 6] there is at least one point in time at which the instantaneous velocity (or speed) of the body is equal to the average speed for the time period [0, 6].

Determining t_0:
$h'(t_0) = 30 \quad \Rightarrow \quad 60 - 10 \cdot t_0 = 30 \quad \Rightarrow \quad t_0 = 3$ s

Answer key:

The point for the guiding question is to be awarded if the average rate of change has been calculated correctly, a correct interpretation of the Mean Value Theorem has been given in the context, and the point t_0 has been determined correctly.

Task 5

Statistical Values

An ordered list of data with values $a_1, a_2, \dots, a_n$ ($n \in \mathbb{N}$, $n > 3$) is given.

Task:

For each of the statements given below, write down if it is true or false and in each case justify your decision.

Statement 1: The median is definitely a value that appears in the ordered list of data.

Statement 2: For the median m of the list of data, the formula $m = \frac{a_1 + a_n}{2}$ holds.

Statement 3: For the mean $\bar{x}$ of the list of data, the formula $n \cdot \bar{x} = a_1 + a_2 + \dots + a_n$ holds.

Guiding question:

Write down an ordered list of data that contains 11 values and satisfies the following conditions: The mean, the median and the range have the value 10.
Explain your reasoning.

Explain how you could add two values to your list of data such that neither the median nor the mean change, but the range increases by 6.

Solution to Task 5

Statistical Values

Expected solution to the statement of the task:

Statements 1 and 2 are false. This can be demonstrated by a counterexample:

$a_1 = 1 \quad a_2 = 2 \quad a_3 = 3 \quad a_4 = 6$

$m = 2.5$ and thus does not belong to the list of data.

$\frac{a_1 + a_4}{2} = 3.5$ and is thus not the median.

Statement 3 is true as by definition for the mean $\bar{x} = \frac{a_1 + a_2 + ... + a_n}{n}$ holds.

Answer key:

The point for the core competencies is to be given if for each of the three statements the decision about whether the statement is true has been made correctly and a correct justification has been given.

Expected solution to the guiding question:

Possible lists of data:

5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15
5, 5, 7, 8, 9, 10, 11, 12, 13, 15, 15
5, 6, 7, 8, 10, 10, 10, 12, 13, 14, 15
4, 5, 7, 8, 9, 10, 12, 13, 14, 14, 14

Possible method:

The middle (sixth) value of the ordered list has to be 10.
The difference between the largest and the smallest numbers has to be 10.
The sum of the 11 values has to be 110.

A value that is 3 smaller than the smallest value and a second value that is 3 larger than the largest value should be added.

Answer key:

The point for the guiding question is to be awarded if a correct ordered list of data with 11 values has been given, a correct method has been explained and the additional numbers have been correctly explained.