Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

January 2019

Mathematics

Supplementary Examination 1
Candidate's Version

■ Bundesministerium
Bildung, Wissenschaft
und Forschung

Instructions for the supplementary examination

Dear candidate,

The following supplementary examination is comprised of five tasks that can be completed independently of one another.

Each task contains two parts: The statement of the task requires you to demonstrate core competencies, and the guiding question that follows it requires you to demonstrate your ability to communicate your ideas.

You will be given preparation time of at least 30 minutes, and the examination will last at the most 25 minutes.

Assessment

Each task can be awarded zero, one or two points. There is one point available for each demonstration of core competencies as well as for each guiding question. A maximum of 10 points can be achieved.

For the grading of the examination the following scale will be used:

Grade	Minimum number of points
Pass	4 points for the core competencies + 0 points for the guiding questions 3 points for the core competencies + 1 point for the guiding questions
Satisfactory	5 points for the core competencies + 0 points for the guiding questions 4 points for the core competencies + 1 point for the guiding questions 3 points for the core competencies + 2 points for the guiding questions
Good	5 points for the core competencies + 1 point for the guiding questions 4 points for the core competencies + 2 points for the guiding questions 3 points for the core competencies + 3 points for the guiding questions
Very good	5 points for the core competencies + 2 points for the guiding questions 4 points for the core competencies + 3 points for the guiding questions

The examination board will decide on the final grade based on your performance in the supplementary examination as well as the result of the written examination.

Good Luck!

Task 1

Three Vectors in $\mathbb{R}^3$

Below, you will see three vectors.

$$\vec{a} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 2 \\ 1 \\ b_z \end{pmatrix}, \quad \vec{c} = \begin{pmatrix} -3 \\ c_y \\ 5 \end{pmatrix}$$

Task:

Determine the components b_z and c_y so that the vectors $\vec{b}$ and $\vec{c}$ are both perpendicular to $\vec{a}$.

Show that the vectors $\vec{b}$ and $\vec{c}$ are also perpendicular to one another for the components you have calculated. Show your method.

Guiding question:

For each of the lines g, h and i, determine a vector equation such that each of the conditions below is fulfilled.

I: The line g has the vector $\vec{a}$ as its direction vector and goes through the origin.

II: The line h has the vector $\vec{b}$ as its direction vector and crosses the line g at exactly one point.

III: The line i is parallel to the line h and skew to the line g (i.e. it does not cross the line g).

Explain your method and show that i is skew to g.

Task 2

Functions

In the coordinate system below, three graphs of functions of the form $x \mapsto a \cdot x^2 + b$ are shown. The points marked in bold have integer coordinates.
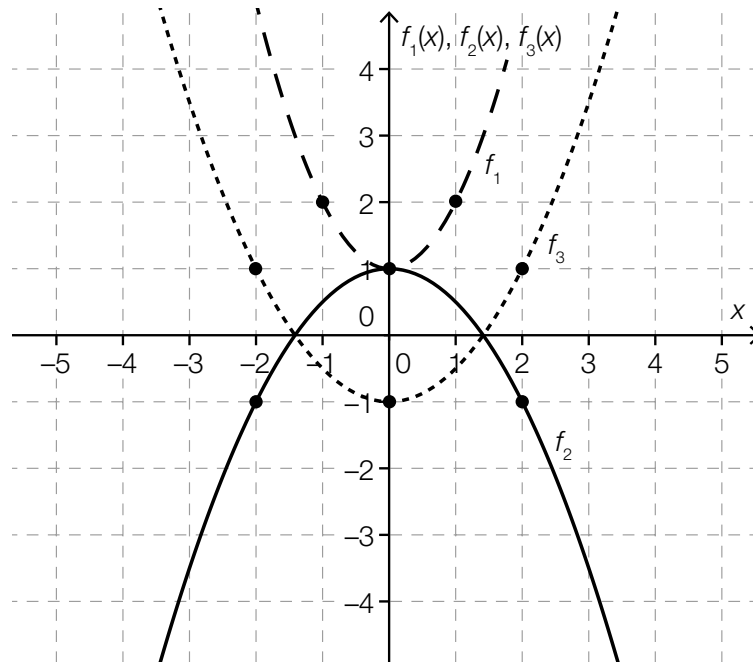
Task:

Determine an equation of the function f_2.

Guiding question:

Explain the influence of the parameters a and b on the behaviour of the graph of a general function f where $f(x) = a \cdot x^2 + b$ and $a \neq 0$.

By comparing the parameters of the three functions f_1, f_2 and f_3, show how your explanation applies to concrete examples.

Task 3

Wild Pigs

According to a newspaper article, the population of wild pigs in Bavaria in the year 2013 increased sharply, even though so many wild pigs had never been shot before. In the hunting season 2012/13, 66 000 wild pigs were shot. In the hunting season 2011/12, only 42 300 wild pigs had been shot.

Task:

Determine the absolute and relative increase of wild pig shootings in Bavaria from the hunting season of 2011/12 to that of 2012/13.

Guiding question:

State the type of functional relationship between time and the number of wild pig shootings that describes a constant annual rate of increase in shootings and corresponds to the relative change in shooting numbers in Bavaria calculated above.

Determine an equation for the function W that describes the number of wild pig shootings in Bavaria as a function of time, t (measured in years), where $W(0)$ gives the number of shootings in the 2012/13 season.

Using this equation, determine the number of wild pig shootings in the hunting season 2022/23. Estimate whether or not it is realistic for the number of wild pig shootings to develop in accordance with this function over a long period of time.

Task 4

Derivatives and Antiderivatives

The diagram below shows a section of a graph of a fourth degree polynomial function, f, with the points of inflexion W_1 and W_2.
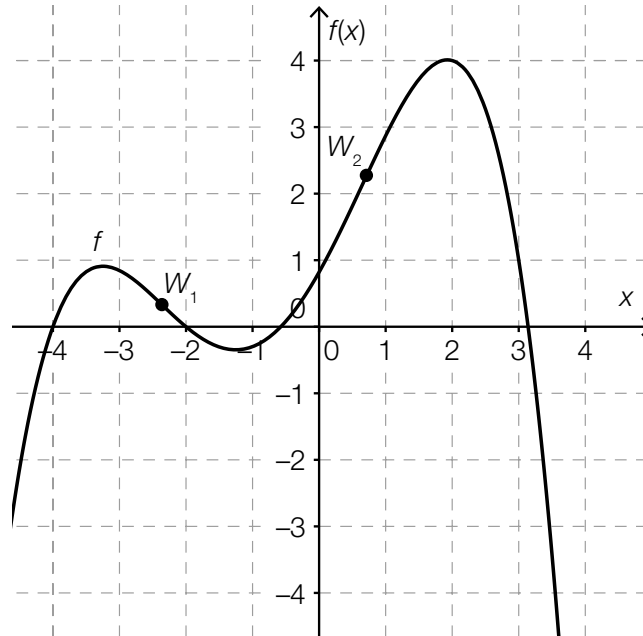
Task:

Determine whether the following statements are true or false. For each statement, justify your answer.

Statement 1: For all $x \in [-1, 1]$, $f'(x) > 0$.

Statement 2: There exists an $x \in [0, 1]$ for which $f'(x) = 0$.

Statement 3: For all $x \in [-4, -2]$, $f''(x) < 0$.

Statement 4: There exists an $x \in [1, 3]$ for which $f''(x) = 0$.

Guiding question:

Determine the intervals in the range $[-4, 3]$ for which an antiderivative of f is strictly monotonically increasing and explain your answer.

Task 5

Medication

According to the information from a pharmaceutical company, only 2 % of people who consume a particular medication experience mild side effects.

The medication is consumed by 50 people.

As a simplification, it should be assumed in the following that the number of people who experience mild side effects is binomially distributed.

Task:

Determine how many people can be expected to experience mild side effects.

Determine the probability that more than two people experience mild side effects.

Guiding question:

Determine the lowest number n ($n \in \mathbb{N}$) of people that would have to take the medication such that there is a probability of at least 90 % that at least one person experiences mild side effects. Explain your method.