Name:	
Class:	

Standardised Competence-Oriented
Written School-Leaving Examination

AHS

20th September 2019

Mathematics

Part 1 and Part 2 Tasks

Bundesministerium
Bildung, Wissenschaft
und Forschung

Advice for Completing the Tasks

Dear candidate!

The following booklet contains Part 1 tasks and Part 2 tasks (divided into sub-tasks). The tasks can be completed independently of one another. You have a total of *270 minutes* available in which to work through this booklet.

Please do all of your working out solely in this booklet and the paper provided to you. Write your name and that of your class on the cover page of the booklet in the spaces provided. Also, write your name and consecutive page numbers on each sheet of paper used. When answering each sub-task, indicate its name/number on your sheet.

In the assessment of your work, everything that is not crossed out will be considered. Your solutions must be marked clearly. If a solution is not clearly marked or if more than one solution is given, the task will be considered to be unsolved.

You may use the official formula booklet for this examination session as well as approved electronic device(s), provided there is no possibility to communicate via internet, Bluetooth, mobile networks, etc. and there is no access to your own data stored on the device.

An explanation of the task types is available in the examination room and can be viewed on request.

Please hand in the task booklet and all used sheets at the end of the examination.

Changing an answer for a task that requires a cross:

1. Fill in the box that contains the cross.
2. Put a cross in the box next to your new answer.

In this instance, the answer “5 + 5 = 9” was originally chosen. The answer was later changed to be “2 + 2 = 4”.

1 + 1 = 3	☐
2 + 2 = 4	☒
3 + 3 = 5	☐
4 + 4 = 4	☐
5 + 5 = 9	■

Selecting an item that has been filled in:

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

In this instance, the answer “2 + 2 = 4” was filled in and then selected again.

1 + 1 = 3	☐
2 + 2 = 4	■
3 + 3 = 5	☐
4 + 4 = 4	■
5 + 5 = 9	☐

Assessment

The tasks in Part 1 will be awarded either 0 points or 1 point or 0, ½ or 1 point, respectively. The points that can be reached in each task are listed in the booklet for all Part 1 tasks. Every sub-task in Part 2 will be awarded 0, 1 or 2 points. The tasks marked with an $\boxed{\text{A}}$ will be awarded either 0 points or 1 point.

Two assessment options

1) If you have reached **at least 16** of the 28 points (24 Part 1 points + 4 $\boxed{\text{A}}$ points from Part 2), a grade will be awarded as follows:

Pass	16–23.5 points
Satisfactory	24–32.5 points
Good	33–40.5 points
Very Good	41–48 points

2) If you have reached **fewer than 16** of the 28 points (24 Part 1 points + 4 $\boxed{\text{A}}$ points from Part 2), but have reached a **total of 24 points or more** (from Part 1 and Part 2 tasks), then a “Pass” or “Satisfactory” grade is possible as follows:

Pass	24–28.5 points
Satisfactory	29–35.5 points

From 36 points upward, the assessment key specified in 1) applies.

If you have reached fewer than 16 points in Part 1 (including the compensation tasks marked with an $\boxed{\text{A}}$ from Part 2) and if the total is less than 24 points, you will not pass the examination.

Good luck!

Task 1

Sets of Numbers

Certain relationships hold between sets of numbers.

Task:

Put a cross next to each of the two correct statements.

$\mathbb{Z}^+ \subseteq \mathbb{N}$	☐
$\mathbb{C} \subseteq \mathbb{Z}$	☐
$\mathbb{N} \subseteq \mathbb{R}^-$	☐
$\mathbb{R}^+ \subseteq \mathbb{Q}$	☐
$\mathbb{Q} \subseteq \mathbb{C}$	☐

[0/1 point]

Task 2

Simultaneous Linear Equations

Below, you will see a pair of simultaneous linear equations in the variables x_1 and x_2. For the parameters a and b: $a, b \in \mathbb{R}$ holds.

$$\begin{array}{ll} \text{I:} & 3 \cdot x_1 - 4 \cdot x_2 = a \\ \text{II:} & b \cdot x_1 + \phantom{4 \cdot{}} x_2 = a \end{array}$$

Task:

Determine the values of the parameters a and b such that the solution to the pair of simultaneous linear equations is $L = \{(2,-2)\}$.

a = ____

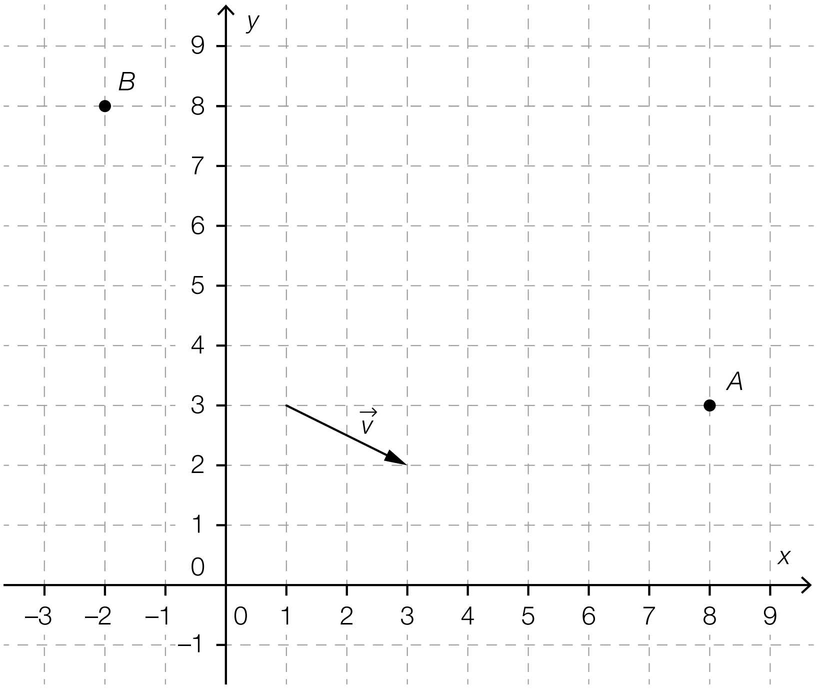
b = ____

[0/1 point]

Task 3

Representation in a Coordinate System

The coordinate system below shows the vector $\vec{v}$ as well as the points A and B. The vector $\vec{v}$ has integer components and both of the points A and B have integer coordinates.



Task:

Determine the value of the parameter t such that the equation $B = A + t \cdot \vec{v}$ is satisfied.

$t =$ ______________________________________

[0/1 point]

Task 4

Equation of a Line

Let $A = (7,6)$, $M = (-1,7)$ and $N = (8,1)$ be points. A line g goes through the point A and is perpendicular to the line connecting the points M and N.

Task:

Write down an equation of the line g.

[0/1 point]

Task 5

Cone

A cone has a height of 6 cm. The angle between the axis of the cone and its curved surface is 32°.

Task:

Determine the radius r of the base of the cone.

$r \approx$ ______________ cm

[0/1 point]

Task 6

Angle with the Same Sine

Let c be a real number where $0 < c < 1$. For the two distinct angles α and β, the following relationship holds: $\sin(\alpha) = \sin(\beta) = c$.
The angle α is an acute angle, and the angle β lies in the interval (0°, 360°).

Task:

Which relationship holds between the angles α and β?
Put a cross next to the correct relationship.

$\alpha + \beta = 90°$	☐
$\alpha + \beta = 180°$	☐
$\alpha + \beta = 270°$	☐
$\alpha + \beta = 360°$	☐
$\beta - \alpha = 270°$	☐
$\beta - \alpha = 180°$	☐

[0/1 point]

Task 7

Quadratic Function

Let $f: \mathbb{R} \to \mathbb{R}$ be a quadratic function where $f(x) = a \cdot x^2 + b \cdot x + c$ ($a, b, c \in \mathbb{R}$ and $a \neq 0$).

Task:

Complete the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

When ___①___ holds, the function f definitely has ___②___.

①	
$a < 0$	☐
$b = 0$	☐
$c > 0$	☐

②	
a graph that is symmetrical about the vertical axis	☐
two real zeros	☐
a local minimum	☐

[0/1 point]

Task 8

Oscillation of a String

The frequency f of the oscillation of a string on a musical instrument can be calculated using the following formula.

$$f = \frac{1}{2 \cdot l} \cdot \sqrt{\frac{F}{\varrho \cdot A}}$$

l … length of the string
A … cross-sectional area of the string
ϱ … density of the material of the string
F … force with which the string is held taut

Task:

Write down how the length l of the string should be changed if the string is to oscillate at double its original frequency and the other values (F, ϱ, A) remain constant.

[0/1 point]

Task 9

Height of a Candle

A burning candle that was lit t hours ago has a height $h(t)$. The height of the candle can be approximated by $h(t) = a \cdot t + b$ where $a, b \in \mathbb{R}$.

Task:

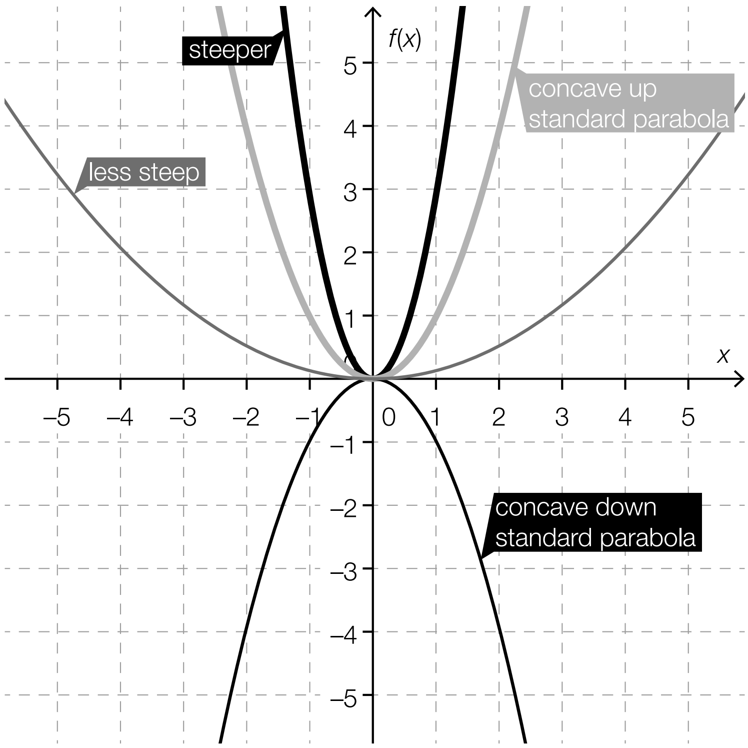
Write down whether each of the coefficients a and b must be positive, negative or exactly zero.

[0/1 point]

Task 10

Parabolas

The graphs of the functions $f: \mathbb{R} \to \mathbb{R}$ where $f(x) = a \cdot x^2$ and $a \in \mathbb{R}\setminus\{0\}$ are parabolas.
For $a = 1$ the graph is often known as the *standard parabola*. Depending on the value of the parameter a, the parabolas obtained are "steeper" or "less steep" than the standard parabola and either "concave down" or "concave up".



Task:

Four parabolas are described below. Match each of the four descriptions to the condition (from A to F) that the parameter a must satisfy.

In comparison to the standard parabola, the parabola is "less steep" and "concave up".	
In comparison to the standard parabola, the parabola is neither "steeper" nor "less steep" but "concave down".	
In comparison to the standard parabola, the parabola is "steeper" and "concave down".	
In comparison to the standard parabola, the parabola is "steeper" and "concave up".	

A	$a < -1$
B	$a = -1$
C	$-1 < a < 0$
D	$0 < a < 1$
E	$a = 1$
F	$a > 1$

[0/½/1 point]

Task 11

Function with a Particular Property

For a non-constant function $f: \mathbb{R} \to \mathbb{R}$ the relationship $f(x + 1) = 3 \cdot f(x)$ holds for all $x \in \mathbb{R}$.

Task:

Write down an equation of one such function f.

$f(x) =$ ____

[0/1 point]

Task 12

Length of a Period

Let $f\colon \mathbb{R} \to \mathbb{R}$ be a function where $f(x) = \frac{1}{3} \cdot \sin\left(\frac{3 \cdot \pi}{4} \cdot x\right)$.

Task:

Determine the length of the (smallest) period p of the function f.

$p =$ ____

[0/1 point]

Task 13

Difference Quotient

The graph of a function f goes through the points $P = (-1,2)$ and $Q = (3,f(3))$.

Task:

Determine the value of $f(3)$ such that the difference quotient of f in the interval $[-1, 3]$ has the value 1.

$f(3) =$ ____

[0/1 point]

Task 14

Derivative and Antiderivative

Let $f\colon \mathbb{R} \to \mathbb{R}$ be a polynomial function.

Task:

Two of the following statements about f are definitely true.
Put a cross next to each of the two correct statements.

The function f has exactly one antiderivative F.	☐
The function f has exactly one derivative f'.	☐
If F is an antiderivative of f, then $f' = F$ holds.	☐
If F is an antiderivative of f, then $F'' = f'$ holds.	☐
If F is an antiderivative of f, then $\int_0^1 F(x)\,\mathrm{d}x = f(1) - f(0)$ holds.	☐

[0/1 point]

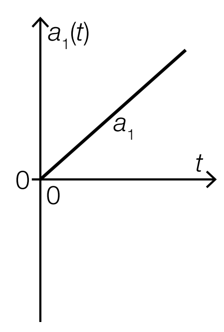
Task 15

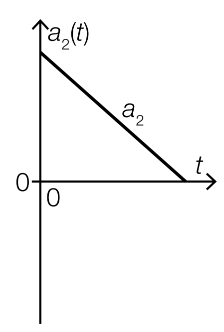
Velocity and Acceleration

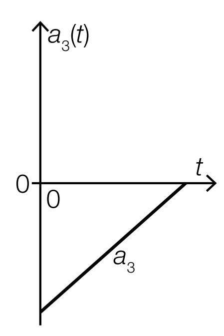
The diagrams below show the graphs of four acceleration functions (a_1, a_2, a_3, a_4) and six velocity functions (v_1, v_2, v_3, v_4, v_5, v_6) in terms of time t.

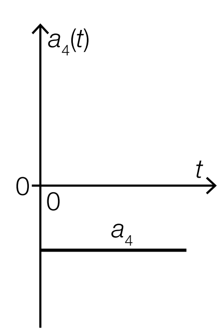
Task:

Match each of the graphs from a_1 to a_4 to the corresponding graph from v_1 to v_6 (from A to F).

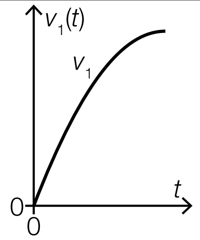








A



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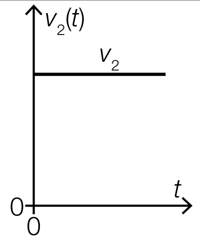

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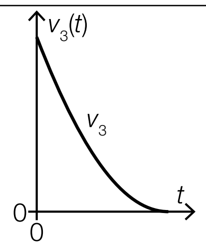

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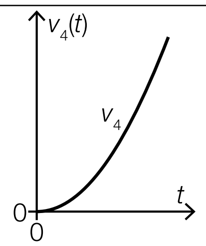

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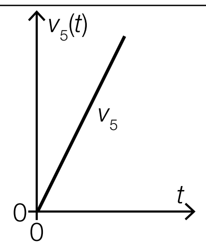
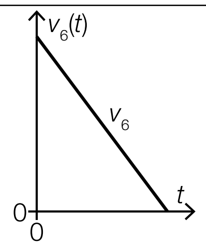


F



[0/½/1 point]

Task 16

Properties of a Third Degree Polynomial Function

Let f be a third degree polynomial function. At the points x_1 and x_2 where $x_1 < x_2$, the following conditions hold:

$f'(x_1) = 0$ and $f''(x_1) < 0$
$f'(x_2) = 0$ and $f''(x_2) > 0$

Task:

Put a cross next to each of the two statements that are definitely true for the function f.

$f(x_1) > f(x_2)$	☐
There exists one further point x_3 where $f'(x_3) = 0$.	☐
In the interval $[x_1, x_2]$ there exists a point x_3 where $f(x_3) > f(x_1)$.	☐
In the interval $[x_1, x_2]$ there exists a point x_3 where $f''(x_3) = 0$.	☐
In the interval $[x_1, x_2]$ there exists a point x_3 where $f'(x_3) > 0$.	☐

[0/1 point]

Task 17

Determining a Coefficient

Let $f\colon \mathbb{R} \to \mathbb{R}$ be a function where $f(x) = a \cdot x^2 + 2$ with $a \in \mathbb{R}$.

Task:

Write down the value of the coefficient a such that the equation $\int_0^1 f(x)\,\mathrm{d}x = 1$ is satisfied.

$a =$ ____

[0/1 point]

Task 18

Height of an Object

An object is thrown vertically upwards from a height of 1 m above the Earth's surface. The velocity of the object after t seconds is modelled by the function v where $v(t) = 15 - 10 \cdot t$ ($v(t)$ in metres per second, t in seconds).

Task:

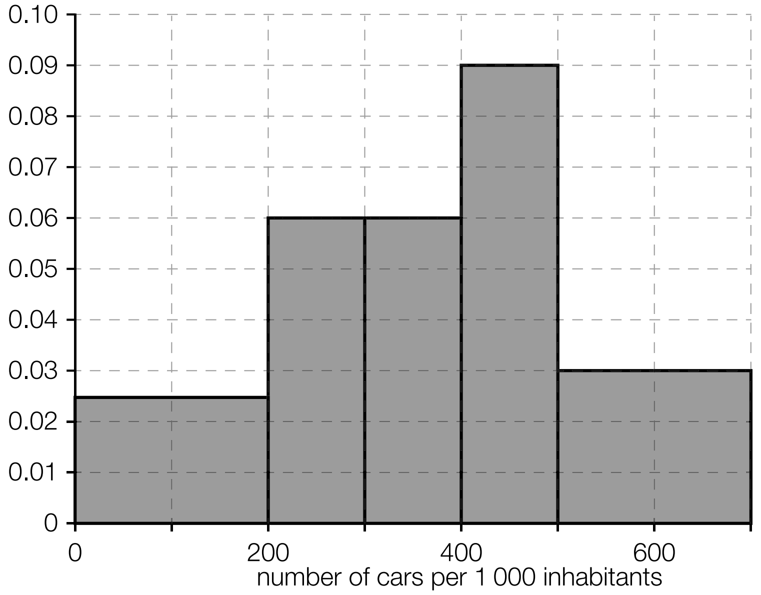
Write down the height of the object (in metres) above the Earth's surface after 2 s.

[0/1 point]

Task 19

Density of Cars

Data about the number of cars per 1 000 inhabitants has been collected in 32 European countries. The histogram below has been created based on this data. The absolute frequencies of the countries are represented by areas of rectangles.



Task:

Determine in how many countries the number of cars per 1 000 inhabitants lies between 500 and 700 cars.

Number of countries = ________________

[0/1 point]

Task 20

Data Set

Below, you will see an ordered data set. One of the values is k where $k \in \mathbb{R}$.

1	2	3	5	k	8	8	8	9	10

Task:

Determine the value of k such that the mean of the whole data set has the value 6.

k = ________________

[0/1 point]

Task 21

Probability of Selection

There are five balls in a container. Two balls are removed from the container one after the other without replacement (it can be assumed that the removal of any two balls is equally likely). Two of the five balls in the container are blue; the other balls are red. The probability of selecting a blue ball second is given by p.

Task:

Write down the probability p.

p = ____

[0/1 point]

Task 22

Playing Cards

Five playing cards (three Kings and two Queens) are shuffled and laid face down on a table. As part of a game, Laura turns the cards over one by one and leaves them face up on the table until the first Queen appears.

The random variable X gives the number of cards lying face up at the end of a game.

Task:

Determine the expectation value of the random variable X.

$E(X) =$ ____

[0/1 point]

Task 23

Rolling Doubles

In a game, two dice are rolled in each round. If the dice land on the same number, then the player has *rolled a double*. The probability of rolling a double is $\frac{1}{6}$.

Image source: BMBWF

Task:

Eight rounds (independent of each other) are played. The random variable *X* gives the number of doubles rolled.
Determine the probability that the number *X* of doubles rolled is less than the expectation value *E*(*X*).

[0/1 point]

Task 24

Opinion Poll

An opinion poll collected responses to the question: "If there were an election this Sunday, which party would you vote for?". The options given in the opinion poll were the parties *A* and *B*, and 234 out of the 1000 people asked said that they would vote for Party *A*. In the election that followed, the actual proportion of people who voted for Party *A* was 29.5 %.

Task:

Based on the results of the opinion poll, write down a symmetrical 95 % confidence interval for the (unknown) proportion of votes for Party *A* and state whether the actual proportion falls within this interval.

[0/1 point]

Task 25 (Part 2)

Braking

The braking distance s_B is the length of the stretch of road a vehicle covers after applying the brakes until it comes to a stop. The variables that determine the braking distance are the velocity v_0 of the vehicle at the moment the brakes are applied and the deceleration due to braking b. The braking distance s_B can be calculated using the formula $s_B = \frac{v_0^2}{2 \cdot b}$ (v_0 in m/s, b in m/s², s_B in m).

The stopping distance s_A takes into account both the braking distance and the distance covered during the reaction time t_R. This so-called *thinking distance* s_R can be calculated using the formula $s_R = v_0 \cdot t_R$ (v_0 in m/s, t_R in s, s_R in m).

The stopping distance s_A is equal to the sum of the thinking distance s_R and the braking distance s_B.

Task:

a) 1) $\boxed{\text{A}}$ Write down a formula that can be used to calculate the velocity v_0 in terms of the braking distance s_B and the deceleration due to braking b.

$v_0 =$ ____

2) Put a cross next to each of the two correct statements.

The thinking distance s_R is directly proportional to the velocity v_0.	☐
The braking distance s_B is directly proportional to the velocity v_0.	☐
The braking distance s_B is indirectly proportional to the deceleration due to braking b.	☐
The stopping distance s_A is directly proportional to the velocity v_0.	☐
The stopping distance s_A is directly proportional to the reaction time t_R.	☐

b) The formulae often used by driving schools to approximate the thinking and braking distances (both in m) are:

$s_R = \frac{v_0}{10} \cdot 3$ and $s_B = \left(\frac{v_0}{10}\right)^2$ with v_0 in km/h and both s_R and s_B in m

1) By rearranging appropriately, show that the formula used to approximate the thinking distance for a reaction time of around one second gives roughly the same result as the formula for s_R given in the introduction.

2) Determine which value is used for the deceleration due to braking in the formula used to approximate the braking distance.

c) The deceleration due to braking b can be assumed to be 8 m/s² in dry conditions, 6 m/s² in wet conditions and at most 4 m/s² in icy conditions.

1) Write down the fraction by which the braking distance is longer in wet conditions than in dry conditions given the same velocity.

A vehicle is being driven with a velocity of v_0 = 20 m/s. The stopping distance in icy conditions is longer than in dry conditions.

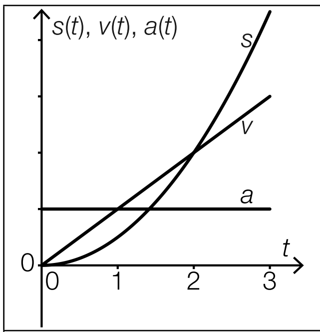
2) Assuming that t_R = 1 s, determine the minimum absolute increase in the stopping distance under icy conditions in comparison to dry conditions

d) A vehicle's brakes are applied at time $t = 0$. The velocity $v(t)$ of the vehicle in the time period [0, 3] can be modelled by the function v, the acceleration $a(t)$ can be modelled by the function a, and the distance $s(t)$ covered in this time period can be modelled by the function s ($v(t)$ in m/s, $a(t)$ in m/s², $s(t)$ in m, t in s).

1) Interpret the meaning of the definite integral $\int_0^3 v(t)\,\mathrm{d}t$ in the given context.

Each of the six diagrams below shows the graph of an acceleration function B, the graph of a velocity function v, and the graph of a distance function s in the time interval [0, 3].

2) Put a cross next to the diagram that shows the three corresponding graphs of a vehicle that is braking for three seconds.



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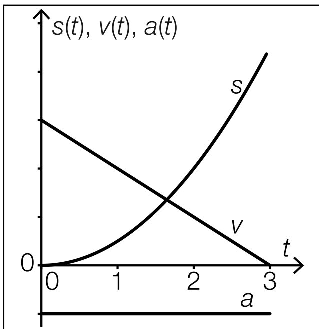

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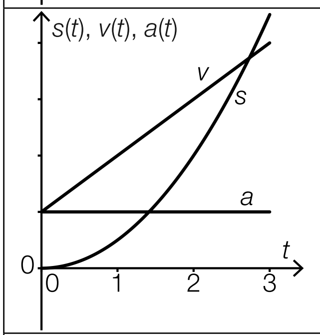

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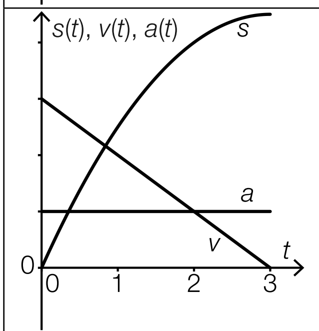

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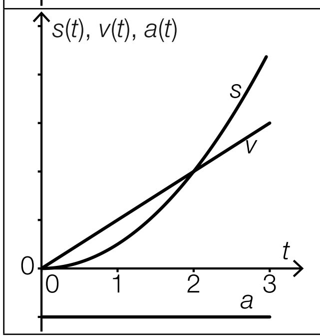

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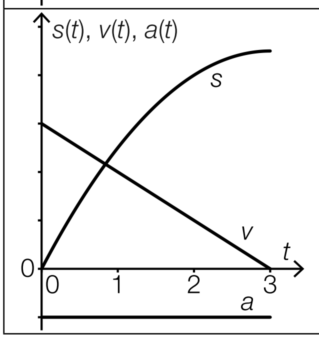

☐

Task 26 (Part 2)

Cost Function

A producer is interested in the monthly costs accrued in the production of a particular product. The production costs for this product in terms of the number x (in units ME) produced (the production volume) can be modelled by a third degree polynomial function K where
$K(x) = 8 \cdot 10^{-7} \cdot x^3 - 7{,}5 \cdot 10^{-4} \cdot x^2 + 0{,}2405 \cdot x + 42$ ($K(x)$ in monetary units, GE).

Task:

a) 1) $\boxed{\text{A}}$ For this product, determine the average increase in costs per additional unit produced in the interval [100 ME, 200 ME].

2) Determine the production volume above which the marginal costs increase.

b) The production volume x_{opt} for which the unit cost function $\overline{K}$ where $\overline{K}(x) = \frac{K(x)}{x}$ is minimal, is the optimal production volume for the cost function K.

1) Determine the optimal production volume x_{opt}.

The producer calculates the production costs for the production volume x_{opt}. He determines that these costs amount to 65 % of the available capital for the production of this product.

2) Determine the capital the producer has available for the production of this product.

c) For the sales price p, the revenue can be modelled in terms of the production volume x by a linear function E where $E(x) = p \cdot x$ ($E(x)$ in GE, x in ME, p in GE/ME). A condition of this model is that every unit produced is also sold.

1) Determine the value of p such that the maximum profit is generated through the sale of 600 ME.

The maximum possible production volume is 650 ME.

2) Determine the range of production volumes for which the producer makes a profit.

d) For another product made by this producer, the production costs (in GE) for various production volumes (in ME) are shown in the table below.

production volume (in ME)	50	100	250		500
production cost (in GE)	197	253	308	380	700

These production costs can be modelled by a third degree polynomial function K_1 where $K_1(x) = a \cdot x^3 + b \cdot x^2 + c \cdot x + d$ with $a, b, c, d \in \mathbb{R}$.

1) Determine the values of a, b, c and d.

2) Determine the production volume that is missing in the table above.

Task 27 (Part 2)

Fibonacci Numbers and the Golden Ratio

The so-called *Fibonacci Numbers* are defined for $n \in \mathbb{N}$ and $n > 2$ by the difference equation $f(n) = f(n-1) + f(n-2)$ with initial values $f(1) = 1$ and $f(2) = 1$.

For large values of n, the ratio $f(n) : f(n-1)$ approaches the *golden ratio* $\phi = \frac{1+\sqrt{5}}{2}$.

Task:

a) 1) $\boxed{\text{A}}$ Write down the value of n for which the ratio $f(n) : f(n-1)$ first corresponds to the golden ratio ϕ to two decimal places.

For Fibonacci numbers, the following equation holds for $k \in \mathbb{N}$ and $k > 2$:
$f(n + k) = f(n-1) \cdot f(k) + f(n) \cdot f(k+1)$

2) Show that this equation holds for $n = 3$ and $k = 5$.

b) One method of approximating Fibonacci numbers using a simple explicit expression is the approximation $f(n) \approx g(n) = \frac{1}{\sqrt{5}} \cdot \left(\frac{1+\sqrt{5}}{2}\right)^n$ where $n \in \mathbb{N}\backslash\{0\}$.

The number 832 040 is a Fibonacci number, which means there exists an $n \in \mathbb{N}$ for which $f(n) = 832\,040$ and $g(n) \approx 832\,040$.

1) Determine the value of this n.

An exact explicit method of calculating the Fibonacci numbers $f(n)$ is the Moivre/Binet formula:
$f(n) = \frac{1}{\sqrt{5}} \cdot (x_1^n - x_2^n)$
In this formula, $x_1 = \phi$ and x_2 are the solutions to the equation $x^2 + a \cdot x - 1 = 0$ where $a \in \mathbb{R}$.

2) Determine the values of a and x_2.

Task 28 (Part 2)

Cinema

A cinema has three screens. The first screen has 185 seats, the second screen has 94, and the third screen has 76.

New films are normally first shown on Thursdays. The owner of the cinema assumes that on a Thursday on which a new film is being shown, each seat in all three screens will be filled with a probability of 95 %.

Task:

a) Let X be a binomially distributed random variable with parameters $n = 355$ and $p = 0.95$.

   1) Describe the meaning of the expression $1 - P(X < 350)$ in the given context.

   At the end of the school year, a school rents all three screens to show the same film at the same starting time. All of the seats have been assigned, and each viewer receives a ticket for a specific seat in one of the three screens. In addition to the seat number, all of the tickets also have a distinct, consecutive ticket number. Directly before the film is shown, two ticket numbers are drawn by lot. The two people who have the corresponding tickets receive a large portion of popcorn each.

   2) Write down the probability that these two people have tickets for the same screen.

b) The owner of the cinema would like to know how satisfied his customers are with the cinema (the selection of food, the cleanliness etc.). He conducts a survey of 628 customers, and 515 of these customers say that they are generally satisfied with the cinema.

   1) $\boxed{\text{A}}$ On the basis of this survey, write down a symmetrical 95 % confidence interval for the relative proportion of all the cinema's customers that are generally satisfied with the cinema.

   A second survey is conducted in which four times as many customers are asked. The relative proportion of customers who are generally satisfied with the cinema in this survey is exactly the same as the result of the first survey.

   2) Write down in numerical terms how this increase in sample size influences the width of the symmetrical 95 % confidence interval calculated using the result of the first survey.