Name:	
Class:	

Standardised Competence-Oriented
Written School-Leaving Examination

AHS

15th January 2019

Mathematics

Part 2 Tasks

Bundesministerium
Bildung, Wissenschaft
und Forschung

Advice for Completing the Tasks

Dear candidate,

The following booklet for Part 2 contains four tasks, each of which contains between two and four sub-tasks. All sub-tasks can be completed independently of one another. You have *150 minutes* available in which to work on these tasks.

Please use a blue or black pen that cannot be rubbed out. You may use a pencil for tasks that require you to draw a graph, vectors or a geometric construction.

When completing these tasks please use this booklet and the paper provided. Write your name on each piece of paper you use as well as on the first page of this task booklet in the space provided. Please show clearly which sub-task each answer relates to.

In the assessment of your work, everything that is not crossed out will be considered. Your solutions must be clearly marked. If a solution is not clearly marked or if more than one solution is given, the task will be considered to be unsolved. Draw a line through any notes you make.

You may use the official formula booklet for this examination session as well as approved electronic device(s).

Please hand in both the task booklet and the separate sheets you have used at the end of the examination.

Assessment

Every task in Part 1 will be awarded either 0 points or 1 point. Every sub-task in Part 2 will be awarded 0, 1 or 2 points. The tasks marked with an $\boxed{\text{A}}$ will be awarded either 0 points or 1 point.

– If at least 16 of the 24 tasks in Part 1 are solved correctly, you will pass the examination.
– If fewer than 16 of the 24 tasks in Part 1 are solved correctly, then the tasks marked with an $\boxed{\text{A}}$ from Part 2 may compensate for the shortfall (as part of the "range of essential skills" outlined by the LVBO).
  If, including the tasks marked with an $\boxed{\text{A}}$ from Part 2, at least 16 tasks are solved correctly, you will pass the examination.
  If, including the tasks marked with an $\boxed{\text{A}}$ from Part 2, fewer than 16 tasks are solved correctly, you will not be awarded enough points to pass the examination.
– If at least 16 tasks are solved correctly (including the compensation tasks marked with an $\boxed{\text{A}}$ from Part 2), a grade will be awarded as follows:

Pass	16–23 points
Satisfactory	24–32 points
Good	33–40 points
Very Good	41–48 points

Explanation of the Task Types

Some tasks require a *free answer*. For these tasks, you should write your answer directly underneath each task in the task booklet or on the paper provided. Other task types used in the examination are as follows:

Matching tasks: For this task type you will be given a number of statements, tables or diagrams, which will appear alongside a selection of possible answers. To correctly answer these tasks, you will need to match each statement, table or diagram to its corresponding answer. You should write the letter of the correct answer next to the statement, table or diagram in the space provided.

Example:
You are given two equations.

Task:
Match the two equations to their corresponding description (from A to D).

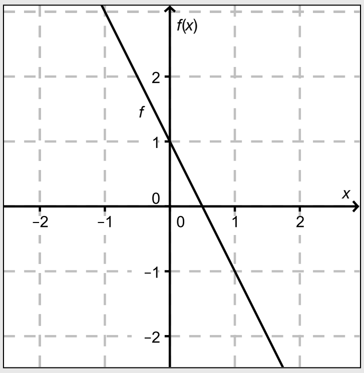
$1 + 1 = 2$	A
$2 \cdot 2 = 4$	C

A	Addition
B	Division
C	Multiplication
D	Subtraction

Construction tasks: This task type requires you to draw points, lines and/or curves in the task booklet.

Example:
Below you will see a linear function *f* where $f(x) = k \cdot x + d$.

Task:
On the axes provided below, draw the graph of a linear function for which $k = -2$ and $d > 0$.



Multiple-choice tasks of the form "1 out of 6": This task type consists of a question and six possible answers. Only **one answer** should be selected. You should put a cross next to the only correct answer in the space provided.

Example:
Which equation is correct?

Task:
Put a cross next to the correct equation.

1 + 1 = 1	☐
2 + 2 = 2	☐
3 + 3 = 3	☐
4 + 4 = 8	☒
5 + 5 = 5	☐
6 + 6 = 6	☐

Multiple-choice tasks of the form "2 out of 5": This task type consists of a question and five possible answers, of which **two answers** should be selected. You should put a cross next to each of the two correct answers in the space provided.

Example:
Which equations are correct?

Task:
Put a cross next to each of the two correct equations.

1 + 1 = 1	☐
2 + 2 = 4	☒
3 + 3 = 3	☐
4 + 4 = 8	☒
5 + 5 = 5	☐

Multiple-choice tasks of the form "x out of 5": This task type consists of a question and five possible answers, of which **one, two, three, four** *or* **five answers** may be selected. The task will require you to: "Put a cross next to each correct statement/equation …". You should put a cross next to each correct answer in the space provided.

Example:
Which of the equations given are correct?

Task:
Put a cross next to each correct equation.

1 + 1 = 2	☒
2 + 2 = 4	☒
3 + 3 = 6	☒
4 + 4 = 4	☐
5 + 5 = 10	☒

Gap-fill: This task type consists of a sentence with two gaps, i. e. two sections of the sentence are missing and must be completed. For each gap you will be given the choice of three possible answers. You should put a cross next to **each of the two answers** that are necessary to complete the sentence correctly.

Example:
Below you will see 3 equations.

Task:
Complete the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

The operation in equation ______①______ is known as summation or ______②______.

①	
1 – 1 = 0	☐
1 + 1 = 2	☒
1 · 1 = 1	☐

②	
Multiplication	☐
Subtraction	☐
Addition	☒

Changing an answer for a task that requires a cross:

1. Fill in the box that contains the cross for your original answer.
2. Put a cross in the box next to your new answer.

1 + 1 = 3	☐
2 + 2 = 4	☒
3 + 3 = 5	☐
4 + 4 = 4	☐
5 + 5 = 9	■

In this instance, the answer “5 + 5 = 9” was originally chosen. The answer was later changed to be “2 + 2 = 4”.

Selecting an answer that has been filled in:

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

1 + 1 = 3	☐
2 + 2 = 4	■
3 + 3 = 5	☐
4 + 4 = 4	■
5 + 5 = 9	☐

In this instance, the answer “2 + 2 = 4” was filled in and then selected again.

If you still have any questions, please ask your teacher.

Good Luck!

Task 1

Third Degree Polynomial Function

Let f_t be a third degree polynomial function with equation $f_t(x) = \frac{1}{t} \cdot x^3 - 2 \cdot x^2 + t \cdot x$. For the parameter t holds: $t \in \mathbb{R}$ and $t \neq 0$.

Task:

a) $\boxed{\text{A}}$ Find the local maxima and minima of f_t in terms of t.

When $x = t$, the equations $f_t(t) = 0$, $f_t'(t) = 0$ and $f_t''(t) = 2$ hold for the function f_t. Describe the shape of the graph of f_t at $x = t$.

b) Determine the point x_0 dependant of t, where the concavity of f_t changes.

Show by calculation that the concavity of the graph of f_t at $x = 0$ is independent of the value chosen for the parameter t.

c) The function A describes the area of the region bounded by the graph of f_t and the x-axis in terms of t in the interval $[0, t]$, where $t > 0$. The function A: $\mathbb{R}^+ \to \mathbb{R}_0^+$, $t \mapsto A(t)$, is a polynomial function.

Write down the equation and the degree of the function A.

Determine the ratio $A(t) : A(2 \cdot t)$.

d) Show by means of calculation that $f_{-1}(x) = f_1(-x)$ holds for all $x \in \mathbb{R}$.

Describe how the graph of the function f_{-1} can be developed from the graph of f_1.

Task 2

Capacitor

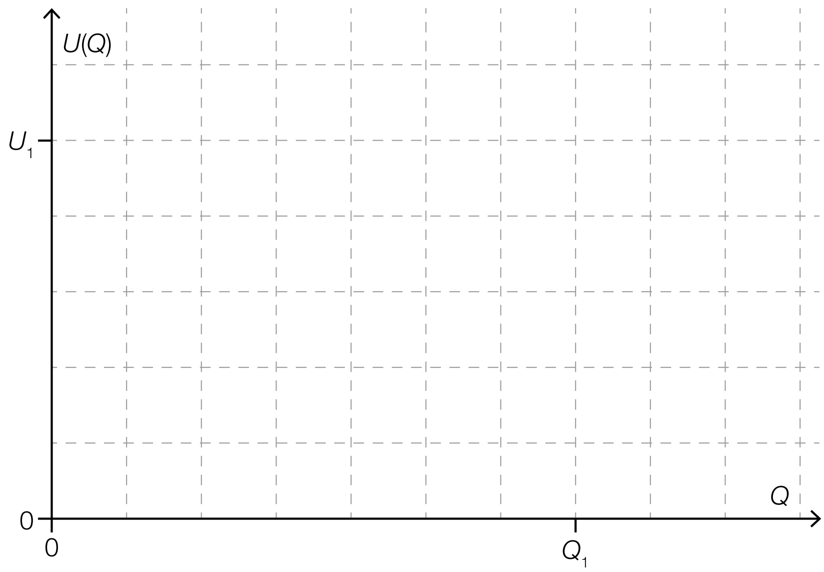
A capacitor is an electrical component that can store electric charge as well as the resulting electric energy.
The so-called *plate capacitor* is a simple version of a capacitor. It consists of two opposing electrically conductive plates, which are called *capacitor plates*.

The ratio between the stored charge Q and the applied (DC) voltage U is called the capacity C.

It is known that $C = \frac{Q}{U}$, where C is given in Farad.

Task:

a) When a capacitor with a particular capacity C is charged up to the charge Q_1, the voltage $U(Q_1)$ measured has the value U_1.
$\boxed{\text{A}}$ Sketch the voltage U with respect to the charge Q, while charging the capacitor.



The energy W that is stored inside the capacitor can be calculated using the formula $W = \int_0^{Q_1} U(Q)\,\mathrm{d}Q$.
Write down a formula for the energy W with respect to U_1 and C.

b) During the process of charging, the voltage U between the capacitor plates in terms of the time t can be described by the equation $U(t) = U^* \cdot \left(1 - e^{-\frac{t}{\tau}}\right)$. Here, $U^* > 0$ is the voltage that is applied to the capacitor and $\tau > 0$ is the characteristic constant of the charging process. The charging process starts at $t = 0$.

The time after which the voltage $U(t)$ between the capacitor plates reaches 99 % of the applied voltage U^* is known as the *charging time*.
Determine the charging time of a capacitor with respect to τ.

Write down a formula for the instantaneous rate of change of the voltage between the capacitor plates with respect to t and show on the basis of this formula that the charge is constantly increasing during the charging process.

Task 3

Wealth Distribution

The total wealth of a country is often distributed very unevenly across its population. A survey conducted in 2012 by the European Central Bank (ECB) provided data for an estimate of which proportion of the Austrian population owns which share of the wealth (in millions of euros). The results of the study based on this information are shown in Diagram 1. The 20 % threshold, for example, means that the most disadvantaged 20 % of the Austrian population in terms of wealth own assets of € 6,086 at most.
In the year 2012, Austria had a total population of around 8.45 million inhabitants.

The so-called *Lorenz curve L* (see Diagram 2) illustrates which relative proportion of the population owns which relative proportion of the total wealth. Thus, according to the ECB study, the most disadvantaged 80 % in terms of wealth own only around 23 % of the total wealth.

Diagram 1:

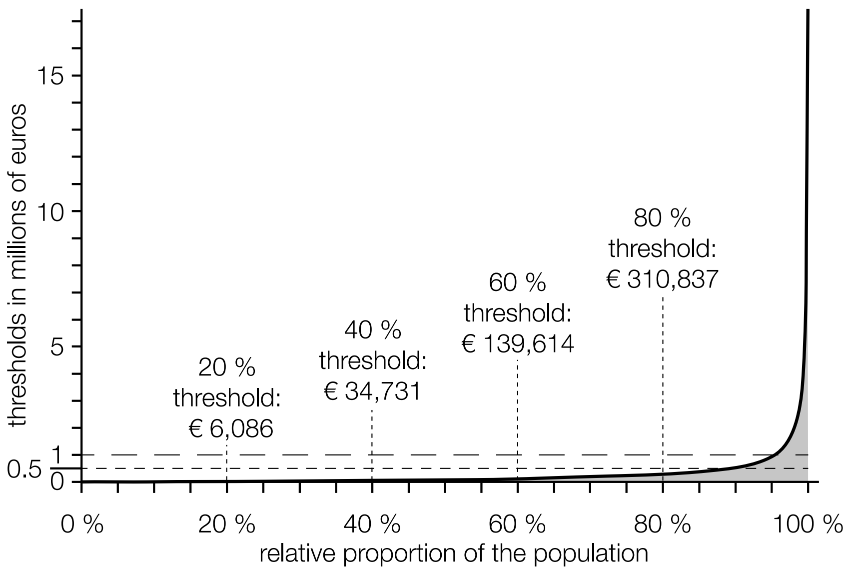
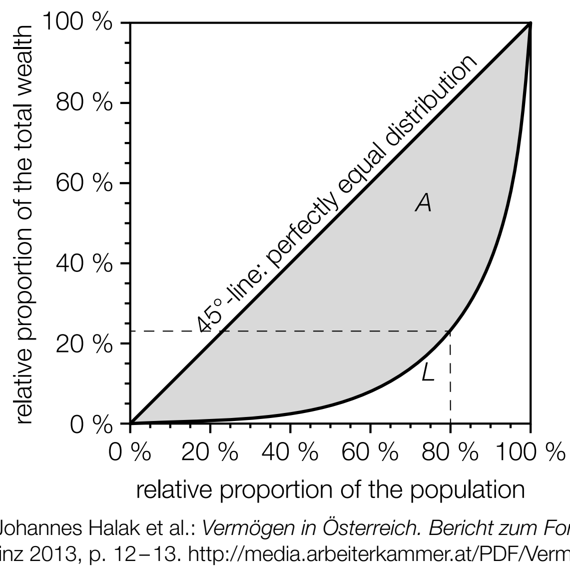


Diagram 2:



Source: Eckerstorfer, Paul, Johannes Halak et al.: *Vermögen in Österreich. Bericht zum Forschungsprojekt „Reichtum im Wandel".* Linz: Johannes-Kepler-Universität Linz 2013, p. 12–13. http://media.arbeiterkammer.at/PDF/Vermoegen_in_Oesterreich.pdf [17.10.2014] (adapted).

The Gini coefficient is a measure of the inequality of a country's distribution of wealth. It equals the quotient of the area of the shaded region A (between the 45°-line and the Lorenz curve L) and the area of the triangle defined by the given points (0 %,0 %), (100 %,0 %) and (100 %,100 %). According to the ECB study, Austria's Gini coefficient for the year 2012 was 0.76.

Task:

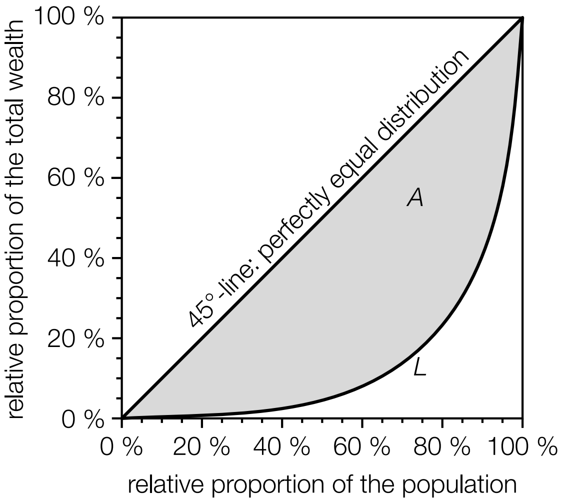
a) $\boxed{\text{A}}$ Using Diagram 1, determine how many Austrian people owned assets worth at least one million euros in 2012.

Under the simplified assumption that the thresholds in the interval [20 %, 40 %] can be expressed as a linear function, determine an approximation of the 25 % threshold.

b) Determine the relative proportion of wealth that is owned by the wealthiest 10 % of the Austrian population.

According to a study carried out by the University of Linz in 2013, the relative proportion of the total wealth owned by the wealthiest 10 % of the Austrian population was much higher than the ECB study claimed.

Considering the results of the study by the University of Linz, a different Lorenz curve L^* than the shown Lorenz curve L is obtained. Sketch one possible shape of a Lorenz curve L^* that shows this results in the diagram below.



c) The Lorenz curve in the interval [0, 1] can be modelled by a real function in terms of x, where x stands for the relative proportion of the population.

Determine the Gini coefficient for a country S, whose Lorenz curve for the year 2012 can be described by the function L_1 with equation $L_1(x) = 0.9 \cdot x^5 + 0.08 \cdot x^2 + 0.02 \cdot x$ in the interval [0, 1].

Compare your result with the Austrian Gini coefficient for the year 2012 and determine whether the total wealth in this year was more equally distributed among the population of Austria or country S.

Task 4

Election Forecast

There are different mathematical methods used to forecast how voters behave in upcoming elections. One popular method is to collect and analyse data from a sample. Another method is to calculate so-called *regression lines* which enable a relatively precise forecast. To determine such regression lines, the results of a so-called *comparable election* are used, ideally of one that has taken place shortly before the election.

4 150 eligible voters of a particular village, consisting of five constituencies, could decide between two candidates, A and B, in a mayoral election. All voters cast their vote and there were no invalid votes. After having counted all the votes from four of the five constituencies, the following interim results were obtained:

Table 1: Mayoral election

	1st Constituency	2nd Constituency	3rd Constituency	4th Constituency	5th Constituency
Candidate A	443	400	462	343	not counted
Candidate B	332	499	466	227	not counted
Eligible Voters	775	899	928	570	978

The relative proportion of votes for candidate A in the first four constituencies for the mayoral election is denoted by h.

Task:

a) $\boxed{\text{A}}$ Determine how many votes for candidate A can be expected in the 5th Constituency, if h is taken as the estimate for the relative proportion of votes for this candidate in this particular constituency.

In the 4th Constituency, the result for candidate A deviates most strongly from h.
Write down how much these values differ in terms of percentage points.

b) The following table shows the results of a comparable election.

Table 2: Comparable election

	1st Constituency	2nd Constituency	3rd Constituency	4th Constituency	5th Constituency	Total
Candidate *A*	390	416	409	383	478	2 076
Candidate *B*	385	483	519	187	500	2 074
Eligible Voters	775	899	928	570	978	4 150

Let x be the number of votes for candidate A in the comparable election and y be the number of votes for candidate A in the mayoral election. Thus, the results of candidate A from the 1st, 2nd, 3rd and 4th Constituency define four points in a Cartesian coordinate system.
The regression line g: $y = 1.5462 \cdot x - 205.71$ goes through this “point cloud” in a way that a linear relationship between the two variables x and y is well-described.

Using this regression line g, determine the expected number of votes for candidate A in the mayoral election in the 5th Constituency.

Interpret the gradient of the regression line g in the given context.

c) In an Austrian-wide election, a third candidate C can be chosen. Based on an earlier election, it is known that the proportion of votes h for candidate A in the constituencies 1 to 4 of the mayoral election is representative for the proportion of votes for candidate C in the Austrian-wide election.
Using the proportion of votes h, determine a symmetrical 95 % confidence interval for the unknown proportion of votes for candidate C.

After all votes have been counted, candidate C received 61 % of all the votes in the Austrian-wide election. Thus, this proportion of votes lies outside the symmetrical 95 % confidence interval determined above.

If a confidence level of 90 % had been chosen, the width of the confidence interval obtained would have been different.
State whether the actual proportion of votes for candidate C would be included in this confidence interval and justify your decision.