Name:	
Class:	

Standardised Competence-Oriented
Written School-Leaving Examination

AHS

15th January 2019

Mathematics

Part 1 Tasks

Bundesministerium
Bildung, Wissenschaft
und Forschung

Advice for Completing the Tasks

Dear candidate,

The following booklet for Part 1 contains 24 tasks. The tasks can be completed independently of one another. You have *120 minutes* available in which to work through this booklet.

Please use a blue or black pen that cannot be rubbed out. You may use a pencil for tasks that require you to draw a graph, vectors or a geometric construction.

Please do all of your working out solely in this booklet. Write your name on the first page of the booklet in the space provided.

All answers must be written in this booklet. In the assessment of your work, everything that is not crossed out will be considered. Your solutions must be clearly marked. If a solution is not clearly marked or if more than one solution is given, the task will be considered to be unsolved. Draw a line through any notes you make.

You may use the official formula booklet for this examination session as well as approved electronic device(s).

Please hand in the task booklet at the end of the examination.

Assessment

Every task in Part 1 will be awarded either 0 points or 1 point. Every sub-task in Part 2 will be awarded 0, 1 or 2 points. The tasks marked with an $\boxed{\text{A}}$ will be awarded either 0 points or 1 point.

- If at least 16 of the 24 tasks in Part 1 are solved correctly, you will pass the examination.
- If fewer than 16 of the 24 tasks in Part 1 are solved correctly, then the tasks marked with an $\boxed{\text{A}}$ from Part 2 may compensate for the shortfall (as part of the "range of essential skills" outlined by the LVBO).
  If, including the tasks marked with an $\boxed{\text{A}}$ from Part 2, at least 16 tasks are solved correctly, you will pass the examination.
  If, including the tasks marked with an $\boxed{\text{A}}$ from Part 2, fewer than 16 tasks are solved correctly, you will not be awarded enough points to pass the examination.
- If at least 16 tasks are solved correctly (including the compensation tasks marked with an $\boxed{\text{A}}$ from Part 2), a grade will be awarded as follows:

Pass	16–23 points
Satisfactory	24–32 points
Good	33–40 points
Very Good	41–48 points

Explanation of the Task Types

Some tasks require a *free answer*. For these tasks, you should write your answer directly underneath each task in the task booklet. Other task types used in the examination are as follows:

Matching tasks: For this task type you will be given a number of statements, tables or diagrams, which will appear alongside a selection of possible answers. To correctly answer these tasks, you will need to match each statement, table or diagram to its corresponding answer. You should write **the letter of the correct answer** next to the statement, table or diagram in the space provided.

Example:
You are given two equations.

Task:
Match the two equations to their corresponding description (from A to D).

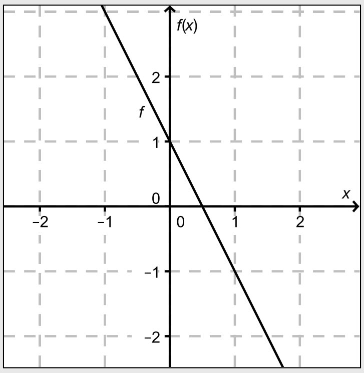
$1 + 1 = 2$	A
$2 \cdot 2 = 4$	C

A	Addition
B	Division
C	Multiplication
D	Subtraction

Construction tasks: This task type requires you to draw points, lines and/or curves in the task booklet.

Example:
Below you will see a linear function f where $f(x) = k \cdot x + d$.

Task:
On the axes provided below, draw the graph of a linear function for which $k = -2$ and $d > 0$.



Multiple-choice tasks of the form "1 out of 6": This task type consists of a question and six possible answers. Only **one answer** should be selected. You should put a cross next to the only correct answer in the space provided.

Example:
Which equation is correct?

Task:
Put a cross next to the correct equation.

1 + 1 = 1	☐
2 + 2 = 2	☐
3 + 3 = 3	☐
4 + 4 = 8	☒
5 + 5 = 5	☐
6 + 6 = 6	☐

Multiple-choice tasks of the form "2 out of 5": This task type consists of a question and five possible answers, of which **two answers** should be selected. You should put a cross next to each of the two correct answers in the space provided.

Example:
Which equations are correct?

Task:
Put a cross next to each of the two correct equations.

1 + 1 = 1	☐
2 + 2 = 4	☒
3 + 3 = 3	☐
4 + 4 = 8	☒
5 + 5 = 5	☐

Multiple-choice tasks of the form "x out of 5": This task type consists of a question and five possible answers, of which **one, two, three, four** *or* **five answers** may be selected. The task will require you to: "Put a cross next to each correct statement/equation …". You should put a cross next to each correct answer in the space provided.

Example:
Which of the equations given are correct?

Task:
Put a cross next to each correct equation.

1 + 1 = 2	☒
2 + 2 = 4	☒
3 + 3 = 6	☒
4 + 4 = 4	☐
5 + 5 = 10	☒

Gap-fill: This task type consists of a sentence with two gaps, i. e. two sections of the sentence are missing and must be completed. For each gap you will be given the choice of three possible answers. You should put a cross next to **each of the two answers** that are necessary to complete the sentence correctly.

Example:
Below you will see 3 equations.

Task:
Complete the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

The operation in equation ______①______ is known as summation or ______②______.

①	
1 – 1 = 0	☐
1 + 1 = 2	☒
1 · 1 = 1	☐

②	
Multiplication	☐
Subtraction	☐
Addition	☒

Changing an answer for a task that requires a cross:

1. Fill in the box that contains the cross for your original answer.
2. Put a cross in the box next to your new answer.

1 + 1 = 3	☐
2 + 2 = 4	☒
3 + 3 = 5	☐
4 + 4 = 4	☐
5 + 5 = 9	■

In this instance, the answer “5 + 5 = 9” was originally chosen. The answer was later changed to be “2 + 2 = 4”.

Selecting an answer that has been filled in:

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

1 + 1 = 3	☐
2 + 2 = 4	■
3 + 3 = 5	☐
4 + 4 = 4	■
5 + 5 = 9	☐

In this instance, the answer “2 + 2 = 4” was filled in and then selected again.

If you still have any questions now, please ask your teacher.

Good Luck!

Task 1

Numbers and Sets of Numbers

Below you will see statements about numbers and sets of numbers.

Task:

Put a cross next to each of the two true statements.

There exists at least one number which is contained in $\mathbb{N}$, but not in $\mathbb{Z}$.	☐
$-\sqrt{9}$ is an irrational number.	☐
The number 3 is an element of the set $\mathbb{Q}$.	☐
$\sqrt{-2}$ is contained in $\mathbb{C}$, but not in $\mathbb{R}$.	☐
The recurring number $1.\dot{5}$ is contained in $\mathbb{R}$, but not in $\mathbb{Q}$.	☐

Task 2

Representing Relationships as Equations

Many relationships can be expressed mathematically as equations.

Task:

Match each of the four descriptions of a possible relationship between two numbers a and b, where $a, b \in \mathbb{R}^+$, with the corresponding equation (from A to F).

a is half as big as *b*.	
b is 2 % of *a*.	
a is 2 % bigger than *b*.	
b is 2 % smaller than *a*.	

A	$2 \cdot a = b$
B	$2 \cdot b = a$
C	$a = 1.02 \cdot b$
D	$b = 0.02 \cdot a$
E	$1.2 \cdot b = a$
F	$b = 0.98 \cdot a$

Task 3

System of Equations

Below you will see a system of two linear equations involving the variables $x, y \in \mathbb{R}$.

$$\begin{aligned} \text{I: } & a \cdot x + \phantom{b \cdot{}} y = -2 \quad \text{where } a \in \mathbb{R} \\ \text{II: } & 3 \cdot x + b \cdot y = 6 \quad \text{where } b \in \mathbb{R} \end{aligned}$$

Task:

Determine the coefficients a and b such that the system of equations has infinitely many solutions.

a = ____

b = ____

Task 4

Parallel Lines

Let g and h be lines with vector equations $g: X = P + t \cdot \vec{u}$ and $h: X = Q + s \cdot \vec{v}$ where $s, t \in \mathbb{R}$ and $\vec{u}, \vec{v} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

Task:

Assuming that the two lines are parallel to each other but not identical, which of the following statements are definitely true?
Put a cross next to each of the two true statements.

$P = Q$	☐
$P \in h$	☐
$Q \notin g$	☐
$\vec{u} \cdot \vec{v} = 0$	☐
$\vec{u} = a \cdot \vec{v}$ where $a \in \mathbb{R} \backslash \{0\}$	☐

Task 5

Relationship between Vectors

Let $\vec{a}$ and $\vec{b}$ be two vectors with $\vec{a} = \begin{pmatrix} 13 \\ 5 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 10 \cdot m \\ n \end{pmatrix}$ where $m, n \in \mathbb{R}\backslash\{0\}$.

Task:

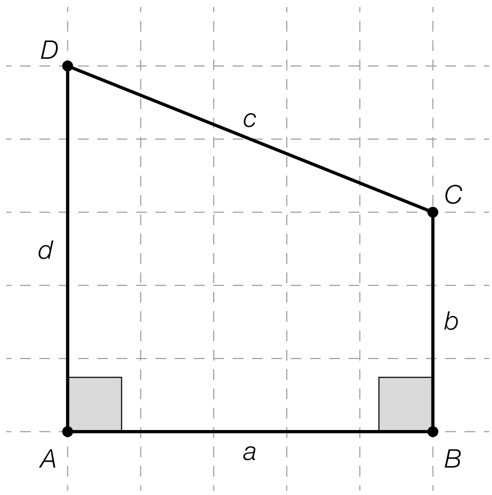
The vectors $\vec{a}$ and $\vec{b}$ will be perpendicular to each other. Write down an expression for n in terms of m that holds in this case.

$n =$ ____________

Task 6

Quadrilateral

Given is the following quadrilateral with side lengths a, b, c and d.



Task:

Draw the angle φ on the diagram above such that $\sin(\varphi) = \frac{d-b}{c}$ holds.

Task 7

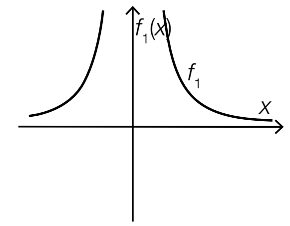
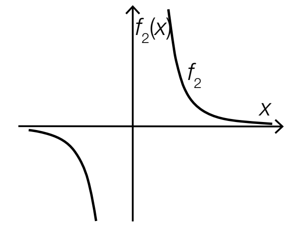
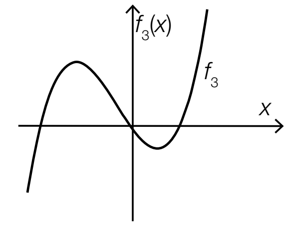
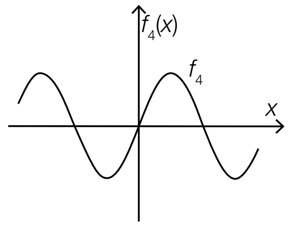
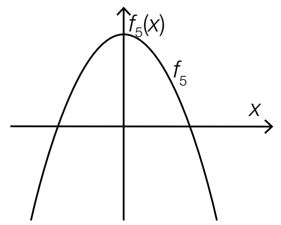
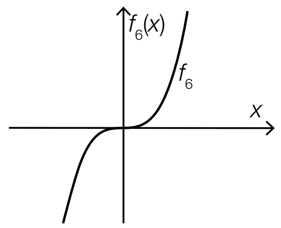
Properties of Graphs of Functions

Properties of functions and characteristic sections of graphs are shown below.

Task:

Match each of the four properties to the corresponding graph (from A to F).

The function is monotonically increasing on its whole domain.	
The function is concave down on its whole domain.	
The function is concave up on the interval $(-\infty, 0)$.	
The function is monotonically decreasing on the interval $(-\infty, 0)$.	

A	
B	
C	
D	
E	
F	

Task 8

Costs and Revenue

The cost function K of a product with $K(x) = 2 \cdot x + 4\,000$ and the revenue function E with $E(x) = 10 \cdot x$ are known. Here, x is the number of units of quantity produced and all units that are produced are sold as well. Costs and revenue are both given in euros.
The point of intersection of the two graphs is $S = (500 | 5\,000)$.

Task:

Interpret the coordinates 500 and 5 000 of the point of intersection S in the given context.

Task 9

Interpreting an Equation

A balloon filled with helium ascends vertically. The balloon's height above a flat surface can be expressed as a linear function h in terms of the time t. The height $h(t)$ is measured in meters, the time t is measured in seconds.

Task:

Interpret the equation $h(t + 1) - h(t) = 2$ in the given context using the correct units.

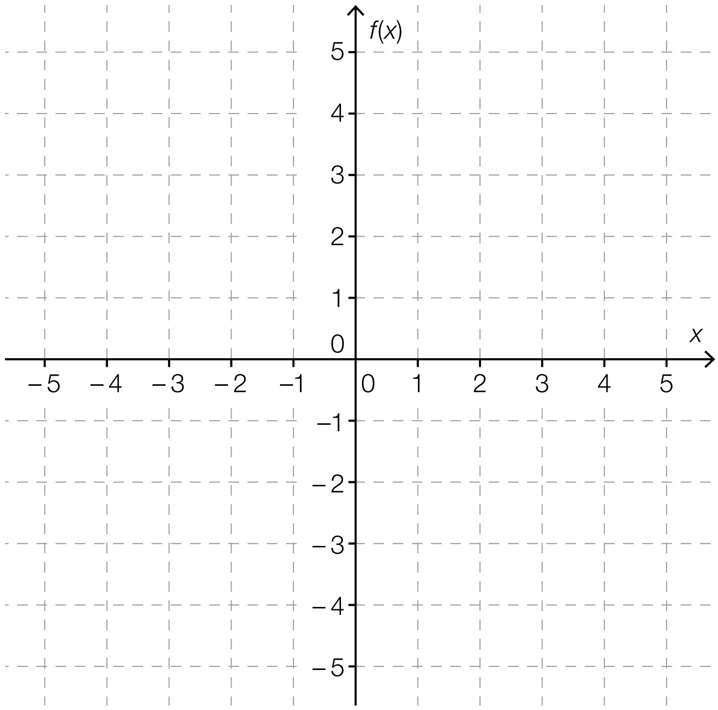
Task 10

Third Degree Polynomial Functions

A third degree polynomial function can change its monotonicity at up to two points.

Task:

In the coordinate system given below, sketch the graph of a third degree polynomial function f that changes its monotonicity when $x = -3$ and $x = 1$.



Task 11

Thickness of a Lead Plate

X-rays are used in the field of medical technology. To protect people against radiation, lead plates are installed. It is assumed that per 1 mm thickness of the lead plate, the intensity of the X-rays decreases by 5 %.

Task:

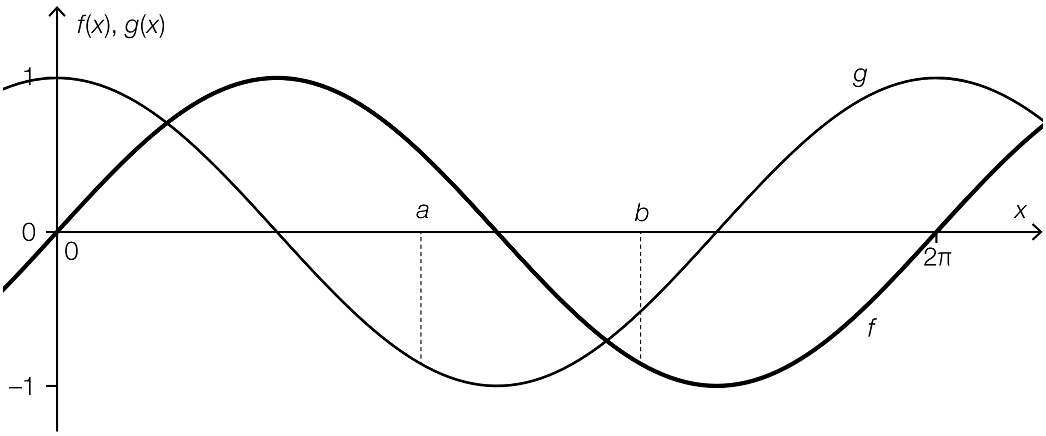
Calculate the thickness x (in mm) of the lead plate necessary to decrease the intensity of radiation to 10 % of the original intensity, with which the rays hit the lead plate.

Task 12

Trigonometric Functions

The diagram below shows the graphs of the functions f and g with equations $f(x) = \sin(x)$ and $g(x) = \cos(x)$.

For the points a and b, shown in the diagram, the following equation holds: $\cos(a) = \sin(b)$.



Task:

Determine $k \in \mathbb{R}$ such that $b - a = k \cdot \pi$ holds.

Task 13

Overnight Stays in Austrian Youth Hostels

The value N_{12} represents the number of overnight stays in Austrian youth hostels in the year 2012, whereas the value N_{13} represents the number of overnight stays in the year 2013.

Task:

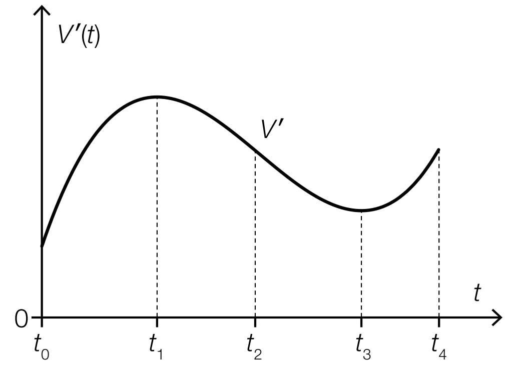
Write down the meaning of the equation $\frac{N_{13}}{N_{12}} = 1.012$ with respect to the change of the number of overnight stays in Austrian youth hostels.

Task 14

Change of the Volume of a Liquid

The volume V of a liquid in a container changes over time t in the time interval $[t_0, t_4]$.

The diagram below shows the graph of the function V', which gives the instantaneous rate of change of the volume of the liquid in the container in the given time interval.



Task:

Put a cross next to each of the two true statements.

The volume of the liquid in the container decreases in the time interval $[t_1, t_3]$.	☐
The volume of the liquid in the container is smaller at time t_2 than at time t_3.	☐
The instantaneous rate of change of the volume of the liquid in the container is lowest at time t_3.	☐
The volume of the liquid in the container is greatest at time t_4.	☐
The volume of the liquid in the container at time t_2 is the same as at time t_4.	☐

Task 15

Relationship between Function and Antiderivatives

The functions g and h are different antiderivatives of a polynomial function f of degree $n \geq 1$.

Task:

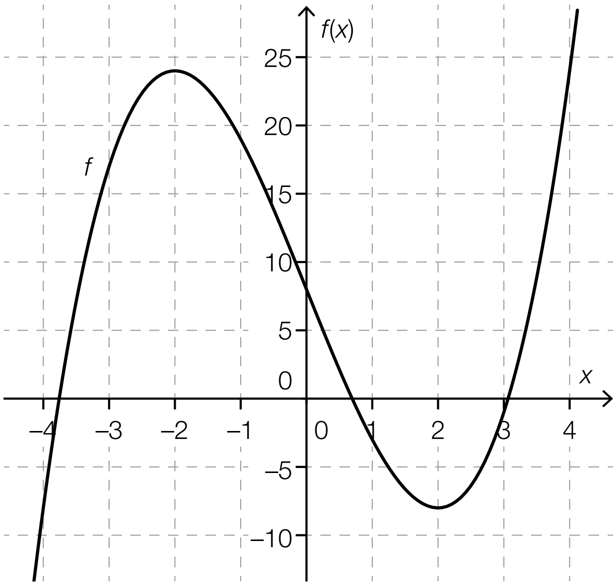
Put a cross next to each of the two true statements.

$g'(x) = h'(x)$	☐
$g(x) + h(x) = c,\ c \in \mathbb{R}\setminus\{0\}$	☐
$\int_0^2 g(x)\,\mathrm{d}x = f(2) - f(0)$	☐
$\int_0^2 f(x)\,\mathrm{d}x = h(2) - h(0)$	☐
$g(x) = c \cdot h(x),\ c \in \mathbb{R}\setminus\{1\}$	☐

Task 16

Properties of a Third Degree Polynomial Function

The graph of a third degree polynomial function f is shown below. The points $x = -2$ and $x = 2$ are local maxima or minima of f.



Task:

Put a cross next to each of the two true statements.

$f'(0) = 0$	☐
$f''(1) > 0$	☐
$f'(-3) < 0$	☐
$f'(2) = 0$	☐
$f''(-2) > 0$	☐

Task 17

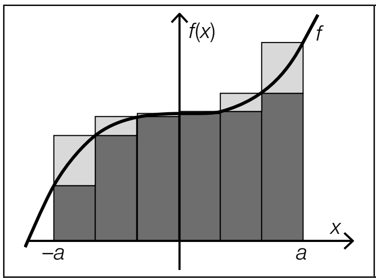
Lower Sum and Upper Sum

The diagrams below each show the graph of a function *f*, as well as one lower sum *U* (= sum of the areas of the rectangles marked in dark grey with the same width) and one upper sum *O* (= sum of the areas of the rectangles marked in light and dark grey with the same width) in the interval [–*a*, *a*].

Task:

For two of the functions shown below, the condition $\int_{-a}^{a} f(x)\,\mathrm{d}x = \frac{O+U}{2}$ holds true for constant rectangle width in the interval [–*a*, *a*].

Put a cross next to each of the two diagrams for which the given condition is fulfilled.



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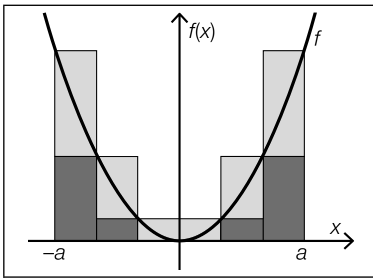

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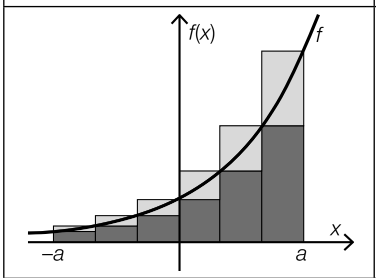

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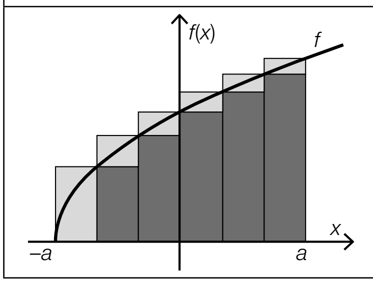

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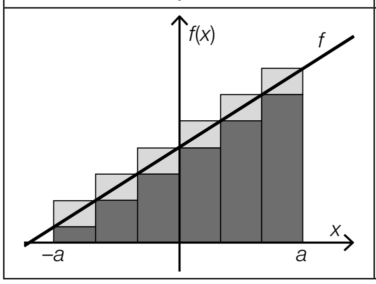
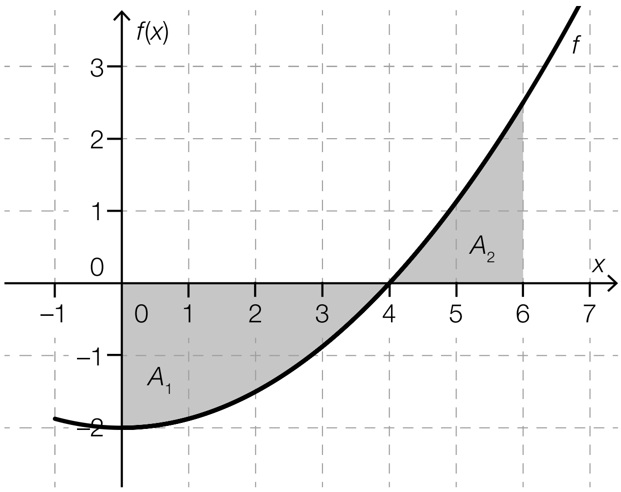


☐

Task 18

Value of a Definite Integral

The diagram below shows the graph of a function $f\colon \mathbb{R} \to \mathbb{R}$. In addition, two areas have been shaded.
The region A_1 is enclosed by the graph of the function f and the x-axis in the interval $[0, 4]$ and has an area of $\frac{16}{3}$ units of area.
The region A_2 is enclosed by the graph of the function f and the x-axis in the interval $[4, 6]$ and has an area of $\frac{7}{3}$ units of area.



Task:

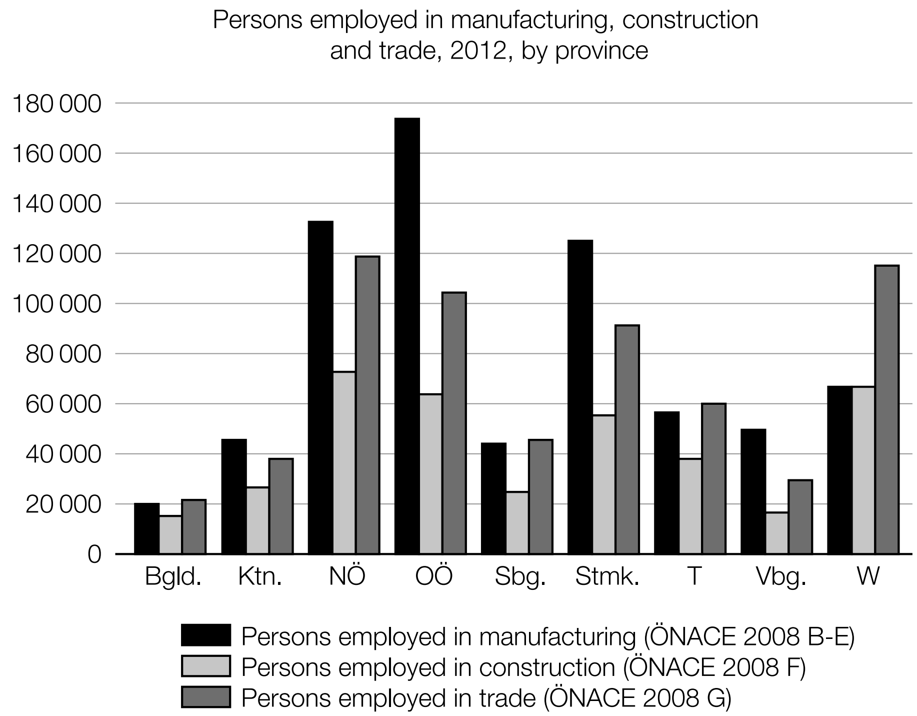
Write down the value of the definite integral $\int_0^6 f(x)\,\mathrm{d}x$.

$\int_0^6 f(x)\,\mathrm{d}x =$ ____________

Task 19

Persons in Employment

The diagram below shows the number of persons in employment in Austria in 2012 across three sectors. The diagram shows the data broken down by Austrian province.



Source: STATISTIK AUSTRIA, Mikrozensus-Arbeitskräfteerhebung 2012. Created on 22.05.2013.

Task:

Which of the following statements about the year 2012 can be deduced from the diagram? Put a cross next to each correct statement.

In every province, there were more persons employed in trade than in construction.	☐
There were more persons employed in manufacturing in Upper Austria (OÖ) than in any other province.	☐
In Vienna (W), there were more persons employed in trade than in manufacturing and construction combined.	☐
In Vorarlberg (Vbg.), there were fewer persons employed in all three sectors combined than in manufacturing in Styria (Stmk.)	☐
There were fewer persons employed in trade in Burgenland (Bgld.) than in any other province.	☐

Task 20

Median Class Size

The following information on class sizes was collected from 24 classes from the lower cycle of a secondary school, i.e. the first four years of secondary school.

class size	20	21	22	23	24	25	26	27	28
number of classes	1	2	1	2	3	2	4	6	3

Task:

Determine the median value of the class sizes in the lower cycle of this secondary school.

Task 21

Gaming Chips

Inside two boxes, there are gaming chips.
Inside box I there are five 2-euro chips and two 1-euro chips.
Inside box II there are four 2-euro chips and five 1-euro chips.
One chip is drawn independently from each of the boxes. For each box, the probability to be drawn is the same for each chip.

Task:

Determine the probability that the same amount of money is left in each box after both chips have been drawn.

Task 22

Computer Chips

A company produces computer chips. Independent of each other, the probability that any of the computer chips produced is fully functioning is 97 %.
On one particular day, the company produces 500 computer chips.

Task:

Determine the expectation value and the standard deviation for the number of fully functioning computer chips that are produced on this day.

Expectation value: ____

Standard deviation: ____

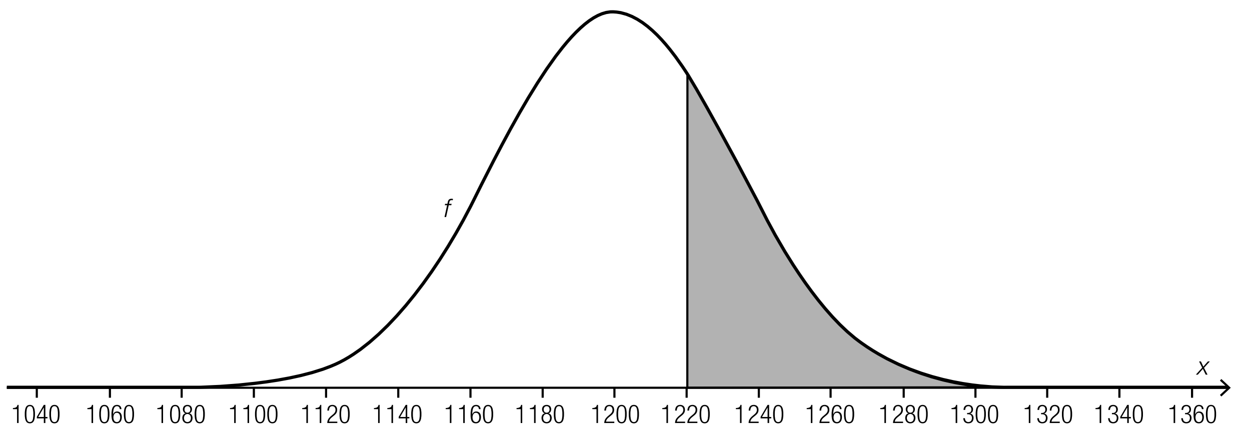
Task 23

Returnable Bottles

The rate of return of the returnable bottles of a specific brand of mineral water is 92 %.

During one month, 15 000 returnable bottles of this brand of mineral water are sold. The random variable *X* gives the number of returnable bottles that are not returned. The random variable *X* can be approximated by a normal distribution.

The following diagram shows the graph of the density function *f* of this normal distribution. The area of the shaded section is around 0.27.



Task:

Interpret the meaning of the value 0.27 in the given context.

Task 24

Telephone Survey

In a representative telephone survey among 400 randomly selected people the value 20 % is obtained for the relative frequency of supporters of shorter summer holidays.

Task:

Demonstrate by means of calculation that the interval [16.0 %, 24.0 %] can be a symmetrical 95 % confidence interval for the relative frequency *p* of supporters in the whole population (where the values for the interval boundaries of the confidence interval are rounded).