Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

May/June 2023

Mathematics

Supplementary Examination 1
Examiner's Version

Bundesministerium
Bildung, Wissenschaft
und Forschung

Instructions for the standardized implementation of the supplementary examination

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Task 3															
Task 4															
Total															

Explanatory notes on assessment

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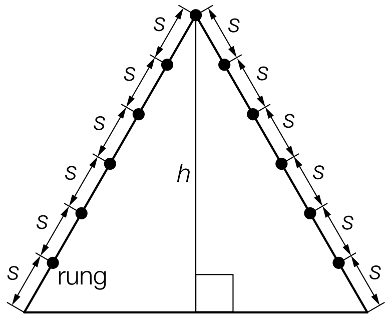
Task 1

Climbing Frame

a) The diagrams below show a climbing frame. The side view shows an equilateral triangle. The rungs are represented as points.

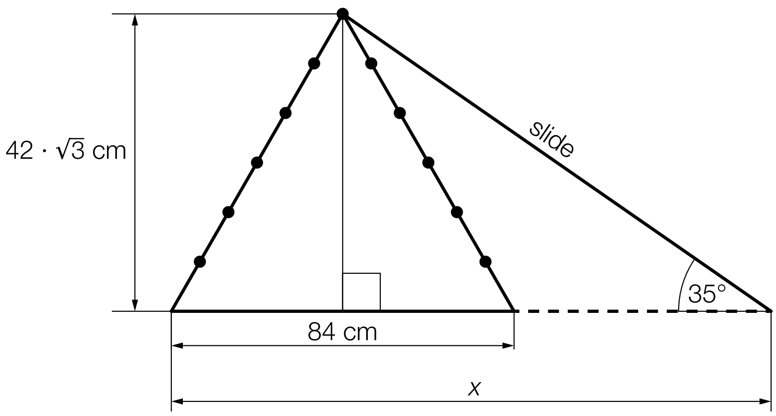
Source: BMBWF



1) Using the distance between the rungs *s*, write down a formula that can be used to calculate the height *h* of this climbing frame.

$h =$ ____

In a toy shop, a climbing frame is also sold together with a straight slide (see the not-to-scale diagram on the right).



2) Determine *x*.

b) A toy shop sells *x* climbing frames without slides and *y* climbing frames with slides in a particular month. In this month, the toy shop takes in a total of € 5760 from the sale of climbing frames with and without slides.

This situation can be described by the system of linear equations shown below.

I: $100 \cdot x + 120 \cdot y = 5760$
II: $x + y = 50$

1) Interpret the values 100, 120 and 50 in the given context.

Solution to Task 1

Climbing Frame

a1) $h = \sqrt{(6 \cdot s)^2 - (3 \cdot s)^2} = \sqrt{27 \cdot s^2} = \sqrt{27} \cdot s$ *or* $h = \frac{6 \cdot s}{2} \cdot \sqrt{3} = 3 \cdot s \cdot \sqrt{3}$

a2) $\tan(35°) = \frac{42 \cdot \sqrt{3}}{x - 42}$

$x = 145.89...$ cm

b1) The price of one climbing frame without a slide is € 100.
The price of one climbing frame with a slide is € 120.
This toy shop sold a total of 50 climbing frames in this month.

Task 2

Play Equipment

A company produces and sells play equipment.
In order to plan financially, the costs, revenue and profit are investigated.

a) The costs can be approximated by the quadratic function K.

$$K(x) = a \cdot x^2 + b \cdot x + c$$

x ... number of play equipment units produced in ME
$K(x)$ ... the cost of producing x play equipment units in monetary units, GE

The following statements hold:

The fixed costs are 22 GE.
The cost of producing 20 ME is 40 GE.
The instantaneous rate of change of the costs when 20 ME are produced is 1.5 GE/ME.

1) Write down a system of equations that can be used to calculate the coefficients of K.

b) The profit can be approximated by the function G.

$$G(x) = -\frac{11}{300} \cdot (x^2 - 70 \cdot x + 600)$$

x ... number of play equipment units sold in ME
$G(x)$ ... the profit from selling x play equipment units in units of currency, GE

1) Determine the zeros of the function G.

c) For a particular x_0, the following statements hold:

$E'(x_0) = 0$
$E''(x_0) < 0$

x ... number of play equipment units sold in ME
$E(x)$ ... the revenue from selling x play equipment units in units of currency, GE

1) Interpret the meaning of x_0 in the given context.

Solution to Task 2

Play Equipment

a1) $K'(x) = 2 \cdot a \cdot x + b$

I: $K(0) = 22$
II: $K(20) = 40$
III: $K'(20) = 1.5$

or:

I: $a \cdot 0^2 + b \cdot 0 + c = 22$
II: $a \cdot 20^2 + b \cdot 20 + c = 40$
III: $2 \cdot a \cdot 20 + b = 1.5$

b1) $G(x) = 0$

calculation using technology:

$x_1 = 10$, $x_2 = 60$

c1) The maximum revenue is obtained for x_0 play equipment units (in ME).

Task 3

Internet Platform

a) The function N models the number of people who use an internet platform in terms of the time t.

$N(t) = 3\,000 \cdot 1.22^t$

t ... time in years since the start of the observations
$N(t)$ ... number of people who use this internet platform at time t

1) Determine the doubling time for the number of people who use this internet platform.
2) Write down the equation of the function N in the form $N(t) = a \cdot e^{\lambda \cdot t}$.

The expression below can be used to calculate the average rate of change of the number of people who use this internet platform in the first 6 years.

$$\frac{3\,000 \cdot 1.22^{\boxed{}} - \boxed{}}{\boxed{} - 0}$$

3) Complete the expression by writing the missing numbers in the boxes provided.

Solution to Task 3

Internet Platform

a1) $6000 = 3000 \cdot 1.22^t$

calculation using technology:

$t = 3.48...$

The doubling time is around 3.5 years.

a2) $\ln(1.22) = 0.1988...$

$N(t) = 3000 \cdot e^{0.199 \cdot t}$ *(coefficient rounded)*

a3) $\dfrac{3000 \cdot 1.22^{\boxed{6}} - \boxed{3000}}{\boxed{6} - 0}$

Task 4

Blood Groups

The table below shows the distribution of blood groups (in Austria).

blood group	0	A	B	AB
frequency	36 %	44 %	14 %	6 %

a) For a study, n people from Austria are selected at random and their blood group is determined.

1) Complete the formula below that can be used to calculate the probability that exactly 5 people have the blood group AB.

$P(\text{“exactly 5 people have the blood group AB”}) = \binom{n}{5} \cdot \boxed{}^{5} \cdot \boxed{}^{\boxed{}}$

b) For another study, 85 people from Austria are selected at random and their blood group is determined.

1) Determine the probability that the number of people with blood group A is at least 25 and at most 30.

c) In yet another study, 2 people from Austria are selected at random.

1) Write down a possible event E in the given context whose probability can be calculated using the expression below.

$P(E) = 2 \cdot 0.36 \cdot 0.14 \approx 0.10$

Solution to Task 4

Blood Groups

a1) P(“exactly 5 people have the blood group AB”) = $\binom{n}{5} \cdot \boxed{0.06}^{5} \cdot \boxed{0.94}^{\boxed{n-5}}$

b1) X ... number of people with blood group A

binomial distribution with $n = 85$ and $p = 0.44$

calculation using technology:

$P(25 \leq X \leq 30) = 0.0627...$

The probability is around 6.3 %.

c1) E ... of these two people, exactly 1 person has the blood group 0 and 1 person has the blood group B

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June 2022

Mathematics

Supplementary Examination 5
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Task 1

Playground

At a playground, there is an assortment of play equipment.

a) Diagram 1 shows a seesaw. In diagram 2, a model of this seesaw is shown from the side.

Diagram 1

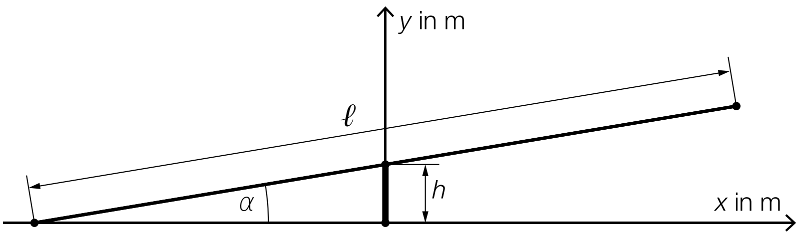


Diagram 2

Image source: Chabe01 – own work, CC BY-SA 4.0, https://commons.wikimedia.org/wiki/File:Aire_Jeux_Rives_Menthon_St_Cyr_Menthon_16.jpg [23.12.2021] (adapted).

The beam has length ℓ and its midpoint has a height h.

1) Using h and ℓ, write down a formula that can be used to calculate the angle α.

α = ____

b) The circular bounce mat of a trampoline has an area of 5 m^2.

1) Determine the diameter of the bounce mat of this trampoline.

c) An old sandpit with a square base with side length a and height h is to be replaced by a new sandpit.

This new sandpit with a square base should have the same height as the old sandpit but the side lengths should be 50 % longer.

1) Show that the volume of the new sandpit is <u>not</u> twice the size of that of the old sandpit.

Solution to Task 1

Playground

a1) $\alpha = \arcsin\left(\frac{h}{\frac{\ell}{2}}\right)$

or:

$\alpha = \arcsin\left(\frac{2 \cdot h}{\ell}\right)$

b1) $d = 2 \cdot \sqrt{\frac{5}{\pi}} = 2.52...$

The bounce mat has a diameter of around 2.5 m.

c1) $V_{old} = a^2 \cdot h$

$V_{new} = (1.5 \cdot a)^2 \cdot h = 2.25 \cdot a^2 \cdot h = 2.25 \cdot V_{old}$

The volume of the new sandpit is therefore not twice the size of that of the old sandpit.

A justification that uses concrete values is also correct.

Task 2

Beer Foam

After a beer has been poured into a glass, the resulting beer foam slowly collapses in upon itself.

a) Thomas measures the height of the beer foam after beer has been poured into a particular glass. The table below shows the results of his measurements.

Time after pouring in s	0	20	60
Height of the beer foam in cm	4	2.5	2

1) Determine the average rate of change of the height of the beer foam for the first 60 seconds after it has been poured. Write down the result with the corresponding unit.

The height of the beer foam is to be described by an exponential function h of the form $h(t) = a \cdot b^t$.

t ... time after pouring in s
$h(t)$ ... height of the beer foam at time t in cm

2) Show that no exponential function h of this form exists whose graph goes through all 3 measurement points.

b) Martin describes the height of the beer foam after pouring beer into a different glass with the function f (see diagrams below).

1) In diagram 2 below, sketch the graph of f'.

Diagram 1

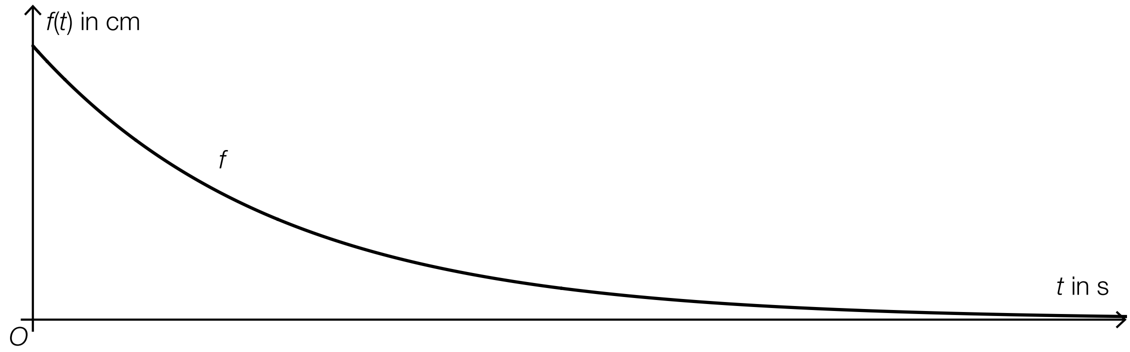


Diagram 2

Solution to Task 2

Beer Foam

a1) $\frac{2-4}{60-0} = -0.0\dot{3}$

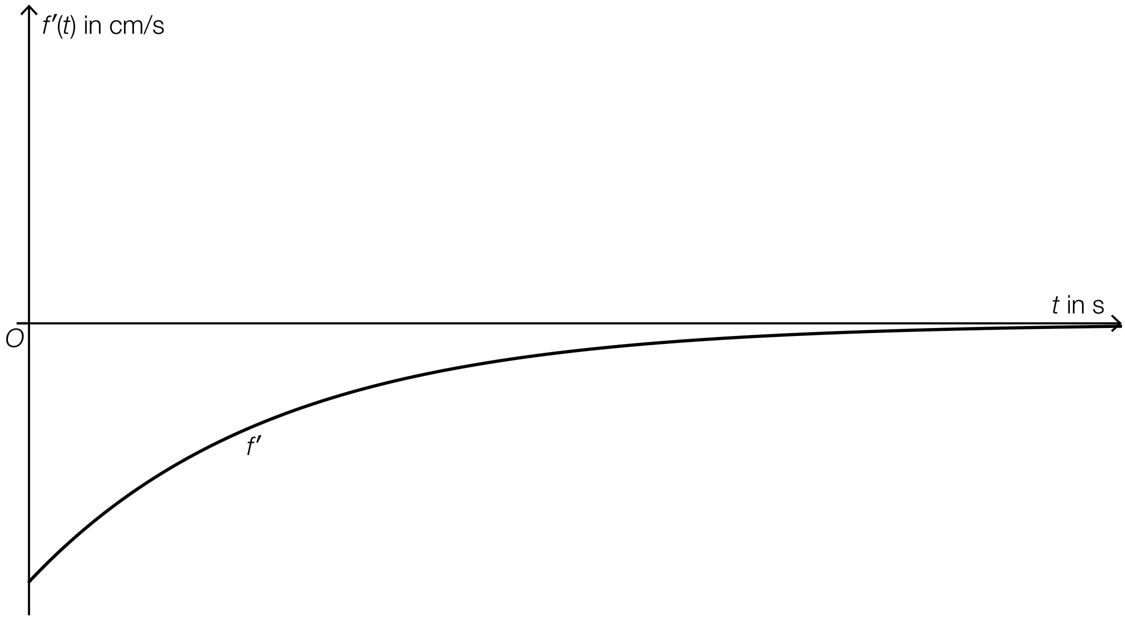
The average rate of change is around –0.03 cm/s.

a2) $4 \cdot b^{20} = 2.5 \quad \Rightarrow \quad b = \sqrt[20]{\frac{2.5}{4}} = 0.976...$

$4 \cdot b^{60} = 2 \quad \Rightarrow \quad b = \sqrt[60]{\frac{2}{4}} = 0.988...$

As the change factors are not the same, there is no exponential function of this form whose graph goes through all 3 measurement points.

b1)



The graph must be monotonically increasing and concave down; it must also approach the horizontal axis asymptotically.

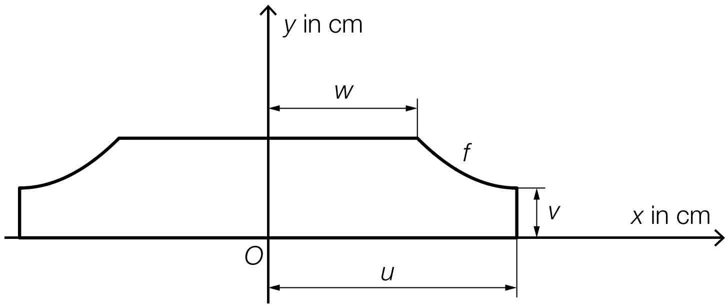
Task 3

Pipe Covering

a) The diagram on the right shows an image of a pipe covering for two heating pipes.

Image source: BMBWF

The diagram below shows a model of the cross-section of this pipe covering from the side.



A part of the boundary line of this cross-section can be modelled by the graph of the quadratic function f with $f(x) = a \cdot x^2 + b \cdot x + c$.

The vertex of the function f has coordinates (u, v).
The angle of the slope when $x = w$ is –45°.

1) Write down a system of equations in terms of u, v and w that can be used to calculate the coefficients a, b and c.
2) On the diagram above, mark the area whose size can be calculated with the expression shown below.
$\int_w^u f(x)\,\mathrm{d}x$

For a particular pipe covering with $u = 5$, the following equation holds:

$f(x) = 0.25 \cdot x^2 - 2.5 \cdot x + 8.75 \quad \text{with} \quad w \leq x \leq u$

3) Determine the length v for this pipe covering.

Solution to Task 3

Pipe Covering

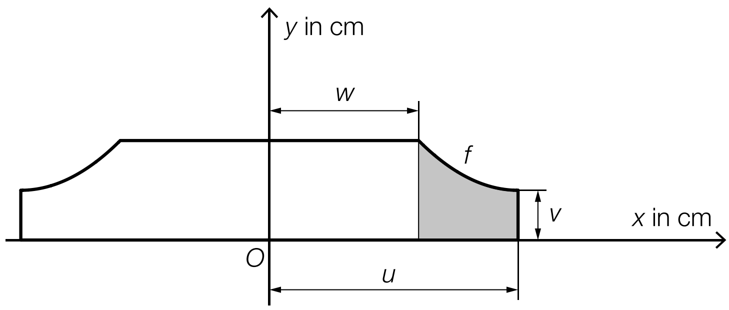
a1) $f(x) = a \cdot x^2 + b \cdot x + c$
$f'(x) = 2 \cdot a \cdot x + b$

I: $f(u) = v$
II: $f'(u) = 0$
III: $f'(w) = -1$

or:

I: $a \cdot u^2 + b \cdot u + c = v$
II: $2 \cdot a \cdot u + b = 0$
III: $2 \cdot a \cdot w + b = -1$

a2)



a3) $f(5) = 2.5$

The length v is 2.5 cm.

Task 4

Parcel Services

Due to the sharp increase in online trade, more and more people are using parcel services.

a) There are complaints offices for registering problems with parcel services.
From long-term observations it is known that 11 % of all complaints at one particular complaints office are due to delivery times that are too long.

On a particular day, a total of 42 independent complaints are received.

1) Determine the probability that exactly 8 of these 42 complaints are received due to delivery times that are too long.

b) For every parcel service, the *first attempt success ratio* is an important metric.
The first attempt success ratio is the probability that a packet selected at random can be delivered on the first attempt. At a particular parcel service, the first attempt success ratio is 90 %.

A delivery driver is to deliver n parcels.

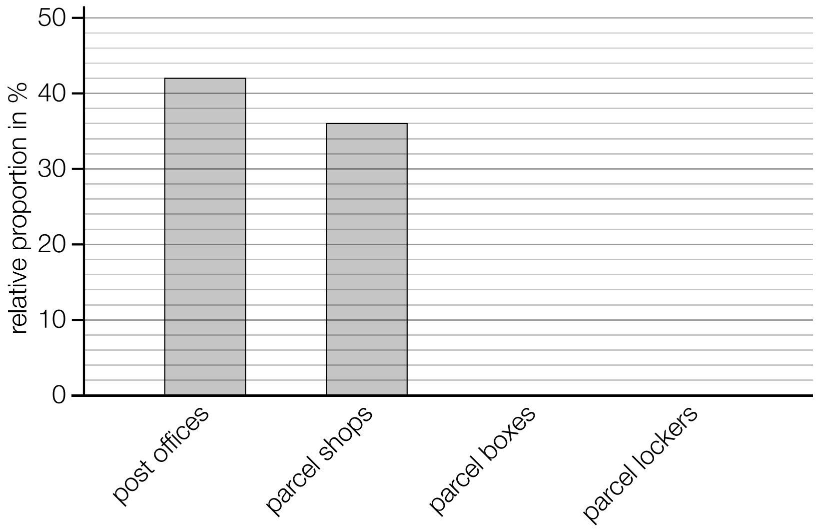
1) Describe the event E whose probability can be calculated with the expression shown below in the given context.

$P(E) = 1 - 0.9^n$

c) In the year 2020, parcels could be sent through a particular parcel service from a total of 31 200 drop-off points.

These 31 200 drop-off points comprised 13 104 post offices, 11 232 parcel shops, 624 parcel boxes and a particular number of parcel lockers.

1) Complete the two missing columns in the bar chart shown below.



Solution to Task 4

Parcel Services

a1) X ... number of complaints due to delivery times that are too long

binomial distribution with $n = 42$, $p = 0.11$

calculation using technology:

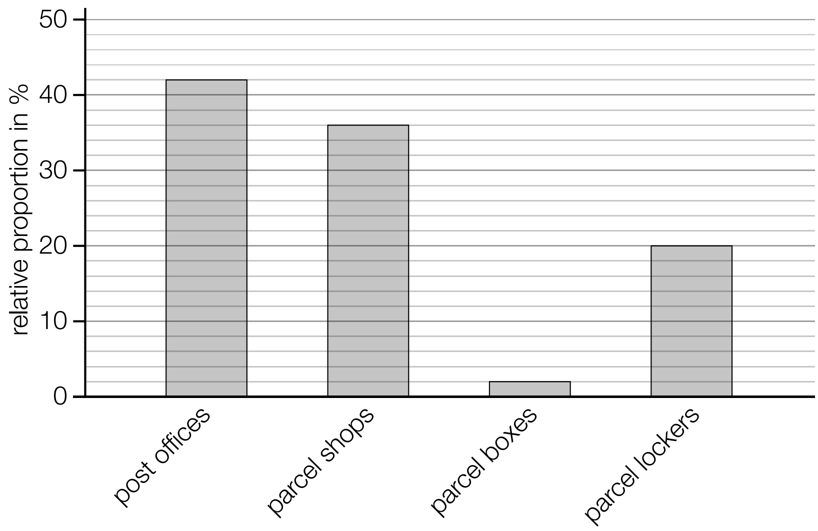
$P(X = 8) = 0.0481...$

The probability is around 4.8 %.

b1) E ... "the delivery driver cannot deliver at least 1 parcel of these n parcels on the first attempt"

c1) $\frac{624}{31\,200} = 0.02$

$\frac{31\,200 - 13\,104 - 11\,232 - 624}{31\,200} = 0.2$



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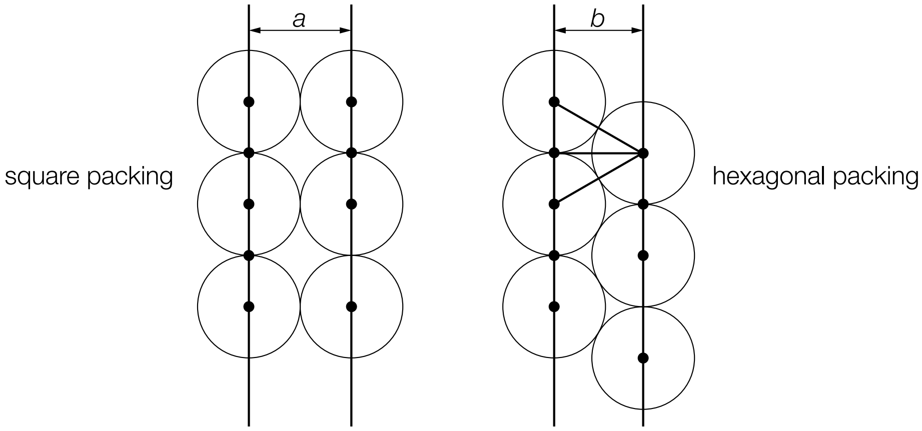
Task 1

Plant Pots

Cylindrical plant pots can be arranged in so-called *square packing* or *hexagonal packing* (see the model bird's-eye view diagrams shown below).



a) The distance b in hexagonal packing is smaller than the distance a in square packing.

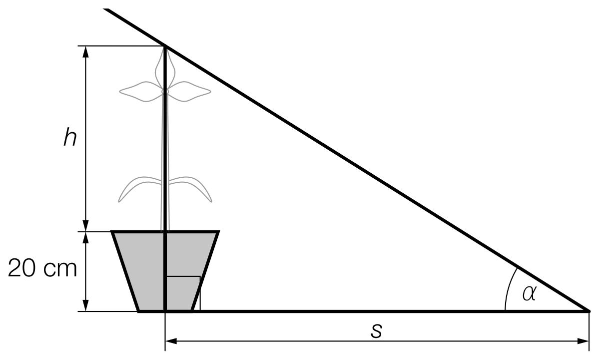
1) Determine the difference $a - b$ for the case when the diameter of the plant pots is 40 cm.

b) Two cylindrical plant pots with a circular base are compared to each other.

Plant pot A has radius r and height h.
The volume of the plant pot is V_A.
Plant pot B has the same height h but a radius that is 10 % larger than plant pot A.

1) Show that the volume V_B of plant pot B is 21 % greater than V_A.

c) In a plant pot with a height of 20 cm, there is a plant with height h (in cm). The rays of sunlight that fall on the plant make an angle of α with the horizontal (see diagram on the right).



1) Write down a formula in terms of h and α that can be used to determine the length s (in cm) of the shadow.

$s =$ ____

Solution to Task 1

Plant Pots

a1) $a = 40$

b is the height of an equilateral triangle with side length 40.

$b = \frac{a}{2} \cdot \sqrt{3} = 34.64...$

or:

$b = \sqrt{40^2 - 20^2} = 34.64...$

$a - b = 40 - 34.64...$
$a - b = 5.35...$ cm

b1) $V_A = r^2 \cdot \pi \cdot h$
$V_B = (1.1 \cdot r)^2 \cdot \pi \cdot h = 1.21 \cdot V_A$

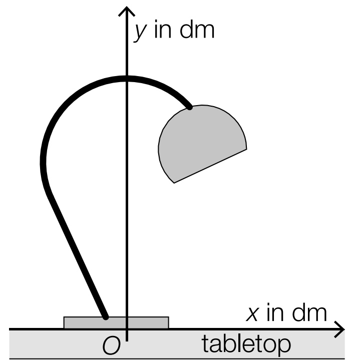
c1) $s = \frac{h + 20}{\tan(\alpha)}$

Task 2

Desk Lamps

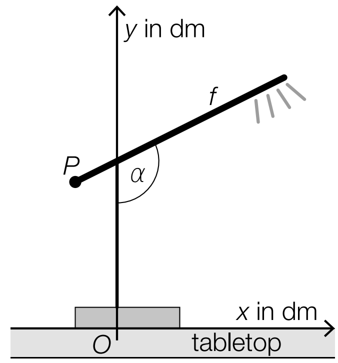
Various models of desk lamp are available. The light source is suspended in different ways depending on design of the model. The suspension methods are modelled by a thick black line in the diagrams below.

a) The suspension of the light source in model *A* is shown in the diagram on the right.



1) Justify why this method of suspension cannot be described by the graph of a single function (y in terms of x).

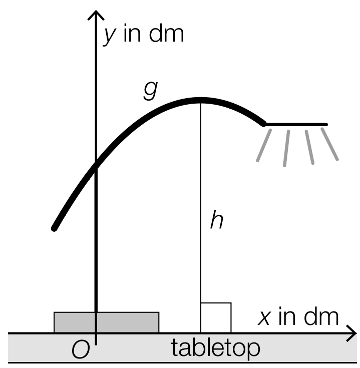
b) The suspension of the light source in model *B* can be described by the graph of the linear function f (see diagram on the right).



1) Using $P = (-1, 3.5)$ and $\alpha = 116.56°$, write down an equation of the function f.

c) The suspension of the light source in model *C* can be described by the graph of the quadratic function g (see diagram on the right).

The following statement holds: $g(x) = -0.25 \cdot x^2 + 1.25 \cdot x + 4$



1) Determine the maximum height h of the lamp above the tabletop.

Solution to Task 2

Desk Lamps

a1) A function assigns each x-value to exactly one y-value. As there is a region for which 2 points of the desk lamp are directly above each other, this method of suspension cannot be described by the graph of a single function.

b1) $f(x) = k \cdot x + d$

$k = \tan(116{,}56° - 90°) = 0.499...$
$-1 \cdot 0.499... + d = 3.5$
$d = 3.99...$

$f(x) = 0.5 \cdot x + 4$ *(coefficients rounded)*

c1) $g'(x) = 0$ *or* $-0.5 \cdot x + 1.25 = 0$

Calculation using technology:

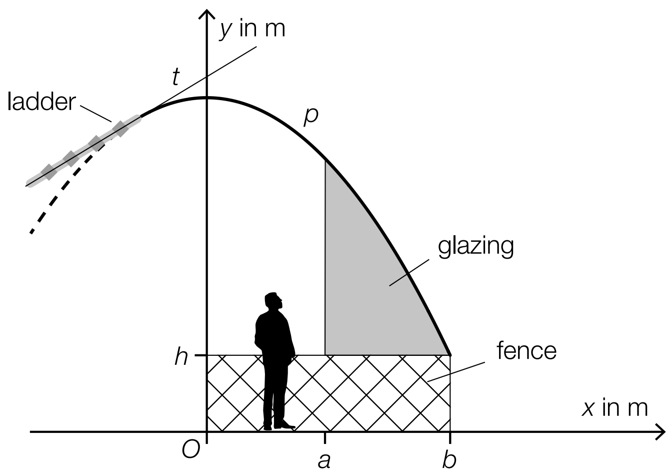
$x = 2.5$
$g(2.5) = 5.56...$

The maximum height h of the desk lamp above the tabletop is around 5.6 dm.

Task 3

Viewing Platform

The diagram below shows a covered viewing platform shown from the side.



a) The roof is modelled by the graph of the quadratic function *p*.

$p(x) = -0.302 \cdot x^2 + 4.8$

x, $p(x)$ ... coordinates in m

For cleaning purposes, a ladder is mounted on the roof. The ladder runs along the tangent *t* to the graph *p* when $x = -1$.

1) Determine the angle of elevation of the tangent *t*.

b) The platform is to be glazed at the side. The glazing will cover the space between the top of the fence and the roof (see diagram above).

1) Write down a formula that can be used to calculate the size *A* of the area shaded grey.

$A =$ ____

c) For safety reasons, the roof requires a supporting beam of length $\ell = p(a) - h$.

1) Draw the length ℓ in the diagram above.

Solution to Task 3

Viewing Platform

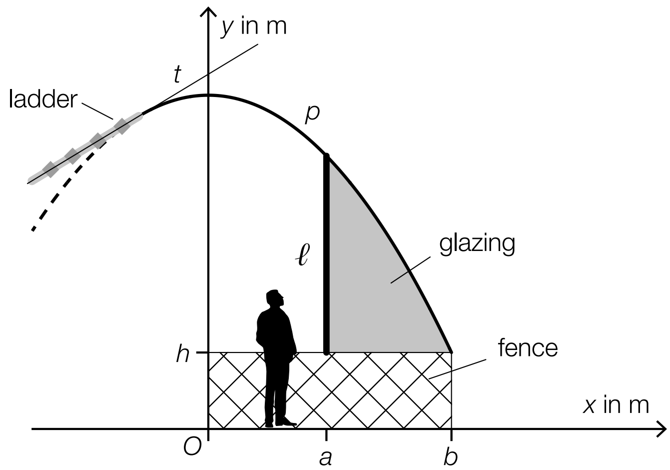
a1) $p'(x) = -0.604 \cdot x$

$p'(-1) = 0.604$
$\alpha = \arctan(0.604) = 31.13...°$

The angle of elevation of the tangent t is around 31.1°.

b1) $A = \int_a^b (p(x) - h)\,\mathrm{d}x$ *or* $A = \int_a^b p(x)\,\mathrm{d}x - (b - a) \cdot h$

c1)



Task 4

Cigarettes

Many of the substances contained in cigarette smoke are hazardous to health.

a) The amount in mg of substances contained in cigarette smoke in the cigarettes of 100 smokers is investigated. These have been sorted into 3 categories (see table below).

class	amount in mg of substances per cigarette	class midpoint	absolute frequency
1	[0, 10[	5	55
2	[10, 30[	20	40
3	[30, 50[	40	5

An estimate of the mean of the amount of substances is to be calculated. The respective class midpoints are determined for this purpose.

1) Determine the mean of the amount of substances.
2) Explain why the median of the amount of substances is in class 1.

b) The probability that a randomly chosen smoker smokes more than one cigarette per day is p.

The probability that exactly 5 out of 100 smokers smoke more than one cigarette per day is to be calculated.

1) Write down a formula that can be used to determine this probability.

Solution to Task 4

Cigarettes

a1) $\frac{5 \cdot 55 + 20 \cdot 40 + 40 \cdot 5}{100} = 12.75$

The mean of the amount of substances is 12.75 mg.

a2) The median of an ordered list always lies in the middle of all values. Of the given 100 values, 55, so more than half, are in class 1. Therefore, the median must also lie in this class.

b1) X ... The number of smokers that smoke more than one cigarette per day

Binomial distribution with $n = 100$ and p

$P(X = 5) = \binom{100}{5} \cdot p^5 \cdot (1 - p)^{95}$

Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

February 2023

Mathematics

Supplementary Examination 1
Examiner's Version

Bundesministerium
Bildung, Wissenschaft
und Forschung

Instructions for the standardized implementation of the supplementary examination

The following supplementary examination booklet contains four tasks that can be completed independently of one another as well as the corresponding solutions.

Each task comprises three competencies to be demonstrated.

The preparation time is to be at least 30 minutes; the examination time is at most 25 minutes.

The use of the official formula booklet that has been approved by the relevant government authority for use in the standardized school-leaving examination in mathematics is allowed. Furthermore, the use of electronic devices (e.g. graphic display calculators or other appropriate technology) is allowed provided there is no possibility to communicate (e.g. via the internet, intranet, Bluetooth, mobile networks etc.) and there is no access to an individual’s data stored on the device.

After the examination, all materials (tasks, extra sheets of paper etc.) from the candidates are to be collected in. The examination materials (tasks, extra sheets of paper, data that has been produced digitally etc.) may only be made public after the time period allocated for the examination has passed.

Evaluation grid for the supplementary examination

The evaluation grid below may be used to assist in assessing the candidates’ performances.

	Candidate 1			Candidate 2			Candidate 3			Candidate 4			Candidate 5		
Task 1															
Task 2															
Task 3															
Task 4															
Total															

Explanatory notes on assessment

Each task can be awarded zero, one, two or three points. A maximum of twelve points can be achieved.

Assessment scale for the supplementary examination

Total number of competencies demonstrated	Assessment of the oral supplementary examination
12	Very good
10–11	Good
8–9	Satisfactory
6–7	Pass
0–5	Fail

Task 1

The Number π

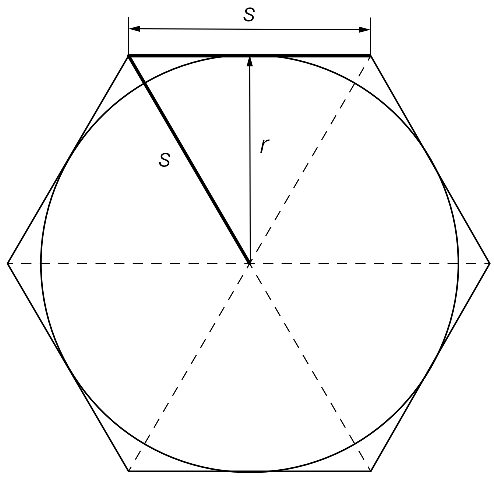
Over the course of history, various methods have been used to determine the value of the number $\pi = 3.141...$ as accurately as possible.

a) In the oldest known book of calculations (*Rhind Mathematical Papyrus*), the following approximation π_N is given:

$\pi_N = \left(\frac{16}{9}\right)^2$

1) Determine in how far the approximation π_N differs from π as a percentage.

b) In another method, the circumference of a circle with radius r is approximated by the perimeter of a circumscribed hexagon (see diagram on the right).

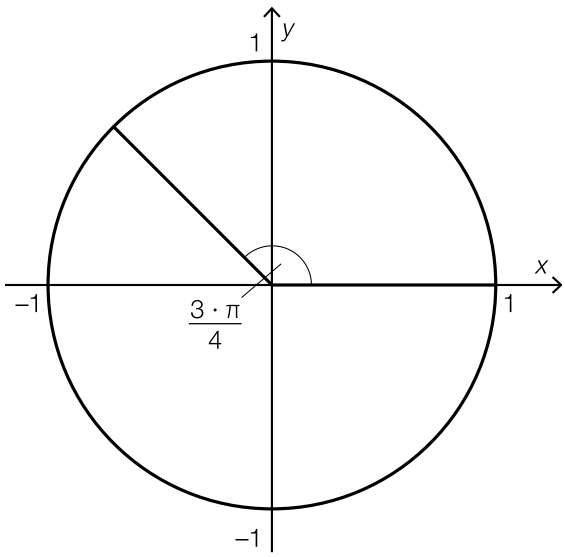


1) Write down a formula in terms of r that can be used to determine the perimeter u of the circumscribed hexagon.

$u =$ ____

c) 1) On the unit circle shown below, sketch the angle α with $\alpha \neq \frac{3 \cdot \pi}{4}$ for which the following statement holds:

$\sin\left(\frac{3 \cdot \pi}{4}\right) = \sin(\alpha)$



Solution to Task 1

The Number π

a1) $\frac{\pi_N - \pi}{\pi} = \frac{0.0189...}{3.1415...} = 0.0060...$

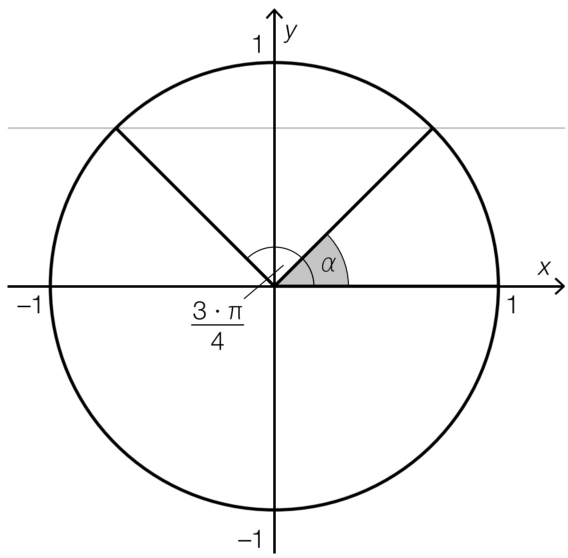
The approximation differs from the number π by around 0.6 %.

b1) $s^2 = \left(\frac{s}{2}\right)^2 + r^2$

$s = \frac{2 \cdot r}{\sqrt{3}}$

$u = 6 \cdot s = \frac{12 \cdot r}{\sqrt{3}}$

c1)



Task 2

Car Journey

a) The diagram below shows the velocity in terms of time for a particular car journey for a time period of 15 s.

The distance covered in the first 5 s is the same as the distance covered in the following 10 s.

1) Write down an equation in terms of v_0 and v_1 that describes this situation correctly.

b) For another car journey, the velocity can be approximated by the function v_A.

$v_A(t) = 70 \cdot t^3 - 260 \cdot t^2 + 230 \cdot t + 80 \quad \text{with} \quad 0 \leq t \leq 1.5$

t ... time in h

$v_A(t)$ ... velocity at time t in km/h

1) Determine the maximum velocity for this car journey.

2) Interpret the result of the calculation shown below in the given context.

$v_A'(0) = 230$

Solution to Task 2

Car Journey

a1) $\frac{(v_0 + v_1) \cdot 5}{2} = 10 \cdot v_1$

b1) $v_A'(t) = 210 \cdot t^2 - 520 \cdot t + 230$
$v_A'(t) = 0$ *or* $210 \cdot t^2 - 520 \cdot t + 230 = 0$

Calculation using technology:

$t_1 = 0.57...$ $(t_2 = 1.89...)$
$v_A(t_1) = 139.59...$

The maximum velocity is around 140 km/h.

b2) The instantaneous rate of change of the velocity (acceleration) at time $t = 0$ is 230 km/h^2.

Task 3

Bacteria

A sample of bacteria is investigated in a laboratory.

The number of bacteria is 1 200 when the observations begin.
After 6 days, the number of bacteria is 1 800.

a) The development of the number of bacteria over time can be modelled by the linear function f.

t ... time in days with $t = 0$ corresponding to the beginning of the observations
$f(t)$ ... number of bacteria at time t

1) Write down an equation of the linear function f.

b) In a different model, the development of the number of these bacteria over time is described by the exponential function g.

t ... time in days with $t = 0$ corresponding to the beginning of the observations
$g(t)$ ... number of bacteria at time t

1) Complete the boxes below with the missing symbol (<, > or =).

$g'(t) \boxed{} g'(t+1)$

$g''(t) \boxed{} 0$

2) Determine the time at which the number of bacteria reaches 6 000 according to this model.

Solution to Task 3

Bacteria

a1) $f(t) = 100 \cdot t + 1\,200$

or:

$f(t) = \frac{600}{6} \cdot t + 1\,200$

b1) $g'(t)\ \boxed{<}\ g'(t + 1)$

$g''(t)\ \boxed{>}\ 0$

b2) $g(t) = a \cdot b^t$
$g(0) = 1\,200$
$g(6) = 1\,800$

Calculation using technology:

$g(t) = 1\,200 \cdot 1.0699^t$ *(coefficient rounded)*

$g(t) = 6\,000$
Calculation using technology:

$t = 23.8...$

The number of bacteria reaches 6 000 around 24 days after the beginning of the observations.

Task 4

Dice

a) Two fair six-sided dice with faces showing the values 1, 2, 3, 4, 5 and 6 are rolled simultaneously. The sum of the values shown is determined.

The random variable X describes the sum of the values shown on the pair of dice.

1) Justify why the following statement holds: $P(X = 11) = 2 \cdot P(X = 12)$.

b) Alex participates in a competition. In this competition, a fair six-sided dice with faces showing the values 1, 2, 2, 3, 3 and 3 is rolled once.

Before Alex rolls the dice, the game master collects an amount of e euros.

If the dice shows the value 1, the game master will pay Alex x euros.
If the dice shows the value 2, the game master will pay Alex 2 euros.
If the dice shows the value 3, the game master will not pay Alex any money.

From experience, the game master knows that he can expect a profit of 0.50 euros per roll of the dice.

1) Write down an equation in terms of e that can be used to calculate x.

c) In the production of a particular six-sided dice with faces showing the values 1, 2, 3, 4, 5 and 6, inaccuracies have occurred. This has resulted in one particular face being rolled with a different probability from the others.

The dice is rolled 500 times. The results are shown in the table below.

value shown on the face	1	2	3	4	5	6
absolute frequency of the value shown on the face	75	98	65	110	80	72

1) Based on these results, determine an estimate for the probability shown below.

P("the value shown on the face is greater than 3 when the dice has been rolled once") = ________________

Solution to Task 4

Dice

a1) The sum 12 is possible in one outcome of the experiment: {(6, 6)}
The sum 11 is possible in two outcomes of the experiment: {(5, 6), (6, 5)}

All of the possible outcomes of the experiment have the same probability, therefore:
$P(X = 11) = 2 \cdot P(X = 12)$

b1) $e - \left(x \cdot \frac{1}{6} + 2 \cdot \frac{1}{3} + 0 \cdot \frac{1}{2}\right) = 0.5$

c1) P(“the value shown on the face is greater than 3 when the dice has been rolled once”) $= \frac{110 + 80 + 72}{500} = 0.524$

Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

Main Examination Session 2021

Mathematics

Supplementary Examination 6
Examiner's Version

Bundesministerium
Bildung, Wissenschaft
und Forschung

Instructions for the standardised implementation of the supplementary examination

The following supplementary examination booklet contains five tasks that can be completed independently of one another. Each task comprises two sub-tasks: the “task” and the “guiding question”.

The preparation time is to be at least 30 minutes; the examination time is at most 25 minutes.

The use of the official formula booklet that has been approved by the relevant government authority for use in the standardised school-leaving examination in mathematics is allowed. Furthermore, the use of electronic devices (e. g. graphic display calculators or other appropriate technology) is allowed provided there is no possibility to communicate (e. g. via the internet, intranet, Bluetooth, mobile networks etc.) and there is no access to an individual’s data stored on the device.

After the examination, all materials (tasks, extra sheets of paper etc.) from all candidates are to be collected in. The examination materials (tasks, extra sheets of paper, digital materials etc.) may only be made public after the time period allocated for the examination has passed.

Evaluation grid for the supplementary examination

The evaluation grid below may be used to assist in assessing the candidates’ performances.

	Candidate 1		Candidate 2		Candidate 3		Candidate 4		Candidate 5	
Task 1										
Task 2										
Task 3										
Task 4										
Task 5										
Total										

Explanatory notes on assessment

Each task can be awarded zero, one or two points. There is one point available for each task for the demonstration of a core competency and one point available for each guiding question. A maximum of ten points can be achieved.

Assessment scale for the supplementary examination

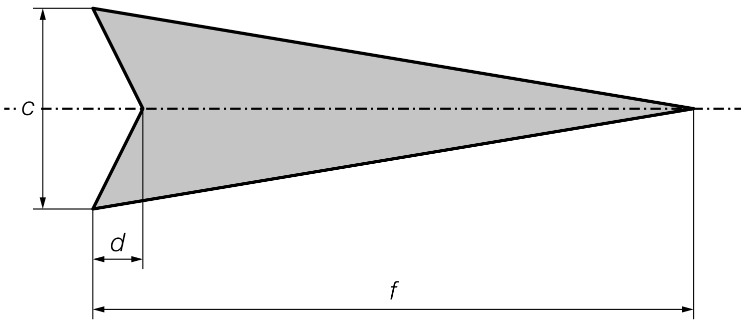
Grade	Number of points achieved (core competencies + guiding questions)
Very good	7 – 10
Good	6
Satisfactory	5
Pass	4

Task 1

Arrow

Task:

The diagram below shows a model of an arrow in 2-dimensional space. The dotted line is the axis of symmetry of this arrow.

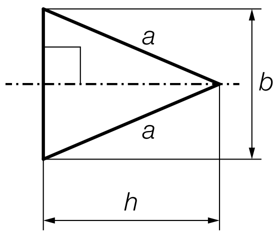


– Using c, d and f, write down a formula that can be used to calculate the area A of the area shaded in grey.

$A =$ ____

Guiding question:

The diagram below shows a model of the tip of an arrow. The dotted line is the axis of symmetry of the tip of the arrow.



The isosceles triangle shown in the diagram above has a base $b = 6$ cm, side length a cm and height $h = 7$ cm. The length of the base is to be kept the same, but the area of the triangle is to be increased by 20 %.

– Determine the side lengths of the triangle after the area has been increased.

Solution to Task 1

Arrow

Expected solution to the statement of the task:

$A = \frac{1}{2} \cdot (c \cdot f - c \cdot d)$

Answer key:

The point for the core competency is to be awarded if the formula has been given correctly.

Expected solution to the guiding question:

$A_{\text{before}} = \frac{6 \cdot 7}{2} = 21$
$A_{\text{after}} = 21 \cdot 1.2 = 25.2$
$A_{\text{after}} = \frac{h_{\text{after}} \cdot 6}{2} \quad \Rightarrow \quad h_{\text{after}} = 8.4 \text{ cm}$
$a_{\text{after}} = \sqrt{8.4^2 + 3^2} = 8.91...$
The sides of the triangle have a length of around 8.9 cm after increasing the area.

Answer key:

The point for the guiding question is to be awarded if the length has been calculated correctly.

Task 2

Mountain Railway

The station at the bottom of a mountain railway is at an altitude of 1 000 m. The horizontal distance between the station at the bottom of the mountain and the station at the top of the mountain is 2 500 m. In this task, the railway line is modelled as a straight line and has a constant gradient of 41 %.

Task:

– Determine the angle of elevation of the railway line.

– Determine the altitude of the station at the top of the mountain.

Guiding question:

The duration of the journey from the station at the bottom of the mountain to the station at the top of the mountain is 5 min.

The function p with $p(h) = 1\,000 \cdot e^{-0.000126 \cdot h}$ can be used to approximate the air pressure at an altitude of h (h in m, $p(h)$ in mbar).

– Determine the average absolute change in air pressure per minute over the course of a journey with this mountain railway from the station at the bottom of the mountain to the station of the top of the mountain.

Solution to Task 2

Mountain Railway

Expected solution to the statement of the task:

α ... angle of elevation of the mountain railway
x ... difference between the altitude of the station at the top of the mountain and the altitude of the station at the bottom of the mountain

$\tan(\alpha) = 0.41 \quad \Rightarrow \quad \alpha = 22.29...°$

$0.41 = \frac{x}{2\,500} \quad \Rightarrow \quad x = 1\,025$

The station at the top of the mountain is at an altitude of 2025 m.

Answer key:

The point for the core competency is to be awarded if the angle of elevation and the altitude of the station at the top of the mountain have been calculated correctly.

Expected solution to the guiding question:

$\frac{p(2\,025) - p(1\,000)}{5} = -21.3...$

The average absolute change in the air pressure is around 21 mbar/min.

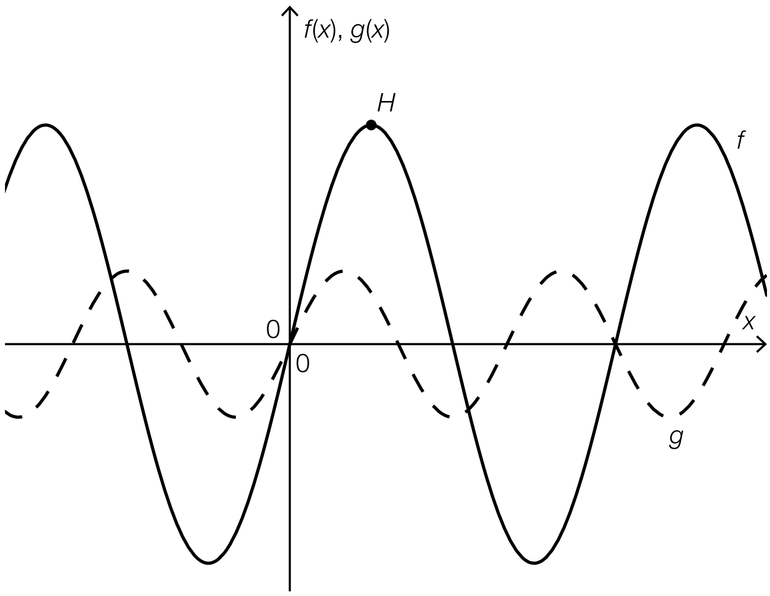
Answer key:

The point for the guiding question is to be awarded if the average absolute change in air pressure has been calculated correctly. The unit “mbar/min” does not need to be given.

Task 3

Trigonometric Functions

The diagram below shows the graphs of the functions *f* and *g* with $f(x) = a \cdot \sin(b \cdot x)$ and $g(x) = c \cdot \sin(d \cdot x)$ with $a, b, c, d \in \mathbb{R}^+$.



Task:

– Complete each of the gaps below with the appropriate symbol “<”, “>” or “=” and justify your answers.

a _____ c

b _____ d

Guiding question:

The maximum point of the graph of *f* labelled *H* in the diagram above has coordinates $H = \left(\frac{\pi}{4}, 3\right)$.

– Determine *a* and *b*.

Solution to Task 3

Trigonometric Functions

Expected solution to the statement of the task:

$a > c$
The function *f* has a larger maximum value than the function *g*.

$b < d$
The function *f* has a larger period length than the function *g*.

Answer key:

The point for the core competency is to be awarded if the correct symbols have been used and correct justifications (that may also be given using the terms “amplitude” and “frequency”) have been given.

Expected solution to the guiding question:

$a = 3$

$$b = \frac{2 \cdot \pi}{\frac{\pi}{4} \cdot 4} = 2$$

Answer key:

The point for the guiding question is to be awarded if *a* and *b* have been determined correctly.

Task 4

Drag Race

Jan and Tom are participating in a drag race. They set off at the same time when $t = 0$. The velocities of their vehicles in the first few seconds can be described by the two functions v_J and v_T.

t ... time in s
$v_J(t)$ ... velocity of Jan's vehicle at time t in m/s
$v_T(t)$ ... velocity of Tom's vehicle at time t in m/s

Task:

For the time-velocity function v_J, the following relationship holds:

$v_J(t) = 0.6 \cdot t^2 \cdot e^{-0.09 \cdot t}$

– Determine the acceleration of Jan's vehicle when $t = 10$.

Guiding question:

At time t_1, Tom's vehicle is ahead of Jan's vehicle. The distance between the vehicles at time t_1 is d metres.

– Using v_J and v_T, write down a formula that can be used to calculate d.

$d =$ ______________________________________

Solution to Task 4

Drag Race

Expected solution to the statement of the task:

$v_J'(10) = 2.68...$

The acceleration is around 2.7 m/s^2.

Answer key:

The point for the core competency is to be awarded if the acceleration has been determined correctly.

Expected solution to the guiding question:

$$d = \int_0^{t_1} v_T(t)\,\mathrm{d}t - \int_0^{t_1} v_J(t)\,\mathrm{d}t$$

Answer key:

The point for the guiding question is to be awarded if the formula has been given correctly.

Task 5

Balls

Task:

A box with 30 balls contains 14 red and 16 yellow balls.

Maria takes 2 balls out of the box at random and without replacement.

– Determine the probability that Maria removes 2 balls of the same colour from the box.

Guiding question:

Another box contains 3 white balls and 1 green ball.
Eva takes balls out of the box at random until she removes the green ball.

The random variable X gives the number of balls removed. If X takes the value 2, this means that the first ball is white and the second ball is green.

– Determine the expectation value of X.

Solution to Task 5

Balls

Expected solution to the statement of the task:

$$\frac{14}{30} \cdot \frac{13}{29} + \frac{16}{30} \cdot \frac{15}{29} = \frac{211}{435} = 0.4850\ldots \approx 48.5\ \%$$

Answer key:

The point for the core competency is to be awarded if the probability has been determined correctly.

Expected solution to the guiding question:

expectation value: $1 \cdot P(X = 1) + 2 \cdot P(X = 2) + 3 \cdot P(X = 3) + 4 \cdot P(X = 4)$

$$= 1 \cdot \frac{1}{4} + 2 \cdot \frac{3}{4} \cdot \frac{1}{3} + 3 \cdot \frac{3}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} + 4 \cdot \frac{3}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} = 2.5$$

Answer key:

The point for the guiding question is to be awarded if the expectation value has been calculated correctly.

Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

October 2021

Mathematics

Supplementary Examination 2
Examiner's Version

Bundesministerium
Bildung, Wissenschaft
und Forschung

Instructions for the standardised implementation of the supplementary examination

The following supplementary examination booklet contains five tasks that can be completed independently of one another. Each task comprises two sub-tasks: the “task” and the “guiding question”.

The preparation time is to be at least 30 minutes; the examination time is at most 25 minutes.

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Evaluation grid for the supplementary examination

The evaluation grid below may be used to assist in assessing the candidates’ performances.

	Candidate 1		Candidate 2		Candidate 3		Candidate 4		Candidate 5	
Task 1										
Task 2										
Task 3										
Task 4										
Task 5										
Total										

Explanatory notes on assessment

Each task can be awarded zero, one or two points. There is one point available for each task for the demonstration of a core competency and one point available for each guiding question. A maximum of ten points can be achieved.

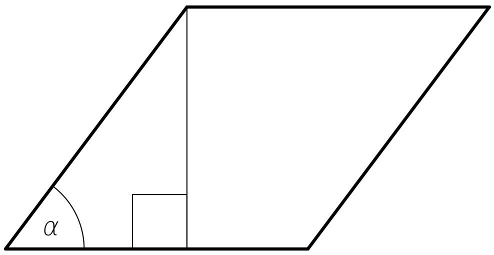
Assessment scale for the supplementary examination

Grade	Number of points achieved (core competencies + guiding questions)
Very good	7–10
Good	6
Satisfactory	5
Pass	4

Task 1

Rhombus

The diagram below shows a rhombus with side length a, height h and angle α ($\alpha < 90°$).



Task:

– Write down a formula in terms of a and α that can be used to calculate h.

h = ________________________________

Guiding question:

– Determine the value of α for which the area A of the rhombus is half the size of the area of a square with the same side length a.

Solution to Task 1

Rhombus

Expected solution to the statement of the task:

$\sin(\alpha) = \frac{h}{a}$
$h = \sin(\alpha) \cdot a$

Answer key:

The point for the core competency is to be awarded if the formula has been written down correctly.

Expected solution to the guiding question:

$A = a^2 \cdot \sin(\alpha)$
$a^2 \cdot \sin(\alpha) = a^2 \cdot 0.5 \quad \Rightarrow \quad \sin(\alpha) = 0.5 \quad \Rightarrow \quad \alpha = 30°$

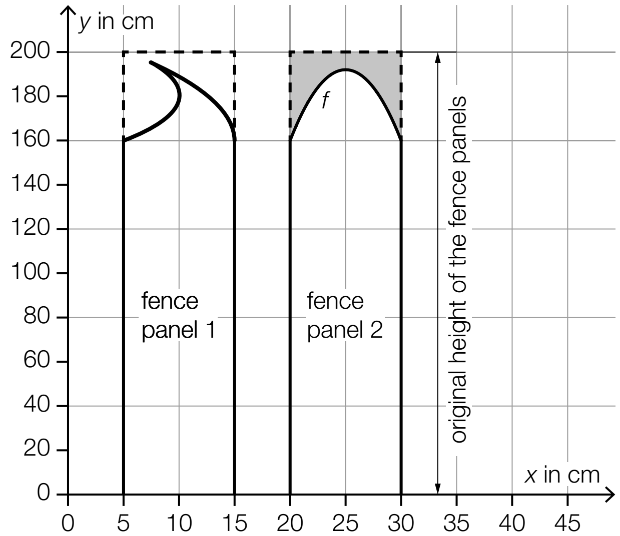
Answer key:

The point for the guiding question is to be awarded if the value of α has been determined correctly.

Task 2

Fence Panels

A carpenter cuts rectangular fence panels creatively.
The original fence panels are rectangular with a height of 200 cm and a width of 10 cm. From these, the carpenter creates the fence panel models shown in the diagram below.



Task:

Fence panel 1: The whole of the upper boundary in the region $5 \leq x \leq 15$ is to be represented by the graph of a function in terms of x.

– Justify why this is not possible.

Guiding question:

Fence panel 2: The upper boundary in the interval [20, 30] is described by the graph of the function f.

x, $f(x)$ ... coordinates in cm

The region marked in grey in the diagram above shows the wastage (i.e. the wood left over when the panel has been cut).

– Write down a formula in terms of f that can be used to calculate the area A of the region marked in grey (in cm^2).

$A =$ ______________________________

Solution to Task 2

Fence Panels

Expected solution to the statement of the task:

This situation cannot be represented by the graph of a function because not all x-values (in the region $5 \leq x \leq 15$) correspond to exactly one y-value.

Answer key:

The point for the core competency is to be awarded if it has been justified correctly why the upper boundary cannot be represented by the graph of a function.

Expected solution to the guiding question:

$$A = 200 \cdot 10 - \int_{20}^{30} f(x)\,\mathrm{d}x$$

Answer key:

The point for the guiding question is to be awarded if the formula has been written down correctly.

Task 3

Noise

Noise can negatively affect a person's health.

Task:

The length of time a person can be exposed to a certain noise level in a day is known as the *exposure time*. This time can be modelled by the function f.

$f(x) = a \cdot 0.8^x$

x ... noise level in decibels (dB)
$f(x)$ ... exposure time for the noise level x in min

At a noise level of 100 dB, the exposure time is 12 min.

– Determine the parameter a.

Guiding question:

On a particular section of road, noise measurements are taken in terms of the number of vehicles per hour. On the basis of these noise measurements, the so-called *average noise level* is calculated (see table below).

number of vehicles per hour	average noise level in dB
10	52
60	58
80	61

– Show by calculation that the relationship between the number of vehicles per hour and the average noise level in dB is <u>not</u> linear.

Solution to Task 3

Noise

Expected solution to the statement of the task:

$f(100) = 12$ *or* $12 = a \cdot 0.8^{100}$

$a = 5.89... \cdot 10^{10}$

Answer key:

The point for the core competency is to be awarded if the parameter *a* has been determined correctly.

Expected solution to the guiding question:

$$k_1 = \frac{58 - 52}{60 - 10} = 0.12$$

$$k_2 = \frac{61 - 58}{80 - 60} = 0.15$$

$$k_3 = \frac{61 - 52}{80 - 10} = 0.128...$$

As the quotients are not the same, the relationship is not linear.
For the award of the point, the comparison of two difference quotients is sufficient.

Answer key:

The point for the guiding question is to be awarded if it has been shown by calculation that there is no linear relationship.

Task 4

Rates of Change

The quadratic function f is given by $f(x) = -x^2 + 2 \cdot x + 3$.

Task:

The average rate of change of f in the interval $[1, a]$ ($a \in \mathbb{R}$, $a > 1$) is -3.

– Determine a.

Guiding question:

– Determine the x-value x_0 for which the instantaneous rate of change of f at x_0 is equal to -3.

Solution to Task 4

Rates of Change

Expected solution to the statement of the task:

$\frac{f(a) - f(1)}{a - 1} = -3 \quad or \quad \frac{-a^2 + 2 \cdot a + 3 - 4}{a - 1} = -3$

Calculation using technology:

$a = 4$

Answer key:

The point for the core competency is to be awarded if a has been determined correctly.

Expected solution to the guiding question:

$f'(x) = -2 \cdot x + 2$
$f'(x_0) = -3 \quad \Rightarrow \quad x_0 = 2.5$

Answer key:

The point for the guiding question is to be awarded if x_0 has been determined correctly.

Task 5

Wheel of Fortune

A wheel of fortune is divided into multiple sectors. The probability of the spinner landing in sector G is p for each spin. The results of the individual spins are independent of each other.

Task:

Marco spins the wheel of fortune n times.

– Write down a formula in terms of p that can be used to calculate the probability shown below.

P("the spinner lands in sector G at least once") = ____________________

Guiding question:

Nina spins the wheel of fortune multiple times.

– State the event E in the given context whose probability can be calculated with the expression shown below.

$$P(E) = \binom{10}{8} \cdot p^8 \cdot (1-p)^2 + \binom{10}{9} \cdot p^9 \cdot (1-p) + p^{10}$$

Solution to Task 5

Wheel of Fortune

Expected solution to the statement of the task:

P(“the spinner lands in sector G at least once”) = $1 - (1 - p)^n$

Answer key:

The point for the core competency is to be awarded if the formula has been written down correctly.

Expected solution to the guiding question:

E ... “the spinner lands in sector G at least 8 times in 10 spins”

Answer key:

The point for the guiding question is to be awarded if the event has been stated correctly.

Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

February 2022

Mathematics

Supplementary Examination 2
Examiner's Version

Bundesministerium
Bildung, Wissenschaft
und Forschung

Instructions for the standardised implementation of the supplementary examination

The following supplementary examination booklet contains five tasks that can be completed independently of one another. Each task comprises two sub-tasks: the "task" and the "guiding question".

The preparation time is to be at least 30 minutes; the examination time is at most 25 minutes.

The use of the official formula booklet that has been approved by the relevant government authority for use in the standardised school-leaving examination in mathematics is allowed. Furthermore, the use of electronic devices (e.g. graphic display calculators or other appropriate technology) is allowed provided there is no possibility to communicate (e.g. via the internet, intranet, Bluetooth, mobile networks etc.) and there is no access to an individual's data stored on the device.

After the examination, all materials (tasks, extra sheets of paper etc.) from all candidates are to be collected in. The examination materials (tasks, extra sheets of paper, digital materials etc.) may only be made public after the time period allocated for the examination has passed.

Evaluation grid for the supplementary examination

The evaluation grid below may be used to assist in assessing the candidates' performances.

	Candidate 1		Candidate 2		Candidate 3		Candidate 4		Candidate 5	
Task 1										
Task 2										
Task 3										
Task 4										
Task 5										
Total										

Explanatory notes on assessment

Each task can be awarded zero, one or two points. There is one point available for each task for the demonstration of a core competency and one point available for each guiding question. A maximum of ten points can be achieved.

Assessment scale for the supplementary examination

Grade	Number of points achieved (core competencies + guiding questions)
Very good	7 – 10
Good	6
Satisfactory	5
Pass	4

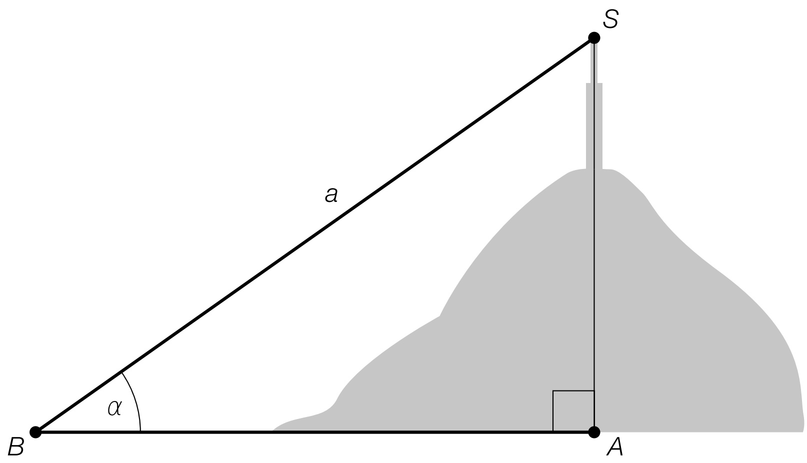
Task 1

Pyramidenkogel

The Pyramidenkogel is a mountain near Lake Wörth. At its peak stands the tallest wooden viewing tower in the world.

Task:

Bettina can see the top S of the viewing tower at an angle of α from point B on the bank of Lake Wörth (see the not-to-scale diagram below).



– Write down a formula in terms of α and a that can be used to calculate $\overline{AS}$.

$\overline{AS}$ = ____

Guiding question:

There is a slide in the viewing tower. This slide is 120 m long and covers a difference in altitude of 52 m. As a simplification, it can be assumed that the gradient of the slide is constant.

– Determine the gradient as a percentage.

Solution to Task 1

Pyramidenkogel

Expected solution to the statement of the task:

$\overline{AS} = a \cdot \sin(\alpha)$

Answer key:

The point for the core competency is to be awarded if the formula has been written down correctly.

Expected solution to the guiding question:

$\frac{52}{\sqrt{120^2 - 52^2}} = 0.480... = 48.0...\ \%$

Answer key:

The point for the guiding question is to be awarded if the gradient has been calculated correctly. A solution for the gradient of –48 % is also correct.

Task 2

Water Temperature

Water is brought to the boil in a pan.

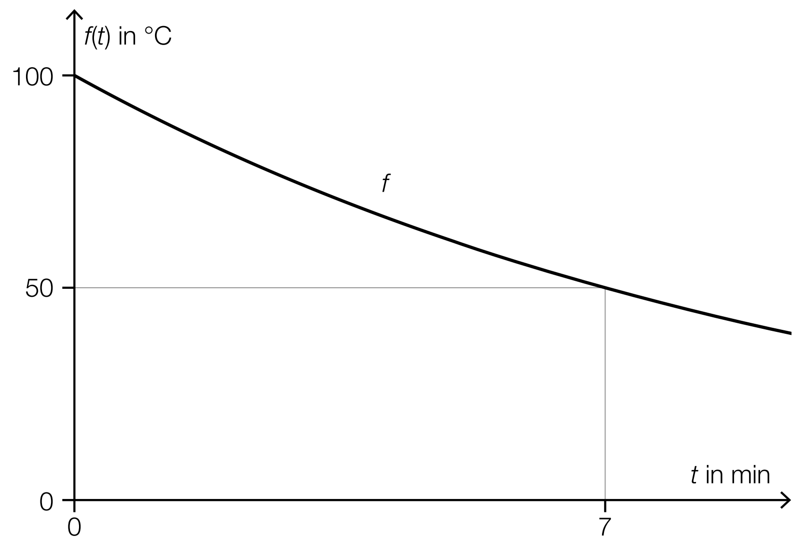
Task:

The initial temperature of the water is 22 °C.
It can be assumed that the temperature of the water increases by 8 °C per minute.

– Determine the time taken in minutes for the water to reach a temperature of 100 °C.

Guiding question:

In an experiment, the temperature of cooling water is measured. The temperature can be modelled by the exponential function *f*. The graph of *f* is shown in the diagram below.



$f(t) = a \cdot b^t$

t ... time in min
$f(t)$ ... temperature of the water at time t in °C
a, b ... parameters

– Determine the parameters a and b.

Solution to Task 2

Water Temperature

Expected solution to the statement of the task:

$\frac{100 - 22}{8} = 9.75$

After 9.75 min, the water has reached a temperature of 100 °C.

Answer key:

The point for the core competency is to be awarded if the time has been calculated correctly.

Expected solution to the guiding question:

$f(0) = 100 \quad \Rightarrow \quad a = 100$

$f(7) = 50 \quad \Rightarrow \quad b = \sqrt[7]{\frac{50}{100}} = 0.9057...$

Answer key:

The point for the guiding question is to be awarded if *a* and *b* have been calculated correctly.

Task 3

Information Plaques

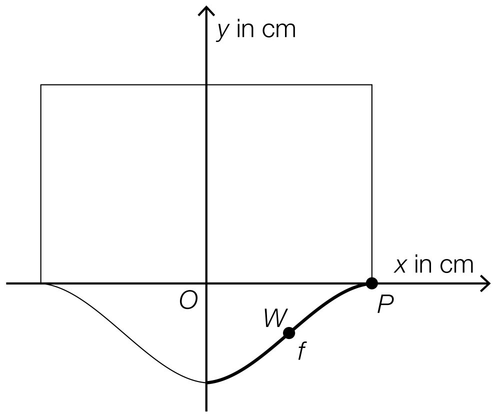
In the town of Steyr, information plaques can be found on historical landmarks. The image on the right shows one such plaque.

Image source: BMBWF

Task:

These plaques are to be replaced with new plaques. The diagram on the right shows the outline of one of the new plaques.

Let f be a function with:
$f(x) = a \cdot x^3 + b \cdot x^2 + c \cdot x + d$ with $x \in [0, 25]$

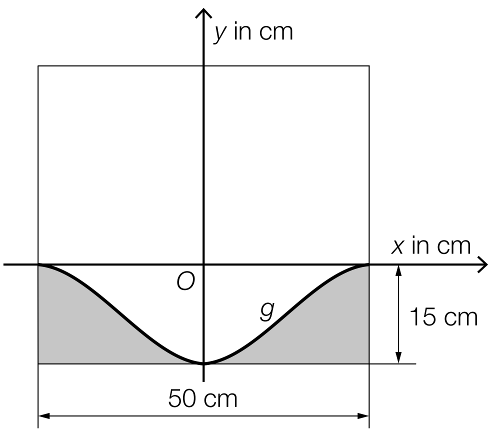


The point P has coordinates (25, 0). The point of inflexion W has coordinates (12.5, –7.5). The gradient of the tangent at the point of inflexion W is 0.8625.

– Using the information about the points P and W, write down a system of equations that can be used to calculate the coefficients a, b, c and d.

Guiding question:

A new plaque is to be cut out of a rectangular metal sheet. This plaque is symmetrical about the y-axis (see diagram below).



– Write down a formula in terms of g that can be used to calculate the area A of the region shaded in grey.

$A =$ ____

Solution to Task 3

Information Plaques

Expected solution to the statement of the task:

$f'(x) = 3 \cdot a \cdot x^2 + 2 \cdot b \cdot x + c$
$f''(x) = 6 \cdot a \cdot x + 2 \cdot b$

I: $f(25) = 0$
II: $f(12.5) = -7.5$
III: $f'(12.5) = 0.8625$
IV: $f''(12.5) = 0$

or:

I: $a \cdot 25^3 + b \cdot 25^2 + c \cdot 25 + d = 0$
II: $a \cdot 12.5^3 + b \cdot 12.5^2 + c \cdot 12.5 + d = -7.5$
III: $3 \cdot a \cdot 12.5^2 + 2 \cdot b \cdot 12.5 + c = 0.8625$
IV: $6 \cdot a \cdot 12.5 + 2 \cdot b = 0$

Answer key:

The point for the core competency is to be awarded if the system of equations has been written down correctly.

Expected solution to the guiding question:

$$A = 2 \cdot \int_0^{25} (g(x) + 15)\,\mathrm{d}x \quad \text{or} \quad A = 50 \cdot 15 + 2 \cdot \int_0^{25} g(x)\,\mathrm{d}x \quad \text{or} \quad A = 50 \cdot 15 + \int_{-25}^{25} g(x)\,\mathrm{d}x$$

Answer key:

The point for the guiding question is to be awarded if the formula has been written down correctly.

Task 4

Body Temperature

Marie has a fever and therefore takes medicine to reduce the fever.

Marie's body temperature after taking the medicine can be modelled by the function T.

t ... time after taking the medicine in h with $0 \leq t \leq 6$
$T(t)$ ... Marie's body temperature at time t in °C

Task:

– Interpret the result of the calculation below in the given context. Write down the corresponding unit.

$$\frac{T(5) - T(0)}{5 - 0} = -0.4$$

Guiding question:

Theo is ill and takes medicine at 9:00.

t ... time after taking the medicine in h with $t = 0$ for 9:00
$K(t)$ ... Theo's body temperature at time t in °C

After taking the medicine, Theo's body temperature is recorded at hourly intervals i.e. at 10:00, 11:00 etc.

A simple model is as follows:

$K(t + 1) = K(t) - 0.5 \cdot (K(t) - 36.5)$
$K(0) = 39$

– Determine the time at which Theo's temperature is first recorded as being below 37.5 °C.

Solution to Task 4

Body Temperature

Expected solution to the statement of the task:

Marie’s body temperature reduces in the first 5 hours after taking the medicine by an average of 0.4 °C per hour.

Answer key:

The point for the core competency is to be awarded if the result has been interpreted correctly with the corresponding unit in the given context.

Expected solution to the guiding question:

$K(1) = 37.75$ °C
$K(2) = 37.125$ °C

At time $t = 2$ (11:00), Theo’s body temperature is recorded as less than 37.5 °C for the first time.

Answer key:

The point for the guiding question is to be awarded if the time has been calculated correctly.

Task 5

Long Jump

Task:

Katja participates in a long jump competition.
5 of Katja's jumps are recorded. The longest jump is 4.3 m and the shortest jump is 3.7 m.
For the 3 other jumps, she achieved the same jump length w.

The mean of the 5 long jumps is 4.06 m.

– Determine the jump length w.

Guiding question:

Chiara is training for a competition. The random variable X gives Chiara's jump length in m. The following probabilities are known:

$P(X \leq 4.5) = 0.5$
$P(4.5 < X \leq 4.6) = 0.3$

Chiara jumps 2 times. The jump lengths she achieves are assumed to be independent of each other.

– Determine the probability that neither of the two jump lengths is greater than 4.6 m.

Solution to Task 5

Long Jump

Expected solution to the statement of the task:

$$\frac{4.3 + 3.7 + 3 \cdot w}{5} = 4.06 \quad \Rightarrow \quad w = 4.1 \text{ m}$$

Answer key:

The point for the core competency is to be awarded if the jump length w has been calculated correctly.

Expected solution to the guiding question:

$0.3^2 + 2 \cdot 0.3 \cdot 0.5 + 0.5^2 = 0.64$

The probability is 64 %.

Answer key:

The point for the guiding question is to be awarded if the probability has been calculated correctly.

Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

May 2020

Mathematics

Supplementary Examination 6
Examiner's Version

Bundesministerium
Bildung, Wissenschaft
und Forschung

Instructions for the supplementary examination

The following supplementary examination is comprised of five tasks that can be completed independently of one another.

Each task contains two parts: The statement of the task requires the candidate to demonstrate core competencies, and the guiding question that follows it requires the candidate to show their ability to communicate their ideas.

In the following document, the examiner will find the tasks as well as the expected solutions and the answer key.

The preparation time shall be at least 30 minutes and the examination time shall be at most 25 minutes.

Assessment

Each task can be awarded zero, one or two points. There is one point available for each demonstration of core competencies as well as for each guiding question. A maximum of 10 points can be achieved.

The following scale will be used for the grading of the examination:

Grade	Number of points
Pass	4 points for the core competencies + 0 points for the guiding questions 3 points for the core competencies + 1 point for the guiding questions
Satisfactory	5 points for the core competencies + 0 points for the guiding questions 4 points for the core competencies + 1 point for the guiding questions 3 points for the core competencies + 2 points for the guiding questions
Good	5 points for the core competencies + 1 point for the guiding questions 4 points for the core competencies + 2 points for the guiding questions 3 points for the core competencies + 3 points for the guiding questions
Very good	5 points for the core competencies + 2 (or more) points for the guiding questions 4 points for the core competencies + 3 (or more) points for the guiding questions

The examination board will decide on the final grade based on the candidate’s performance in the supplementary examination as well as the result of the written examination.

Evaluation grid for the supplementary examination

This evaluation grid may be used to assist the examiner in assessing the candidate's performance.

	Point for core competencies reached	Point for the guiding question reached
Task 1		
Task 2		
Task 3		
Task 4		
Task 5		

Task 1

Angle of a Slope

In order to determine the danger of avalanches, it is important to know the angle of a slope.

Task:

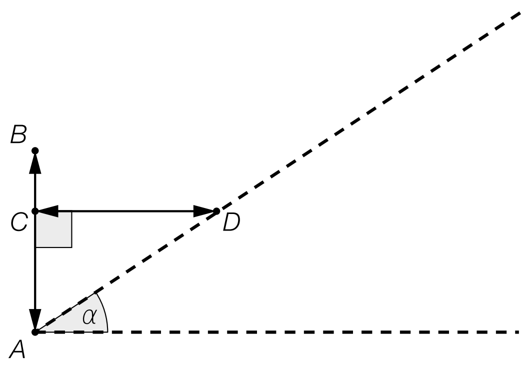
A particular slope has an angle of 30° to the horizontal.

– Determine the gradient of the slope as a percentage.

Guiding question:

The diagram below shows a method used to estimate the angle of a slope using ski poles. The angle of the slope, α, is determined using two ski poles of equal length, *AB* and *CD*.

The ski pole *CD* is held horizontally to the slope; the ski pole *AB* is held vertically at the end of the pole *CD* (as in the diagram).



– Write down the angle of the slope if using this method it is found that the points *B* and *C* have the same position as each other.

– Determine the angle of the slope, α, when the length of the line segment $\overline{BC}$ is one third of the length of the ski pole $\overline{AB}$.

Solution to Task 1

Angle of a Slope

Expected solution to the statement of the task:

$\tan(30°) = 0.57735... \approx 57.74\ \%$

Answer key:

The point for the core competencies is to be awarded if the gradient of the slope is given correctly as a percentage.

Expected solution to the guiding question:

If $B = C$, then the angle of the slope is 45°.

$\tan(\alpha) = \dfrac{\overline{AC}}{\overline{CD}} = \dfrac{2}{3} \quad \Rightarrow \quad \alpha \approx 33.7°$

Answer key:

The point for the guiding question is to be awarded if the angle of the slope is given correctly in both cases.

Task 2

Ideal Gas Equation

The equation $p \cdot V = n \cdot R \cdot T$ models the relationship between the pressure p, the volume V, the amount of the substance n, and the absolute temperature T of an ideal gas. In the equation, R is a constant.

Task:

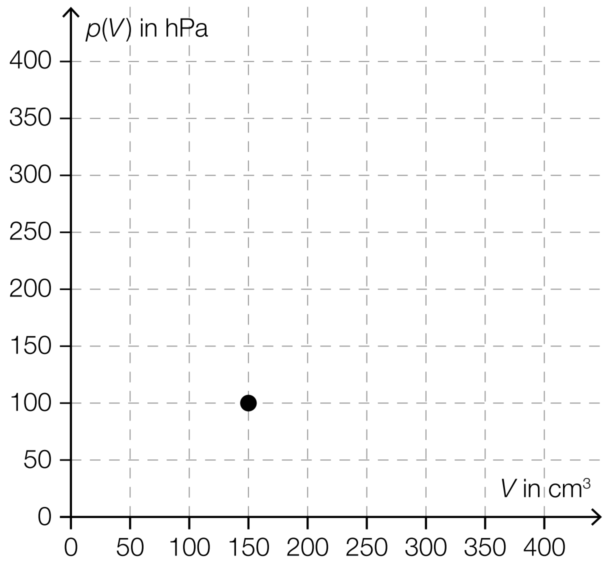
– Justify why the relationship between how the pressure p changes with respect to the temperature T can be modelled by a linear function of the form $p(T) = k \cdot T + d$ (where $k, d \in \mathbb{R}$) if the other values are constant.
– Write down the parameters k and d of this linear function (in terms of the values given above).

Guiding question:

The pressure p of an ideal gas can be given as a function of the volume V if the values of n, R and T are constant.

– Complete the table of values shown below, sketch the graph of the function p in the coordinate system and write down which type of function p is.

V in cm³	50	100	150	200	300
$p(V)$ in hPa			100		



Solution to Task 2

Ideal Gas Equation

Expected solution to the statement of the task:

If n, R and V are constant, then $p(T) = \frac{n \cdot R}{V} \cdot T$. This equation corresponds to a linear function with parameters $k = \frac{n \cdot R}{V}$ and $d = 0$.

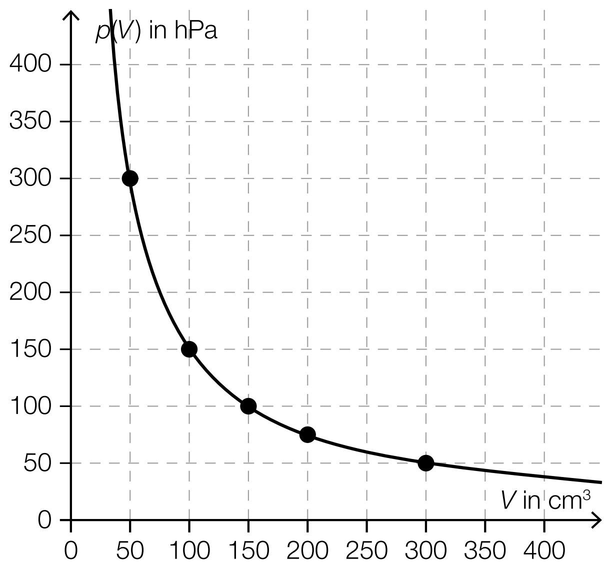
Answer key:

The point for the statement of the task is to be awarded if a correct justification has been given and the parameters of the corresponding linear function k and d have been given correctly.

Expected solution to the guiding question:

$p(V) = \frac{n \cdot R \cdot T}{V} \quad \Rightarrow \quad p(V) \cdot V = \text{constant} \quad \Rightarrow \quad p(V) \cdot V = 15\,000$

V in cm^3	50	100	150	200	300
$p(V)$ in hPa	300	150	100	75	50



The function p is a power function (or reciprocal function or indirectly proportional function).

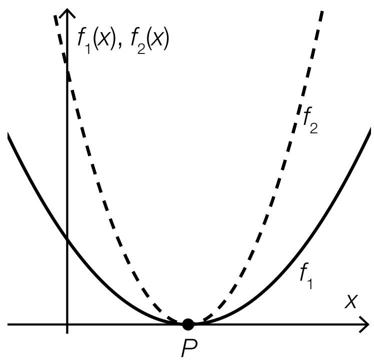
Answer key:

The point for the guiding question is to be awarded if the table of values has been completed correctly, a correct graph has been sketched and a correct function type has been given.

Task 3

Two Parabolas

The graphs of two functions f_1 and f_2 where $f_1(x) = a_1 \cdot x^2 + b_1 \cdot x + c_1$ and $f_2(x) = a_2 \cdot x^2 + b_2 \cdot x + c_2$ are shown in the diagram below.
The graphs of the two functions only have the point P on the positive x-axis in common.



Task:

– Complete each of the gaps shown below with a correct symbol "<"," >" or "=" so that the statements are true and justify your answer.

a_1 ______ a_2

c_1 ______ c_2

Guiding question:

– Given that $a_1 = 0.25$ and $P = (2,0)$, determine the values of the parameters b_1 and c_1 and explain your method.

Solution to Task 3

Two Parabolas

Expected solution to the statement of the task:

$a_1 < a_2$ because the graph corresponding to f_1 is “flatter” than the graph corresponding to f_2.
$c_1 < c_2$ because $f_1(0) < f_2(0)$.

Answer key:

The point for the core competencies is to be awarded if the symbols have been inserted correctly and correct justifications have been given.

Expected solution to the guiding question:

$b_1 = -1, \ c_1 = 1$

Possible method:

$f_1(x) = 0.25 \cdot x^2 + b_1 \cdot x + c_1$

$f_1'(x) = 0.5 \cdot x + b_1$

$f_1'(2) = 0 \quad \Rightarrow \quad 1 + b_1 = 0 \quad \Rightarrow \quad b_1 = -1$

$f_1(2) = 0 \quad \Rightarrow \quad 1 - 2 + c_1 = 0 \quad \Rightarrow \quad c_1 = 1$

Answer key:

The point for the guiding question is to be awarded if the values of both parameters have been given correctly and a correct method has been shown.

Task 4

Velocity of a Vehicle

The velocity of a vehicle between two sets of traffic lights in the time interval $[0, t_1]$ is modelled by the function v with $v(t) = -\frac{4}{15} \cdot t^2 + 4 \cdot t$ where t is given in s and $v(t)$ is given in m/s.
At the time $t = 0$, the vehicle is at the first set of traffic lights.

Task:

At time t_1 the vehicle comes to a stop at the second set of traffic lights.

– Write down the time t_1 and determine the distance covered by the vehicle in this time interval.

Guiding question:

– Determine the time $t_0 \in [0, t_1]$ at which the vehicle reaches its maximum velocity and write down this maximal velocity.

– The time t_2 is the time at which the vehicle has covered 80 % of the distance between the two sets of traffic lights. Using v, write down an equation with which the time t_2 can be found and determine the value of t_2.

Solution to Task 4

Velocity of a Vehicle

Expected solution to the statement of the task:

$v(t) = -\frac{4}{15} \cdot t^2 + 4 \cdot t = 0 \quad \Rightarrow \quad t_1 = 15 \text{ s}$

$s(t) = \int v(t)\,\mathrm{d}t = -\frac{4}{45} \cdot t^3 + 2 \cdot t^2 + c$

$s(0) = 0 \quad \Rightarrow \quad c = 0$

$s(15) = 150 \text{ m}$

Answer key:

The point for the core competencies is to be awarded if both the time as well as the distance covered have been given correctly.

Expected solution to the guiding question:

$v'(t_0) = 0 \quad \Rightarrow \quad -\frac{8}{15} \cdot t_0 + 4 = 0 \quad \Rightarrow \quad t_0 = 7.5 \text{ s}$

$v(7.5) = 15 \text{ m/s}$

Possible equation:

$\int_0^{t_2} v(t)\,\mathrm{d}t = 0.8 \cdot 150 \quad \Rightarrow \quad t_2 \approx 10.7 \text{ s}$

Answer key:

The point for the guiding question is to be awarded if both the correct time t_0 and the correct velocity $v(t_0)$ have been given along with a correct equation and the correct time t_2.

Task 5

Expanding a Data Set

A set of data that consists of six numbers is shown below:

$x_1 = 4,\ x_2 = 8,\ x_3 = 2,\ x_4 = 7,\ x_5 = 4,\ x_6$

The mean of the set of data is $\bar{x} = 5$.

Task:

– Determine the value of x_6 as well as the median of the set of data.

Guiding question:

– Expand the data set by writing down two whole numbers such that both of the following conditions are fulfilled and justify your answer:
  - The mean of the new data set is the same as the mean of the original data set.
  - The median of the new data set is greater than the original median.

Solution to Task 5

Expanding a Data Set

Expected solution to the statement of the task:

$\frac{4 + 8 + 2 + 7 + 4 + x_6}{6} = 5 \quad \Rightarrow \quad x_6 = 5$

Median: $\frac{4 + 5}{2} = 4.5$

Answer key:

The point for the core competencies is to be awarded if both the value of x_6 and the median have been given correctly.

Expected solution to the guiding question:

Numbers to be added to the data set: 5 and 5

Possible justification:

So that the mean $\bar{x}$ stays the same, the extra numbers must be of the form $\bar{x} - c$ and $\bar{x} + c$ with $c \in \mathbb{N}$.

Only when $c = 0$ and therefore only when the data set is expanded with the values 5 and 5 does the value of the median increase. With these values, the median of the data set 2, 4, 4, 5, 5, 5, 7, 8 is 5 > 4.5. For all values $c \in \mathbb{N}\backslash\{0\}$ the value of $5 - c$ lies below the original median and the value of $5 + c$ lies above the original median, which means that the expanded data set also has a median of 4.5.

Answer key:

The point for the guiding question is to be awarded if the correct values of both numbers have been given and the choice of the values has been justified correctly.

Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

October 2020

Mathematics

Supplementary Examination 2
Examiner's Version

Bundesministerium
Bildung, Wissenschaft
und Forschung

Instructions for the supplementary examination

The following supplementary examination is comprised of five tasks that can be completed independently of one another.

Each task contains two parts: The statement of the task requires the candidate to demonstrate core competencies, and the guiding question that follows it requires the candidate to show their ability to communicate their ideas.

In the following document, the examiner will find the tasks as well as the expected solutions and the answer key.

The preparation time shall be at least 30 minutes and the examination time shall be at most 25 minutes.

Assessment

Each task can be awarded zero, one or two points. There is one point available for each demonstration of core competencies as well as for each guiding question. A maximum of 10 points can be achieved.

The following scale will be used for the grading of the examination:

Grade	Number of points
Pass	4 points for the core competencies + 0 points for the guiding questions 3 points for the core competencies + 1 point for the guiding questions
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The examination board will decide on the final grade based on the candidate's performance in the supplementary examination as well as the result of the written examination.

Evaluation grid for the supplementary examination

This evaluation grid may be used to assist the examiner in assessing the candidate's performance.

	Point for core competencies reached	Point for the guiding question reached
Task 1		
Task 2		
Task 3		
Task 4		
Task 5		

Task 1

Flight Paths

The three aeroplanes F_1, F_2 and F_3 fly at the same height for a particular period of time. Their flight paths over this period of time can be modelled by the three lines f_1, f_2 and f_3.

The following statements hold: f_1: $X = A + r \cdot \vec{v}_1$ with $r \in \mathbb{R}^+$

f_2: $X = B + s \cdot \vec{v}_2$ with $s \in \mathbb{R}^+$

f_3: $X = C + u \cdot \vec{v}_3$ with $u \in \mathbb{R}^+$

At the time $t = 0$ the aeroplane F_1 is at point $A = (a, 40)$ with $a \in \mathbb{R}$ and the aeroplane F_2 is at point $B = (–30, 20)$. The velocity vectors of the aeroplanes are given by $\vec{v}_1 = \vec{v}_3 = \begin{pmatrix} 10 \\ -10 \end{pmatrix}$ and $\vec{v}_2 = \begin{pmatrix} 5 \\ b \end{pmatrix}$ with $b \in \mathbb{R}$.
The parameters r, s and u give the flight times in minutes from the time $t = 0$. The velocities of the aeroplanes are given in km/min.

Task:

– Determine the values of a and b for which the flight paths of F_1 and F_2 are identical.

Guiding question:

At the time $t = 0$ the aeroplane F_3 is at point $C = (–20, 40)$.

– For this case, determine the value of b such that the flight paths of F_2 and F_3 cross each other at right angles.
– Determine the point S of intersection of these flight paths and justify why there is no collision between the two aeroplanes.

Solution to Task 1

Flight Paths

Expected solution to the statement of the task:

possible method:

$f_1\colon X = \begin{pmatrix} a \\ 40 \end{pmatrix} + r \cdot \begin{pmatrix} 10 \\ -10 \end{pmatrix}$

$f_2\colon X = \begin{pmatrix} -30 \\ 20 \end{pmatrix} + s \cdot \begin{pmatrix} 5 \\ b \end{pmatrix}$

$\vec{v}_1 = \begin{pmatrix} 10 \\ -10 \end{pmatrix}$ is parallel to $\vec{v}_2 = \begin{pmatrix} 5 \\ b \end{pmatrix} \quad \Rightarrow \quad b = -5$

$\begin{pmatrix} -30 \\ 20 \end{pmatrix} = \begin{pmatrix} a \\ 40 \end{pmatrix} + r \cdot \begin{pmatrix} 10 \\ -10 \end{pmatrix} \quad \Rightarrow \quad r = 2,\ a = -50$

$a = -50$
$b = -5$

Answer key:

The point for the core competencies is to be awarded if the values of a and b have been correctly determined.

Expected solution to the guiding question:

possible method:

$\vec{v}_2 \cdot \vec{v}_3 = 0 \quad \Rightarrow \quad \begin{pmatrix} 5 \\ b \end{pmatrix} \cdot \begin{pmatrix} 10 \\ -10 \end{pmatrix} = 0 \quad \Rightarrow \quad b = 5$

$\begin{pmatrix} -20 \\ 40 \end{pmatrix} + u \cdot \begin{pmatrix} 10 \\ -10 \end{pmatrix} = \begin{pmatrix} -30 \\ 20 \end{pmatrix} + s \cdot \begin{pmatrix} 5 \\ 5 \end{pmatrix} \quad \Rightarrow \quad u = 0.5;\ s = 3 \quad \Rightarrow \quad S = (-15, 35)$

There is no collision because the flight times until point S (with $u = 0.5$ min and $s = 3$ min) are not the same.

Answer key:

The point for the guiding question is to be awarded if the value of the parameter b and the point S of intersection have been correctly determined and a correct justification has been given.

Task 2

Triangle

For an $a \in \mathbb{R}^+$, the linear function g with $g(x) = -2 \cdot a \cdot x + 2$ is given. A triangle is bounded by the graph of g and the two coordinate axes.

Task:

– Write down the area A of this triangle in terms of a.

$A(a) =$ ______________________

Guiding question:

A change in the value of a results in a change of the area A of the triangle.

– Write down how A changes when a is doubled.

– Write down by which percentage A changes if a is reduced by 20 %.

Solution to Task 2

Triangle

Expected solution to the statement of the task:

possible method:

Points of intersection with the axes: $\left(\frac{1}{a}, 0\right)$ and (0, 2)

$A(a) = \frac{1}{a}$

Answer key:

The point for the core competencies is to be awarded if a correct function has been given.

Expected solution to the guiding question:

possible method:

$A(2 \cdot a) = \frac{1}{2 \cdot a} = \frac{1}{2} \cdot \frac{1}{a}$

If a is doubled, then A is halved.

$A(0.8 \cdot a) = \frac{1}{0.8 \cdot a} = 1.25 \cdot \frac{1}{a}$

If a is reduced by 20 %, then A becomes 25 % larger.

Answer key:

The point for the guiding question is to be awarded if both of the changes in area have been given correctly.

Task 3

Examination

In a school, class 8a has 27 pupils and class 8b has 24 pupils.

The last examination was held simultaneously in all 8th classes and all students from 8a and 8b were present. In 8a, one more "very good" grade was awarded than in 8b. The relative proportion of "very good" grades was the same in both classes.

Task:

– Determine the number of examinations that were awarded the grade "very good" in 8a.

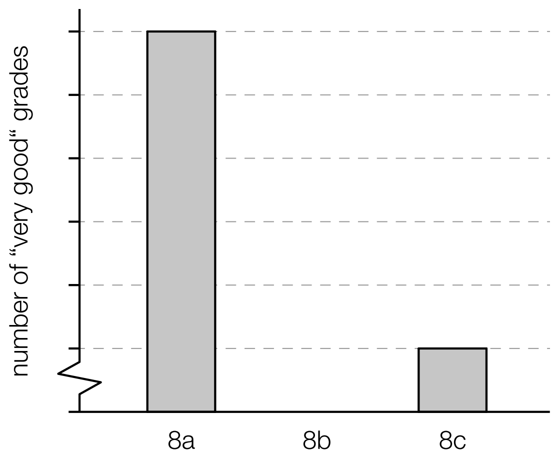
Guiding question:

A group of 9 pupils from 8c also completed this examination.

In total, 35 % of all pupils that took this examination were awarded a "very good" grade.

– Write down the number of pupils in 8c who were awarded a "very good" grade for this examination.

The bar chart shown below shows a graphical representation of the results of this examination.



– In the diagram above, label the vertical axis with a scale so that the situation described is represented correctly and complete the diagram by drawing the bar for the results in 8b.

– Write down a reason why this diagram could be regarded as biased.

Solution to Task 3

Examination

Expected solution to the statement of the task:

possible method:

n ... number of “very good” grades in class 8a
$n - 1$ ... number of “very good” grades in class 8b

$\frac{n}{27} = \frac{n-1}{24} \quad \Rightarrow \quad n = 9$

Nine examinations from 8a were awarded a “very good” grade.

Answer key:

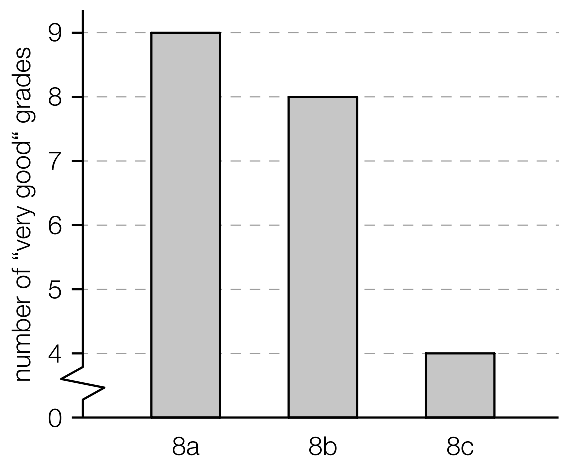
The point for the core competencies is to be awarded if the number of “very good” grades has been correctly determined.

Expected solution to the guiding question:

possible method:

$0.35 \cdot (27 + 24 + 9) = 21$
$21 - 9 - 8 = 4$

Four pupils in 8c were awarded a ”very good” grade.



possible reasons:

As only 9 pupils from class 8c participated in the examination, the relative proportions would be more informative than the absolute frequencies.

or:

The contraction of the vertical axis gives the impression that a lot fewer “very good” grades were awarded in 8c than in 8a.

Answer key:

The point for the guiding question is to be awarded if the correct number of pupils has been given, the diagram has been completed correctly, and a correct reason has been given.

Task 4

Integral

Task:

Let f be a linear (non-constant) function for which $\int_0^3 f(x)\,\mathrm{d}x = 0$ holds.

– Write down the zero of f and justify your answer.

Guiding question:

Let g be a quadratic function with $g(x) = a \cdot x^2 + b$ $(a, b \in \mathbb{R}, a \neq 0)$ for which $\int_0^3 g(x)\,\mathrm{d}x = 0$ holds.

– Based on the behaviour of the graph of g, justify why a and b must have different signs.

– Write down the relationship between a and b by using an equation.

Solution to Task 4

Integral

Expected solution to the statement of the task:

The zero of f is 1.5.

The graph of f must enclose two equally large areas with the x-axis (one above the x-axis and the other below the x-axis). Therefore the zero must lie exactly in the middle of the interval [0, 3].

Answer key:

The point for the core competencies is to be awarded if the correct zero and a correct justification have been given.

Expected solution to the guiding question:

The parabola must cross the x-axis (between 0 and 3). Either the parabola is concave up and the vertex is below the x-axis ($a > 0$, $b < 0$), or the parabola is concave down and the vertex is above the x-axis ($a < 0$, $b > 0$).

$$\int_0^3 g(x)\,\mathrm{d}x = 0$$

$$\int_0^3 (a \cdot x^2 + b)\,\mathrm{d}x = \frac{a \cdot x^3}{3} + b \cdot x\Big|_0^3$$

$$\Rightarrow \quad 3 \cdot a + b = 0$$

Answer key:

The point for the guiding question is to be awarded if a correct justification and a correct equation have been given.

Task 5

Driving Test

The theoretical part of the driving test comprises solely multiple-choice questions. Each question has four possible answers, and at least one of these possible answers is correct. A question is considered to be answered if at least one possible answer has been selected. A question has been answered correctly if the correct possible answer(s) has/have been selected.

Task:

– Determine the number of possible ways that a multiple choice question of this type could be answered.

Guiding question:

Assume that Elias has to guess for every question of the theoretical part of his driving test and for each question chooses one way of answering the question at random from all the possible ways that a question can be answered. The probability that he chooses one particular way of answering a question is the same for all possible ways.

– Write down the probability that Elias answers a multiple choice question correctly.

For the theoretical part of the driving test, 20 multiple choice questions about *basic knowledge* are asked first.

– Determine the probability that Elias answers less than 20 % of the multiple choice questions about *basic knowledge* correctly and therefore has to attend the theory course again.

Solution to Task 5

Driving Test

Expected solution to the statement of the task:

$\binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{4}{4} = 4 + 6 + 4 + 1 = 15$

There are 15 possible ways of answering a multiple choice question of this type.

Answer key:

The point for the core competencies is to be awarded if the number has been calculated correctly.

Expected solution to the guiding question:

$p = \frac{1}{15}$

The number X of multiple choice questions that have been answered correctly is binomially distributed with parameters $n = 20$ and $p = \frac{1}{15}$.

$P(X < 4) = P(X \leq 3) = 0.959... \approx 96\ \%$

Answer key:

The point for the guiding question is to be awarded if the probabilities have been determined correctly.

Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

January 2021

Mathematics

Supplementary Examination 2
Examiner's Version

Bundesministerium
Bildung, Wissenschaft
und Forschung

Instructions for the supplementary examination

The following supplementary examination is comprised of five tasks that can be completed independently of one another.

Each task contains two parts: The statement of the task requires the candidate to demonstrate core competencies, and the guiding question that follows it requires the candidate to show their ability to communicate their ideas.

In the following document, the examiner will find the tasks as well as the expected solutions and the answer key.

The preparation time shall be at least 30 minutes and the examination time shall be at most 25 minutes.

Assessment

Each task can be awarded zero, one or two points. There is one point available for each demonstration of core competencies as well as for each guiding question. A maximum of 10 points can be achieved.

The following scale will be used for the grading of the examination:

Grade	Number of points
Pass	4 points for the core competencies + 0 points for the guiding questions 3 points for the core competencies + 1 point for the guiding questions
Satisfactory	5 points for the core competencies + 0 points for the guiding questions 4 points for the core competencies + 1 point for the guiding questions 3 points for the core competencies + 2 points for the guiding questions
Good	5 points for the core competencies + 1 point for the guiding questions 4 points for the core competencies + 2 points for the guiding questions 3 points for the core competencies + 3 points for the guiding questions
Very good	5 points for the core competencies + 2 (or more) points for the guiding questions 4 points for the core competencies + 3 (or more) points for the guiding questions

The examination board will decide on the final grade based on the candidate's performance in the supplementary examination as well as the result of the written examination.

Evaluation grid for the supplementary examination

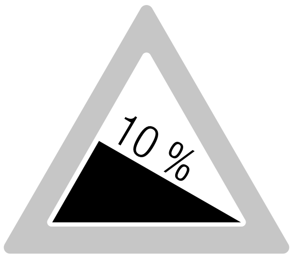
This evaluation grid may be used to assist the examiner in assessing the candidate's performance.

	Point for core competencies reached	Point for the guiding question reached
Task 1		
Task 2		
Task 3		
Task 4		
Task 5		

Task 1

Angle of Depression

The gradient of steeply ascending or steeply descending roads is given as a percentage. The traffic sign shown below states that the height of this road decreases by 10 m for each horizontal distance of 100 m.



Task:

Sonja claims: "If a road has a gradient of 10 %, then the angle of depression of this road is approximately twice as large as a road with a gradient of 5 %."

– Determine both angles of depression.

– Write down whether Sonja's claim is correct or incorrect.

Guiding question:

Martin writes down the following relationship for small angles α:

$\tan(2 \cdot \alpha) \approx 2 \cdot \tan(\alpha)$

– Interpret this expression in the given context.

– Justify why this relationship cannot hold for $\alpha = 45°$.

Solution to Task 1

Angle of Depression

Expected solution to the statement of the task:

$\tan(\alpha_1) = \frac{10}{100} \quad \Rightarrow \quad \alpha_1 = 5.710...° \approx 5.71°$

$\tan(\alpha_2) = \frac{5}{100} \quad \Rightarrow \quad \alpha_2 = 2.862...° \approx 2.86°$

Sonja's claim is correct because $2 \cdot \alpha_2 \approx \alpha_1$ holds.

Answer key:

The point for the core competency is to be awarded if the angles of depression have been calculated correctly and the correctness of Sonja's claim has been recognised.

Expected solution to the guiding question:

possible interpretation:

This expression means that if the angle of depression is doubled then the gradient also approximately doubles.
For $\alpha = 45°$, $\tan(2 \cdot \alpha)$ is not defined.

Answer key:

The point for the guiding question is to be awarded if the expression has been correctly interpreted in the context and it has been justified why this relationship cannot hold for $\alpha = 45°$.

Task 2

Test Tracks

From 1st August 2018 to 29th February 2020, the maximum speed on sections of a motorway in Upper Austria and Lower Austria was increased to 140 km/h for a trial period. The time saved in comparison to the usual permitted maximum speed of 130 km/h (assuming each value is an average speed) is shown in the diagram below.

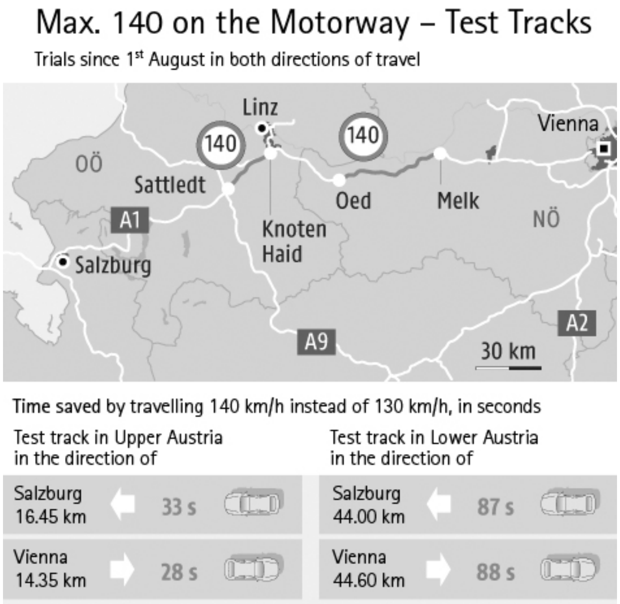


Image source: https://ooe.orf.at/v2/news/stories/2947525/ [26.09.2019] (adapted).

Task:

– Show by calculation that the value of 33 s given for the time saved on the test track in Upper Austria in the direction of Salzburg is correct.

Guiding question:

Michael drives from Vienna to Salzburg with a constant speed of 140 km/h on both of these test tracks. In total, the time saved is 87 s + 33 s = 2 min.
If a different constant speed v (in km/h) is chosen for both of these test tracks, then the time saved is e.

– Write down an expression that can be used to calculate the corresponding constant speed v (in km/h) in terms of the time saved e in minutes (on the route from Vienna to Salzburg).

$v =$ ____________________

Solution to Task 2

Test Tracks

Expected solution to the statement of the task:

$\left(\frac{16.45}{130} - \frac{16.45}{140}\right) \cdot 3\,600 = 32.5... \approx 33 \quad \Rightarrow$ The given value of 33 s is correct.

Answer key:

The point for the core competency is to be awarded if a correct calculation has been given as a justification.

Expected solution to the guiding question:

possible expression:

$$\left(\left(\frac{16.45}{130} - \frac{16.45}{v}\right) + \left(\frac{44}{130} - \frac{44}{v}\right)\right) \cdot 60 = e$$

$$\Rightarrow \quad v = \frac{-36\,270}{10 \cdot e - 279}$$

Answer key:

The point for the guiding question is to be awarded if a correct expression has been given.

Task 3

Polynomial Functions

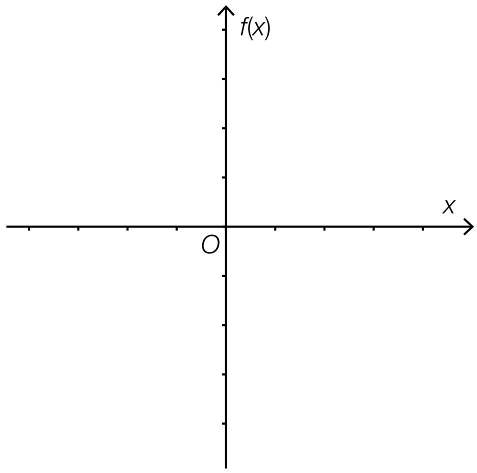
The number of zeros, local maxima and minima and points of inflexion is dependent, among other things, on the degree of a polynomial function.

Task:

– In the coordinate system shown below, sketch the graph of a polynomial function *f* such that exactly one zero and exactly three local maxima or minima are shown.

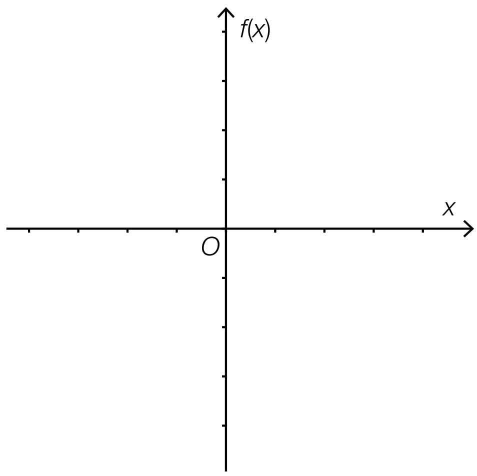
All polynomial functions that fulfil the condition given above are at least of degree *n*.

– Write down *n*.



Guiding question:

– Write down how the number of zeros for the given section of the graph you sketched above changes through vertical translation of the graph and how the equations of these polynomial functions (with different numbers of zeros) differ from each other.
– Sketch the graph of a fourth degree polynomial function *f* that has the smallest possible number of zeros, local maxima and minima and points of inflexion.

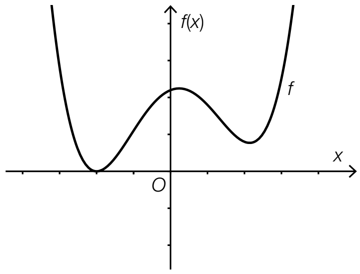


Solution to Task 3

Polynomial Functions

Expected solution to the statement of the task:

possible graph:



$n = 4$

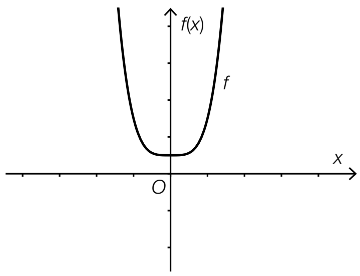
Answer key:

The point for the core competency is to be awarded if a possible graph has been sketched correctly and the correct value of n has been given.

Expected solution to the guiding question:

By translating the sketched graph in the vertical direction, the graph could have 0, 1, 2, 3, or 4 zeros.
The equations of these polynomial functions only differ by a constant value.

possible graph:



The graph shown has no zeros, only one local maximum or minimum and no points of inflexion.

Answer key:

The point for the guiding question is to be awarded if the number of zeros and the difference between the equations of the functions have been given correctly and a possible graph has been sketched correctly.

Task 4

Cooling of a Liquid

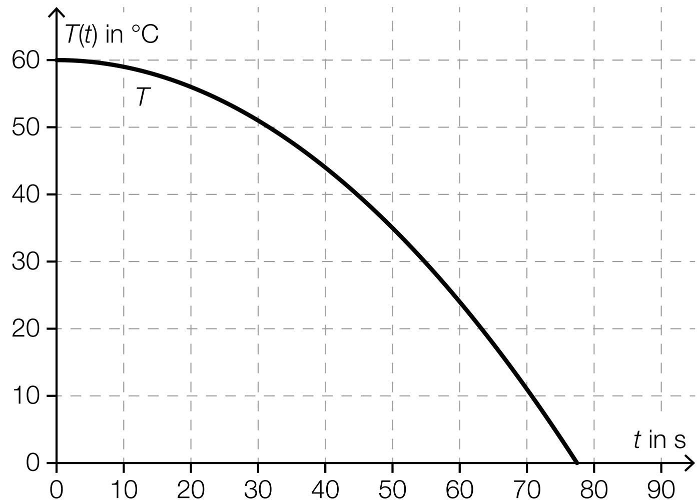
The temperature T of a cooling liquid can be approximated in terms of the time t by the function $T(t) = 60 - 0.01 \cdot t^2$ (t in s, $T(t)$ in °C).

Task:

– Write down the average rate of change of the temperature in the interval [30, 70] and interpret the result in the given context.

Guiding question:

– Sketch the average rate of change calculated above graphically (using the diagram shown below).
– Explain how the point t_1 on the graph of T for which the instantaneous rate of change is equal to the average rate of change calculated above can be determined. Write down the value of t_1.



Solution to Task 4

Cooling of a Liquid

Expected solution to the statement of the task:

average rate of change: $\frac{T(70) - T(30)}{70 - 30} = \frac{11 - 51}{40} = -1$

possible interpretation:

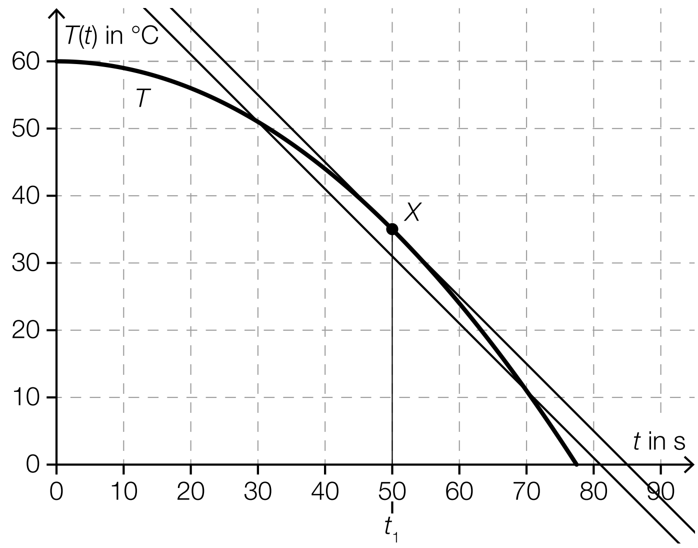
In the time interval [30, 70], the temperature reduces by an average of 1 °C per second.

Answer key:

The point for the core competency is to be awarded if the correct average rate of change and a correct interpretation have been given.

Expected solution to the guiding question:

The average rate of change calculated above is equal to the gradient of the secant function of T over the interval [30, 70].



The point t_1 can be determined graphically by finding the point X on T for which the gradient of the tangent is equal to the gradient of the secant line (i. e. these lines must be parallel).

Calculation of t_1: $T'(t_1) = -0.02 \cdot t_1 = -1 \quad \Rightarrow \quad t_1 = 50$

Answer key:

The point for the guiding question is to be awarded if the average rate of change has been correctly identified as the gradient of the secant function, a method for determining t_1 has been correctly explained, and the correct value for t_1 has been calculated.

Task 5

Normally Approximated Random Variable

The normal approximation of a binomially distributed random variable X results in a random variable Y with an expectation value μ and a standard deviation σ.

Task:

– Describe and determine the probabilities given below.

- $P(Y < \mu - \sigma)$
- $P(\mu - 2 \cdot \sigma \leq Y \leq \mu + 2 \cdot \sigma)$

Guiding question:

– Draw the probabilities determined in the task above graphically (as areas under the graph of an appropriate function) and explain the shape of the graph of the function by referring to local maxima/minima and the symmetry of the graph.

Solution to Task 5

Normally Approximated Random Variable

Expected solution to the statement of the task:

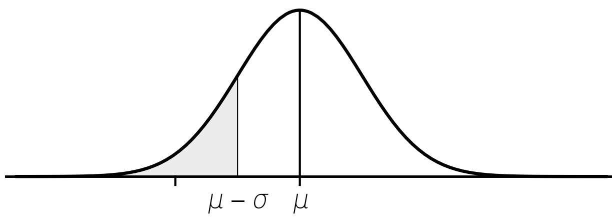
- The expression describes the probability of the random variable taking a value that is less than one standard deviation below the expectation value.
  $P(Y < \mu - \sigma) \approx 0.159$
- The expression describes the probability of the random variable taking a value that is at most two standard deviations above or below the expectation value.
  $P(\mu - 2 \cdot \sigma \leq Y \leq \mu + 2 \cdot \sigma) \approx 0.954$

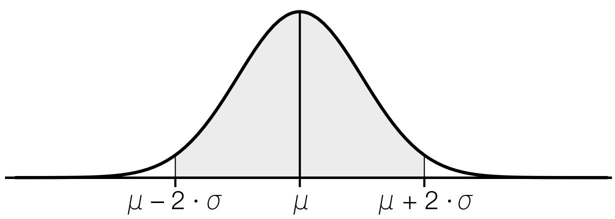
Answer key:

The point for the core competency is to be awarded if both of the probabilities have been determined and described correctly.

Expected solution to the guiding question:

density of Y:





The graph (the Gaussian bell curve) is symmetrical about the expectation value and has a local maximum at this value.

Answer key:

The point for the guiding question is to be awarded if both of the probabilities have been represented correctly on a graph and the shape of the graph has been explained correctly.

Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

May 2019

Mathematics

Supplementary Examination 6
Examiner's Version

Bundesministerium
Bildung, Wissenschaft
und Forschung

Instructions for the supplementary examination

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Each task contains two parts: The statement of the task requires the candidate to demonstrate core competencies, and the guiding question that follows it requires the candidate to show their ability to communicate their ideas.

In the following document, the examiner will find the tasks as well as the expected solutions and the answer key.

The preparation time is to be at least 30 minutes and the examination time is to be at most 25 minutes.

Assessment

Each task can be awarded zero, one or two points. There is one point available for each demonstration of core competencies as well as for each guiding question. A maximum of 10 points can be achieved.

For the grading of the examination the following scale should be used:

Grade	Number of points
Pass	4 points for the core competencies + 0 points for the guiding questions 3 points for the core competencies + 1 point for the guiding questions
Satisfactory	5 points for the core competencies + 0 points for the guiding questions 4 points for the core competencies + 1 point for the guiding questions 3 points for the core competencies + 2 points for the guiding questions
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The examination board will decide on the final grade based on the candidate’s performance in the supplementary examination as well as the result of the written examination.

Evaluation grid for the supplementary examination

This evaluation grid may be used to assist the examiner in assessing the candidate's performance.

	Point for core competencies reached	Point for the guiding question reached
Task 1		
Task 2		
Task 3		
Task 4		
Task 5		

Task 1

Lines in $\mathbb{R}^2$

Let A and B be points where $A = (5,1)$ and $B = (1,2)$.

Task:

The lines p, n and s each go through the point A and can be described as follows:

- The line p is parallel to the x-axis.
- The line n is perpendicular to the x-axis.
- The line s has a gradient of 1.

Write down an equation for each of the three lines.

Guiding question:

A line g: $y = k \cdot x + d$ goes through the point B and makes an angle of α to the horizontal.

Write down expressions for k and d in terms of the angle α.

k = ______________________________

d = ______________________________

Write down the size of the angle $\alpha \in [0°, 90°)$ for which the lines g and p do not intersect.

The line g_1 goes through the point B and has the same gradient as the line s. Determine the point of intersection, S, of the line n and the line g_1.

Solution to Task 1

Lines in $\mathbb{R}^2$

Expected solution to the statement of the task:

Possible equations:
p: $y = 1$
n: $x = 5$
s: $y = x - 4$

Answer key:

The point for the core competencies is to be awarded if correct equations or vector equations for the lines p, n, and s have been given. Equivalent equations or vector equations are also to be marked as correct.

Expected solution to the guiding question:

$k = \tan(\alpha)$
$d = 2 - \tan(\alpha)$

The slope of both lines is zero, therefore $\alpha = 0°$.

Thus, g_1: $y = x + 1 \quad \Rightarrow \quad S = (5, 6)$.

Answer key:

The point for the guiding question is to be awarded if k and d have been given correctly and the angle α as well as the point of intersection of the lines g_1 and n have been given correctly.

Task 2

Barometric Altitude Formula

The relationship between the height h above sea level and the air pressure $p(h)$ at that height can be approximated by the barometric altitude formula:

$p(h) = p_0 \cdot e^{-\frac{h}{7991}}$

h ... height above sea level in metres (m)
$p(h)$ ... air pressure at a height of h in hectopascals (hPa)
p_0 ... air pressure at sea level (at $h = 0$); $p_0 > 0$

Task:

Determine the height h_1 at which the air pressure is only 80 % of p_0.

Guiding question:

The relationship between the height above sea level and the air pressure in the interval [0 m, 3500 m] can be approximated by a linear function f (in terms of h).

On a particular day the following values were obtained:

Height above sea level in m	Air pressure in hPa
1500	840
2000	790

Determine an equation of this function f so that it reflects the measured values.

For the height h_1 found above, determine the difference (in hPa) between the value of $p(h_1)$ calculated using the barometric altitude formula and the value calculated using the linear function f. In your calculations, assume that the value of p_0 in the function p is 1013 hPa.

Solution to Task 2

Barometric Altitude Formula

Expected solution to the statement of the task:

$0.8 \cdot p_0 = p_0 \cdot e^{-\frac{h_1}{7991}}$

$h_1 \approx 1\,783$ m

Answer key:

The point for the core competencies is to be awarded if the correct height h_1 has been determined.
Tolerance interval: [1 780 m, 1 790 m]

Expected solution to the guiding question:

Possible method:

$f(h) = -0.1 \cdot h + 990$

$p(h) = 1\,013 \cdot e^{-\frac{h}{7991}}$

$f(1\,783) = 811.7$ hPa

$p(1\,783) \approx 810.4$ hPa

The difference is around 1.3 hPa.

Answer key:

The point for the guiding question is to be awarded if a correct function of *f* has been given and the difference between the two values has been calculated correctly.
Tolerance interval: [1 hPa, 2 hPa]

Task 3

Rates of Change

Let $f: \mathbb{R} \to \mathbb{R}$ be a function where $f(x) = x^2 - 2 \cdot x - 1$.

Task:

Determine the differential quotient of f when $x = 1$ and write down the meaning of this value in terms of the behaviour of the graph.

Guiding question:

Write down an expression in terms of the parameter a ($a \in \mathbb{R}$ and $a < 3$) that can be used to calculate the difference quotient of f in the interval $[a, 3]$.

Determine the value of the parameter a such that this difference quotient is equal to the differential quotient of f when $x = 1$.

Solution to Task 3

Rates of Change

Expected solution to the statement of the task:

$f'(x) = 2 \cdot x - 2 \quad \Rightarrow \quad f'(1) = 0$

Possible meaning:
The graph of the function f has a horizontal tangent when $x = 1$.

Answer key:

The point for the core competencies is to be awarded if the differential quotient has been determined correctly and the meaning of the value for the shape of a graph has been explained correctly.

Expected solution to the guiding question:

Possible expression:

$$\frac{f(3) - f(a)}{3 - a} = \frac{2 - (a^2 - 2 \cdot a - 1)}{3 - a} = \frac{-a^2 + 2 \cdot a + 3}{3 - a} \ (= a + 1)$$

$-a^2 + 2 \cdot a + 3 = 0 \quad \Rightarrow \quad a = -1$

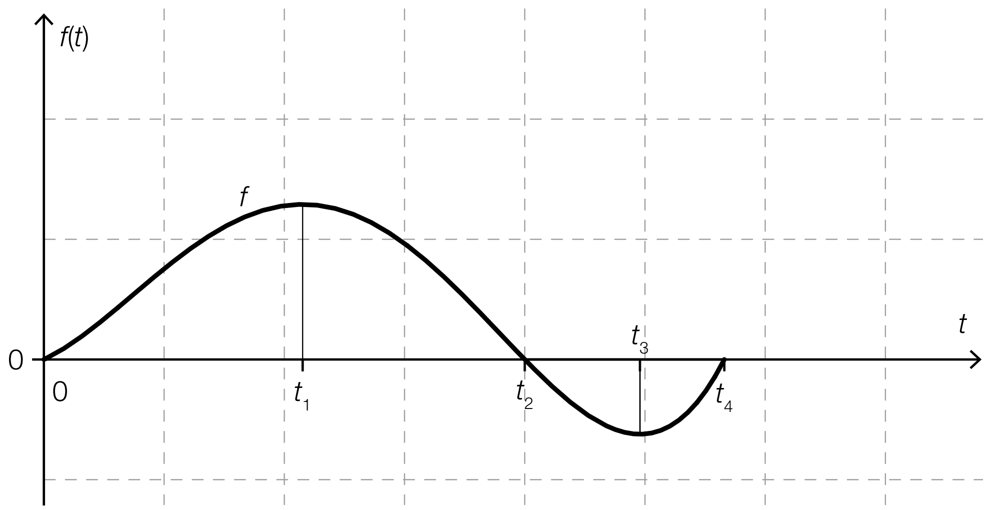
Answer key:

The point for the guiding question is to be awarded if a correct expression and the correct value for a have been given.

Task 4

Rain Water

There are 20 litres of water in a water butt.
From the time $t = 0$ the amount of water in the butt changes. The function f describes the instantaneous rate of change of the amount of water in the butt in terms of time ($f(t)$ is measured in litres per hour, t is measured in hours).
The diagram below shows the graph of the function f.



Task:

Write down at which point in time marked in the diagram above (from t_1 to t_4) the amount of water in the butt is highest and justify your decision.

Guiding question:

Write down a formula for the amount of water M in the butt at time t_4.

In the diagram above in the interval $[0, t_4)$, label the approximate time t^* at which there is exactly the same amount of water in the butt as at t_4. Explain your method.

Solution to Task 4

Rain Water

Expected solution to the statement of the task:

At time t_2 the amount of water in the butt is the highest because the value of f is positive until this time, which means that the amount of water in the butt increases until this time (afterwards the value of f is negative, which means that the amount of water in the butt decreases after this point).

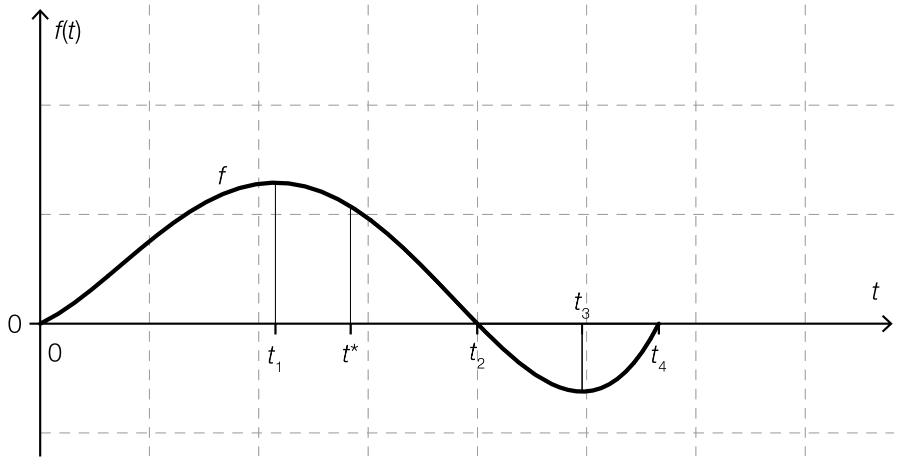
Answer key:

The point for the core competencies is to be awarded if the time t_2 has been stated and this decision has been justified correctly.

Expected solution to the guiding question:

Possible formula:

$M = 20 + \int_0^{t_4} f(t)\,dt$



The areas bounded by the graph of f and the x-axis in the intervals $[t^*, t_2]$ and $[t_2, t_4]$ must be the same size because the amount of water in the butt in the interval $[t^*, t_2]$ has to increase by the same amount as it decreases in the interval $[t_2, t_4]$.

Answer key:

The point for the guiding question is to be awarded if a correct formula for M has been given, the time t^* has been labelled in approximately the correct place, and the method has been explained correctly.

Task 5

Multi-Step Random Experiment

In a game three urns, each containing six balls, are used. Urn *A* contains five white balls and one black ball, urn *B* contains four white and two black balls, and urn *C* contains three white and three black balls. A player wins the game if a white ball is selected.

Victoria participates in this game and thus has to complete the following steps:
First, she chooses one urn and takes a ball out of this urn. In both cases (the choice of the urn and the choice of the ball) the selections are made at random.

Task:

Determine the probability that Victoria wins the game.

Guiding question:

In one option of the game, twelve white and six black balls are being distributed anew, whereby there are six balls again in each of the three urns. In the first urn there are x white balls now, in the second urn there are y white balls, and in the third urn there are z white balls.
Show by calculation that the probability of Victoria winning does not change.

Solution to Task 5

Multi-Step Random Experiment

Expected solution to the statement of the task:

$P(\text{“Victoria wins”}) = \frac{1}{3} \cdot \frac{5}{6} + \frac{1}{3} \cdot \frac{4}{6} + \frac{1}{3} \cdot \frac{3}{6} = \frac{2}{3}$

Answer key:

The point for the core competencies is to be awarded if the probability has been calculated correctly.

Expected solution to the guiding question:

$P(\text{“Victoria wins”}) = \frac{1}{3} \cdot \frac{x}{6} + \frac{1}{3} \cdot \frac{y}{6} + \frac{1}{3} \cdot \frac{z}{6} = \frac{1}{3} \cdot \frac{x + y + z}{6} = \frac{1}{3} \cdot \frac{12}{6} = \frac{2}{3}$

Answer key:

The point for the guiding question is to be awarded if it has been correctly shown by calculation that the probability of Victoria winning is $\frac{2}{3}$ again.

Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

October 2019

Mathematics

Supplementary Examination 2
Examiner's Version

Bundesministerium
Bildung, Wissenschaft
und Forschung

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In the following document, the examiner will find the tasks as well as the expected solutions and the answer key.

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Assessment

Each task can be awarded zero, one or two points. There is one point available for each demonstration of core competencies as well as for each guiding question. A maximum of 10 points can be achieved.

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The examination board will decide on the final grade based on the candidate's performance in the supplementary examination as well as the result of the written examination.

Evaluation grid for the supplementary examination

This evaluation grid may be used to assist the examiner in assessing the candidate's performance.

	Point for core competencies reached	Point for the guiding question reached
Task 1		
Task 2		
Task 3		
Task 4		
Task 5		

Task 1

Sets of Numbers

Numbers are contained in one or more sets of numbers.

Task:

– For each of the numbers shown below, put a cross next to each set of numbers that the number is an element of.

	$\mathbb{Z}^-$	$\mathbb{Q}$	$\mathbb{R}^+$
$\frac{\pi}{2}$			
$3 \cdot \sqrt{3}$			
$-\frac{16}{8}$			
$1.23 \cdot 10^{-3}$			

Guiding question:

If the result of an operation on any two numbers that are members of a particular set is also a member of this set, then this set is said to be closed under this operation.
For example: for any $a, b \in \mathbb{N}$ holds: $a \cdot b \in \mathbb{N}$. Therefore, the set of natural numbers is closed under multiplication.

Consider the operations subtraction, multiplication and taking a square root.

– Write down whether the set of numbers $\mathbb{Q}^-$ is closed under the operations given above and justify your answers.

Solution to Task 1

Sets of Numbers

Expected solution to the statement of the task:

	$\mathbb{Z}^-$	$\mathbb{Q}$	$\mathbb{R}^+$
$\frac{\pi}{2}$			×
$3 \cdot \sqrt{3}$			×
$-\frac{16}{8}$	×	×	
$1.23 \cdot 10^{-3}$		×	×

Answer key:

The point for the core competencies shall be awarded if the correct set(s) of numbers has/have been crossed for each of the numbers.

Expected solution to the guiding question:

A set of numbers is not closed under an operation if at least one counter-example exists.

The set of numbers $\mathbb{Q}^-$ is not closed under any of the operations listed because, for example:

- Subtraction: $-\frac{1}{3} - \left(-\frac{1}{2}\right) = \frac{1}{6} \notin \mathbb{Q}^-$
- Multiplication: $-\frac{1}{3} \cdot \left(-\frac{1}{2}\right) = \frac{1}{6} \notin \mathbb{Q}^-$
- Taking a square root: $\sqrt{-\frac{1}{6}} \notin \mathbb{Q}^-$

Answer key:

The point for the guiding question shall be awarded if it has been correctly determined for each of the operations that the set of numbers is not closed and this has been justified correctly in each case.

Task 2

Solutions to Quadratic Equations

Let $x^2 - 2 \cdot x = p$ with $p \in \mathbb{R}$ be a quadratic equation.

Task:

– Write down all values of p for which the equation shown above has solutions in the set $\mathbb{R}$.

Guiding question:

– Write down the possible solution scenarios for a quadratic equation of the form $a \cdot x^2 + b \cdot x + c = 0$ $(a, b, c \in \mathbb{R}; a \neq 0)$ and show these scenarios graphically by sketching an appropriate graph of a quadratic function for each scenario.

Solution to Task 2

Solutions to Quadratic Equations

Expected solution to the statement of the task:

$x^2 - 2 \cdot x - p = 0$
$x_{1,2} = 1 \pm \sqrt{1 + p}$
$1 + p \geq 0 \quad \Rightarrow \quad p \geq -1$

Answer key:

The point for the core competencies shall be awarded if all correct values of p have been given.

Expected solution to the guiding question:

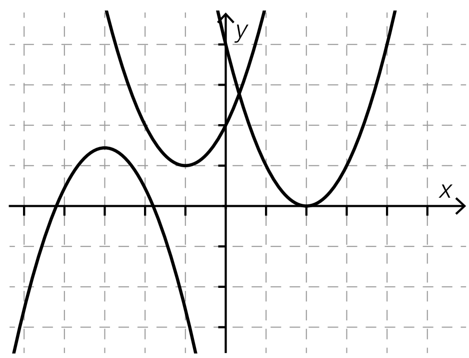
A quadratic equation has either no real solutions, one real solution or two real solutions.

Possible graphical representation:

Graphs of quadratic functions are parabolas, which either do not cross the x-axis, touch it, or cross the x-axis twice.

The quadratic equation has no real solutions if the corresponding parabola does not cross the x-axis.
The quadratic equation has exactly one real solution if the corresponding parabola touches the x-axis.
The quadratic equation has two real solutions if the corresponding parabola crosses the x-axis twice.

Possible sketch:



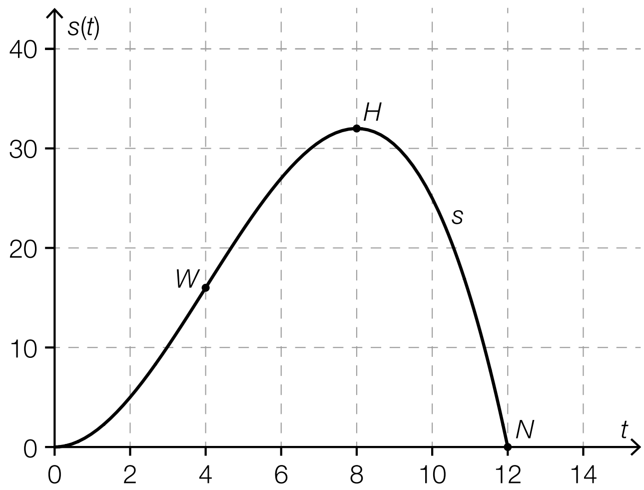
Answer key:

The point for the guiding question shall be awarded if the three solution scenarios have been given correctly and a correct graphical representation has been shown.

Task 3

Movement of an Object

An object moves along a straight path. The distance (in metres) of the object from its starting point is modelled in terms of time t (in seconds) by the third degree polynomial function s. The graph of this function s is shown in the diagram below; the point of inflexion W, the maximum H, and the zero N have integer coordinates.



Task:

– Describe the movement of the object in words, and explain the significance of the coordinates of the points W, H and N.

Guiding question:

The function v describes the velocity of the object in the time interval [0, 12].

– Write down the value of the area enclosed by the graph of the function v and the time axis in the time interval [0, 8].
– Based on the diagram shown above, explain why the maximum speed is greater than 4 m/s.

Solution to Task 3

Movement of an Object

Expected solution to the statement of the task:

Possible description:

The object accelerates for 4 s. After 4 s, when it is 16 m away from its starting point, the object begins to move more slowly. After 8 s, when it is 32 m away from its starting point, the object changes its direction of movement, and after a total of 12 s it is back at its starting point.

Answer key:

The point for the core competencies shall be awarded if a correct description of the movement of the object, including the coordinates of the points, has been given.

Expected solution to the guiding question:

The size of the area between the graph of v and the time axis in the interval [0, 8] has the value 32.

Possible justification:

The maximum of v is at $t = 4$.
The gradient of the tangent at point W on the graph of s is greater than 4 as this tangent is steeper than, for example, the straight line connecting the origin and H, whose gradient is exactly 4.

Answer key:

The point for the guiding question shall be awarded if the correct value for the size of the area has been determined and if a correct explanation of why the maximum must have a value greater than 4 has been given.

Task 4

Pellet Consumption

In 2016 in Germany, 8.1 % more pellets were used than in 2015.
In 2017, 5 % more were used than in 2016.
In 2018, the consumption was 4.8 % higher than in 2017.

In 2017, 2.1 million tonnes of pellets were used.

Task:

– Write down the absolute and relative change in pellet consumption from 2015 to 2018.

Guiding question:

– Determine the annual percentage rate of change p in pellet consumption from 2015 to 2018 if a constant increase is assumed.
– Using the consumption value for 2017 and the annual percentage rate of change p determined above, write down the number of years after which the pellet consumption first reaches 2.5 million tonnes.

Solution to Task 4

Pellet Consumption

Expected solution to the statement of the task:

Consumption in 2015: $\frac{2.1}{1.05 \cdot 1.081} = 1.850...$
Consumption in 2018: $2.1 \cdot 1.048 = 2.2...$
$\Rightarrow$ The absolute change is approximately 0.35 million tonnes.

$1.081 \cdot 1.05 \cdot 1.048 = 1.189...$
$\Rightarrow$ The pellet consumption increased by around 19 % in this time period.

Answer key:

The point for the core competencies shall be awarded if the correct absolute change and the correct percentage change have been given.

Expected solution to the guiding question:

$1.85 \cdot \left(1 + \frac{p}{100}\right)^3 = 2.2 \quad \Rightarrow \quad p = 5.94... \quad \Rightarrow \quad p \approx 6\ \%$

The pellet consumption increases by an average of 6 % per year.

$2.1 \cdot 1.06^t = 2.5 \quad \Rightarrow \quad t \approx 3$
The pellet consumption will reach 2.5 million tonnes for the first time after around 3 years.

Answer key:

The point for the guiding question shall be awarded if the correct value for the rate of change and the correct time period have been given.

Task 5

Gladiolas

Gladiolas are popular cut flowers that grow from gladiola bulbs. By looking at the gladiola bulb, it is impossible to say which colour the gladiola's flowers will be. It is assumed that for a particular type of gladiola, 12 % of all gladiolas have red flowers.

Task:

A hobby gardener plants *n* randomly selected gladiola bulbs in the ground.

– Determine the value of *n* if 6 gladiola plants with red flowers are expected.
– Write down the probability that there are at least 5 gladiola plants with red flowers that grow from the *n* gladiola bulbs that have been planted.

Guiding question:

A wholesaler sells gladiola bulbs in sacks that each contain 200 bulbs. He would like to guarantee that the number of gladiolas with red flowers in a sack does not differ from the expected value by more than a particular number *c*. He would like to be able to keep the promise of this guarantee with a probability of at least 95 %.

– Write down the smallest value that the deviation *c* must be.

Solution to Task 5

Gladiolas

Expected solution to the statement of the task:

$n \cdot 0.12 = 6 \quad \Rightarrow \quad n = 50$

The random variable X gives the number of gladiola plants that grow to have red flowers.

$P(X \geq 5) = 0.732...$

The probability is around 73 %.

Answer key:

The point for the core competencies shall be awarded if the correct sample size n and the correct probability have been given.

Expected solution to the guiding question:

$$\left.\begin{array}{l} n = 200 \\ p = 12\ \% \end{array}\right\} \; E(X) = 24$$

$P(24 - c \leq X \leq 24 + c) \geq 0.95$

$c \geq 9 \quad \Rightarrow \quad c$ must be at least 9

Answer key:

The point for the guiding question shall be awarded if the correct deviation c has been given.

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Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

January 2020

Mathematics

Supplementary Examination 2
Examiner's Version

Bundesministerium
Bildung, Wissenschaft
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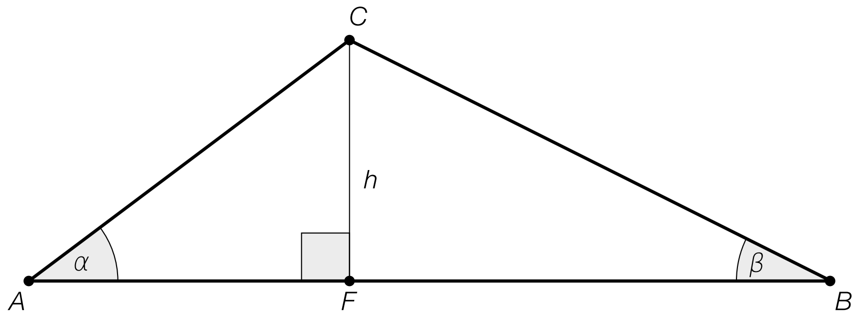
Task 4		
Task 5		

Task 1

Trigonometry

The triangle ABC is shown below. The foot F of the altitude h is closer to the vertex A and divides the line segment AB at a ratio of 2 : 5.



Task:

– Determine the size of the angle β when $h = 7$ cm and $\overline{AB} = 21$ cm.

Guiding question:

– Show by calculation that the triangle ABC is not a right-angled triangle.

The point C is moved so that the foot F of the altitude h lies to the left of the vertex A. The length of the altitude h and the length of the line segment AB remain the same.

– Write down whether this change causes the value of $\tan(\beta)$ to increase or decrease and justify your answer.

Solution to Task 1

Trigonometry

Expected solution to the statement of the task:

$\overline{FB} = \frac{21}{7} \cdot 5 = 15$

$\tan(\beta) = \frac{h}{\overline{FB}} = \frac{7}{15} \quad \Rightarrow \quad \beta = 25.016...° \approx 25°$

The size of the angle is approximately 25°.

Answer key:

The point for the core competencies shall be awarded if the correct angle is given.

Expected solution to the guiding question:

Possible justification by calculation with an indirect proof:

If it is assumed that *ABC* is a right-angled triangle, then according to the Pythagorean theorem, the following statement must hold:

$225 + h^2 + 36 + h^2 = 21^2 \quad \Rightarrow \quad 261 + 2 \cdot h^2 = 441 \quad \Rightarrow \quad h^2 = 90$

This contradicts $h = 7 \quad \Rightarrow$ the triangle *ABC* is not a right-angled triangle.

The value of $\tan(\beta)$ decreases.

Possible justification:

As the size of β decreases and the value of the tangent of $\beta \in [0°, 90°]$ is strictly monotonically increasing, then the value of $\tan(\beta)$ also decreases.

Answer key:

The point for the guiding question shall be awarded if it has been correctly shown by calculation that the triangle *ABC* is not a right-angled triangle and the correct change in the value of $\tan(\beta)$ has been given along with a correct justification.

Task 2

Powder Dye

If 500 g of powder dye is put into a jug of water, then after one minute 70 g of this powder will have dissolved.

The amount of powder dye that has dissolved is modelled by the function p where
$p(t) = 500 - 500 \cdot e^{k \cdot t}$ in terms of the time t (t in min, $p(t)$ in g).

Task:

– Determine the value of *k.*

Guiding question:

The function p fulfils the difference equation $p(t + 1) - p(t) = a \cdot (500 - p(t))$ with $a \in \mathbb{R}$.

– Determine the value of a and interpret your result in the context given.

Solution to Task 2

Powder Dye

Expected solution to the statement of the task:

$70 = 500 - 500 \cdot e^{k \cdot 1}$
$k = \ln\left(\frac{43}{50}\right) = -0.150823... \approx -0,15082$

Answer key:

The point for the core competencies shall be awarded if the correct value of *k* has been given.

Expected solution to the guiding question:

$p(1) - p(0) = a \cdot (500 - p(0))$
$70 = a \cdot (500 - 0) \quad \Rightarrow \quad a = \frac{70}{500} = 0.14$

Every minute, 14 % of the powder that has not yet dissolved dissolves in the water.

Answer key:

The point for the guiding question shall be awarded if the correct value of *a* has been determined and a correct interpretation has been given.

Task 3

Maxima and Minima of a Fourth Degree Polynomial Function

The equation of a fourth degree polynomial function f is $f(x) = a \cdot x^4 + b \cdot x^3 + c \cdot x^2 + d \cdot x + e$ with $a, b, c, d, e \in \mathbb{R}$ and $a > 0$.

Task:

– Justify why f can have at most 3 maxima or minima.

Guiding question:

The following applies: $g(x) = p \cdot x^4 + q \cdot x^2 + r$ with $p, q, r \in \mathbb{R}$ and $p > 0$.

– Write down every number of local maxima or minima that g can have.
– Demonstrate by calculation how the sign of q influences the number of maxima or minima and for each case, sketch a typical graph of the function.

Solution to Task 3

Maxima and Minima of a Fourth Degree Polynomial Function

Expected solution to the statement of the task:

Possible justification:

$f'(x) = 4 \cdot a \cdot x^3 + 3 \cdot b \cdot x^2 + 2 \cdot c \cdot x + d$
The equation $4 \cdot a \cdot x^3 + 3 \cdot b \cdot x^2 + 2 \cdot c \cdot x + d = 0$ is a third degree equation and has a maximum of three solutions $\Rightarrow$ there are at most three maxima or minima.

Answer key:

The point for the core competencies shall be awarded if it has been correctly explained why at most three maxima or minima are possible.

Expected solution to the guiding question:

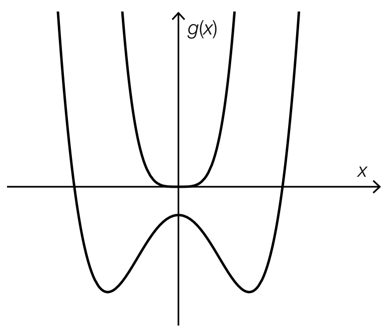
The number of local maxima or minima can only be 1 or 3.

Demonstration by calculation:

$g(x) = p \cdot x^4 + q \cdot x^2 + r$
$g'(x) = 4 \cdot p \cdot x^3 + 2 \cdot q \cdot x = 0$
$x \cdot (4 \cdot p \cdot x^2 + 2 \cdot q) = 0 \quad \Rightarrow \quad x_1 = 0 \quad x_{2,3} = \pm\sqrt{-\frac{q}{2 \cdot p}}$

For $q \geq 0$, there is no second solution and therefore only one maximum or minimum.
For $q < 0$, there are two further solutions and therefore three maxima or minima.

Possible graphs:



Answer key:

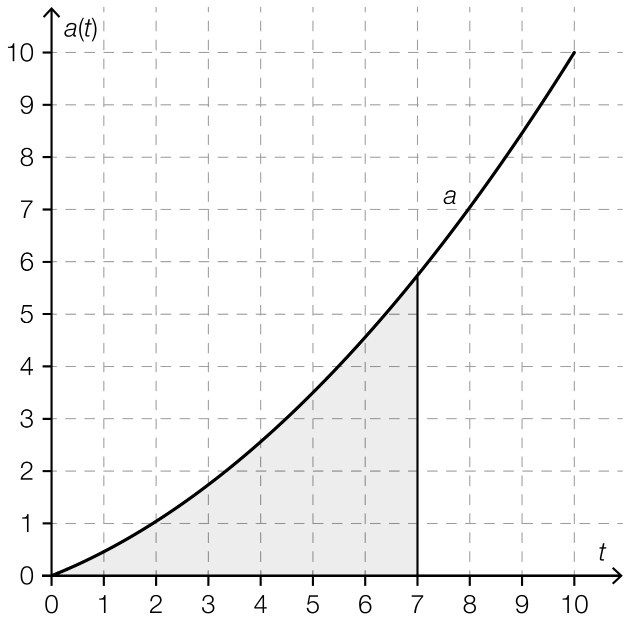
The point for the guiding question shall be awarded if the correct number of possible maxima or minima has been given and the influence of the sign of q has been correctly demonstrated. A correct sketch also must have been shown in which the number of maxima or minima and the symmetry of the graph must be recognisable.

Task 4

Acceleration

An object with an initial speed of 5 m/s accelerates for 10 s.
The acceleration of the object is modelled by the function a in terms of time t.
The equation of the function a is: $a(t) = 0.06 \cdot t^2 + 0.4 \cdot t$ with t in s and $a(t)$ in m/s^2.

The diagram below shows the graph of the function a with a shaded area.



Task:

– Determine the size of the shaded area and explain what this value means in terms of the velocity of the object.

Guiding question:

– Determine the length of the distance covered during this 10 s long acceleration and explain your method.

Solution to Task 4

Acceleration

Expected solution to the statement of the task:

Size of the shaded area:
$\int_0^7 (0.06 \cdot t^2 + 0.4 \cdot t) \, \mathrm{d}t = 16.66$

This value gives the increase in the velocity of the object in the first 7 s.

Answer key:

The point for the core competencies shall be awarded if the correct size of the shaded area has been determined and this value has been correctly interpreted.

Expected solution to the guiding question:

Possible method:
By integrating the acceleration function, the velocity function is obtained.
By integrating again, the length of the distance covered can be calculated.

$\int a(t) \, \mathrm{d}t = \int (0.06 \cdot t^2 + 0.4 \cdot t) \, \mathrm{d}t = 0.02 \cdot t^3 + 0.2 \cdot t^2 + c = v(t)$
$v(0) = 5 \quad \Rightarrow \quad v(t) = 0.02 \cdot t^3 + 0.2 \cdot t^2 + 5$
$s(10) = \int_0^{10} (0.02 \cdot t^3 + 0.2 \cdot t^2 + 5) \, \mathrm{d}t = 166.\dot{6}$

Length of the distance covered: around 167 m

Answer key:

The point for the guiding question shall be awarded if the correct length of the distance covered has been calculated and a correct method has been given.

Task 5

Fitness Training

The list of data shown below gives the number of hours per week that eight young people spend training at a gym.

3, 3, 5, 6, 7, 8, 9, x

The median and the mean of the training times have the same value.

Task:

– Assuming that x is the largest value of the list of data, write down the training time x.

Guiding question:

– Assuming that x is any integer value of the list of data, write down a second possible value for the training time x.

Three out of the eight young people are chosen at random.

– For each of the possible values of x, determine the probability that exactly two out of the three young people train at least five hours per week.

Solution to Task 5

Fitness Training

Expected solution to the statement of the task:

$$\frac{3+3+5+6+7+8+9+x}{8} = 6.5 \quad \Rightarrow \quad x = 11$$

Answer key:

The point for the core competencies shall be awarded if the correct training time x has been given.

Expected solution to the guiding question:

In order to obtain a different median value, then a value of $x \leq 6$ must be chosen.
For $x = 6 \quad \Rightarrow \quad$ median = 6, this does not correspond to the mean (5.875).
Therefore a value of $x \leq 5$ must be chosen $\quad \Rightarrow \quad$ median = 5.5.

$$\frac{x+3+3+5+6+7+8+9}{8} = 5.5 \quad \Rightarrow \quad x = 3$$

$$\frac{6}{8} \cdot \frac{5}{7} \cdot \frac{2}{6} \cdot 3 = \frac{15}{28}$$

$$\frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6} \cdot 3 = \frac{15}{28}$$

The probability for both possible values of x is 53.6 %.

Answer key:

The point for the guiding question shall be awarded if the correct second value of x and the correct probability for both possible values of x have been given.

Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

June 2018

Mathematics

Supplementary Examination 6
Examiner's Version

BMBWF
BUNDESMINISTERIUM
FÜR BILDUNG, WISSENSCHAFT
UND FORSCHUNG

Instructions for the supplementary examination

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In the following document, the examiner will find the tasks as well as the expected solutions and the answer key.

The preparation time is to be at least 30 minutes and the examination time is to be at most 25 minutes.

Assessment

Each task can be awarded zero, one or two points. There is one point available for each demonstration of core competencies as well as for each guiding question. A maximum of 10 points can be achieved.

For the grading of the examination the following scale should be used:

Grade	Minimum number of points
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The examination board will decide on the final grade based on the candidate's performance in the supplementary examination as well as the result of the written examination.

Evaluation grid for the supplementary examination

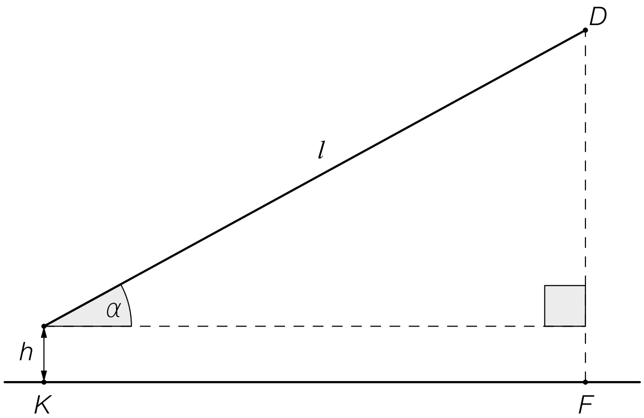
This evaluation grid may be used to assist the examiner in assessing the candidate's performance.

	Point for core competencies reached	Point for the guiding question reached
Task 1		
Task 2		
Task 3		
Task 4		
Task 5		

Task 1

Flying a Kite

A child is flying a kite. An approximation of the positions of the child (*K*) and the kite (*D*) at a particular time is shown in the diagram below.



The position of the child, *K*, as well as the point *F* lie in a horizontal plane. The child is holding the kite at a height *h* of 1.5 metres above the ground. The length of the taut string is $l = 50$ m.

Task:

Write down a formula that can be used to calculate the height $\overline{FD}$ of the kite above the horizontal plane (in metres) in terms of the angle α.

Guiding question:

Write down a function to calculate the size of the angle α for which the horizontal distance $\overline{KF}$ is equal to the height $\overline{FD}$ of the kite and determine the size of α.

Solution to Task 1

Flying a Kite

Expected solution to the statement of the task:

$\overline{FD} = 50 \cdot \sin(\alpha) + 1.5$

Answer key:

The point for the core competencies is to be awarded if a correct formula has been given. Equivalent formulae are also to be accepted.

Expected solution to the guiding question:

$\overline{KF} = \overline{FD} \quad \Rightarrow \quad 50 \cdot \cos(\alpha) = 50 \cdot \sin(\alpha) + 1.5$
$\alpha \approx 43.78°$

Answer key:

The point for the guiding question is to be awarded if a correct equation to the calculation of α and the angle α has been given correctly.
Tolerance interval: [43°, 44°]

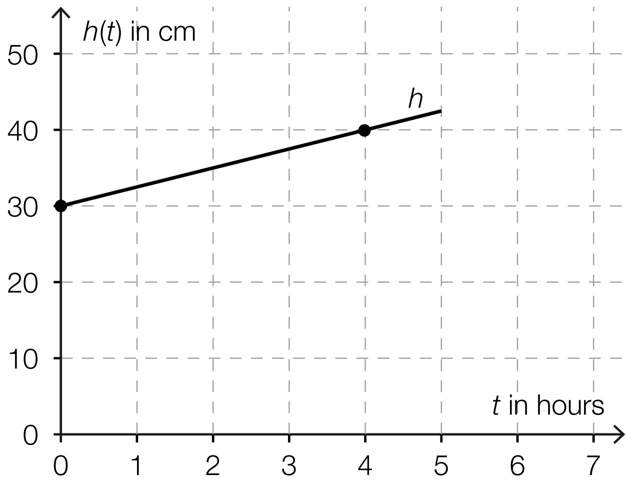
Task 2

Snowfall

The height of the snow level during a five-hour snowfall can be modelled by a linear function, h. The height of the snow, $h(t)$, is measured in cm and the time, t, is measured in hours where $0 \leq t \leq 5$.

Task:

The graph shown below shows the height of the snow level during this five-hour snowfall. The points shown in bold have integer coordinates.



Write down the equation of the function that gives the height of the snow level, h, in terms of the time t and write down the meaning of the numbers that appear in the equation.

Guiding question:

For a function h_1, write down all of the conditions that must be fulfilled so that h_1 describes a directly proportional relationship between the height of the snow level, $h_1(t)$ (in cm), and the time t (in hours).

Write down the equation of the directly proportional function h_1 if the snow level is 20 cm high after a five-hour snowfall.

Solution to Task 2

Snowfall

Expected solution to the statement of the task:

$h(t) = 30 + 2.5 \cdot t$

At the beginning of the observations ($t = 0$), the height of the snow level is 30 cm. The height of the snow level increases by 2.5 cm per hour.

Answer key:

The point for the core competencies is to be given if a correct equation for the function has been given and the meaning of the numbers has been given correctly.

Expected solution to the guiding question:

At the beginning of the observations (when $t = 0$), the height of the snow level has to be zero and the (absolute) height must increase at a constant rate.

$h_1(t) = 4 \cdot t$

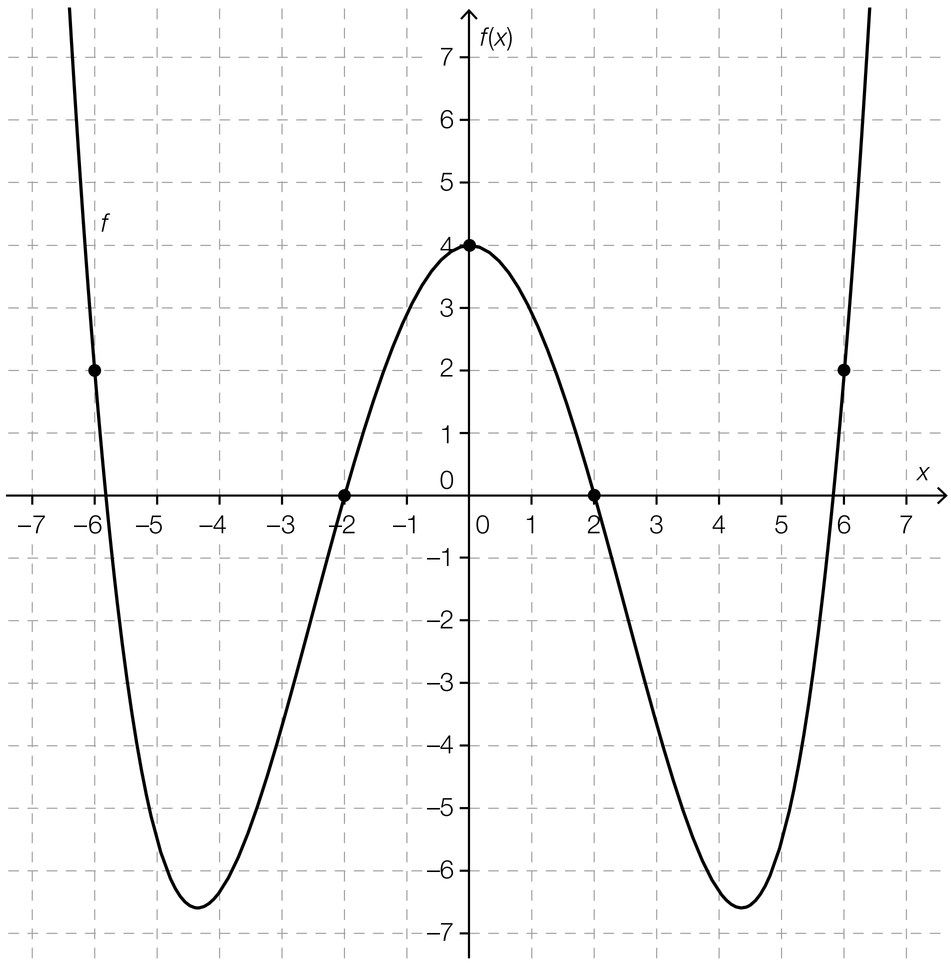
Answer key:

The point for the guiding question is to be awarded if the conditions given in the expected solution have been given correctly and the equation of the function has been given correctly.

Task 3

Fourth Degree Polynomial Function

The diagram below shows the graph of a fourth degree polynomial function, f, with equation $f(x) = a \cdot x^4 + b \cdot x^2 + c$ where $a, b, c \in \mathbb{R}$. The points shown in bold have integer coordinates.



Task:

Determine the parameters a, b and c of the function f.
Write down the intervals for which $f'(x) > 0$ holds and explain your method.

Guiding question:

Write down a value of $k \in \mathbb{R}$ where $k > 2$ such that the equation shown below is generally valid and explain your reasoning.

$$\int_{-3}^{0} f(x)\,\mathrm{d}x - \int_{0}^{k} f(x)\,\mathrm{d}x = f'(0)$$

There is a further value $h \in \mathbb{R}$, $0 \leq h \leq 2$ for which the equation $\int_{-3}^{0} f(x)\,\mathrm{d}x - \int_{0}^{h} f(x)\,\mathrm{d}x = f'(0)$ is satisfied. Determine this value.

Solution to Task 3

Fourth Degree Polynomial Function

Expected solution to the statement of the task:

$a \approx 0.0295$ $b \approx -1.1181$ $c = 4$
intervals: $(-4.35, 0)$ and $(4.35, \infty)$

Possible explanation:
In an interval (x_1, x_2) $f'(x) > 0$ holds, if for all $x \in (x_1, x_2)$ the tangent to the graph of the function f is increasing. By finding the maxima and minima of the function, the boundaries of the intervals can be determined.

Answer key:

The point for the core competencies is to be given if the parameters a, b and c and also both intervals have been given correctly and a correct method has been explained.
Half-open or closed intervals as well as equivalent notation are also to be considered correct.
Tolerance intervals for the lower interval boundaries: $[-4.4, -4.3]$ and $[4.3, 4.4]$ respectively

Expected solution to the guiding question:

For $k = 3$ the equation is generally valid.

Possible explanation:
As $x = 0$ is a local maximum, $f'(0) = 0$ holds. So that the difference between the definite integrals is also zero, both of the non-zero integrals (under the condition that $k > 2$) have to be symmetrical about the origin (as the function f is an even function and the graph of the function is symmetrical about the vertical axis).

Also for $h \approx 0.91$ the equation is generally valid.

Answer key:

The point for the guiding question is to be awarded if the parameters k and h have been given correctly and correct methods have been explained.

Task 4

Number of Inhabitants

The number of inhabitants in a particular country in year t is represented by $B(t)$.

Task:

Interpret both of the equations below with respect to the number of inhabitants in this country.

- $\frac{B(2015)}{B(1950)} = 2$
- $\frac{B(2015) - B(2000)}{B(2000)} = 0.1$

Guiding question:

Interpret the equation $\frac{B(2015) - B(2000)}{15} = 100\,000$ in the given context.
Using the equations given, determine the number of inhabitants in this country in the year 2015 and explain your reasoning.

Solution to Task 4

Number of Inhabitants

Expected solution to the statement of the task:

Possible interpretations:

- The number of inhabitants in the country is twice as high in 2015 than in 1950.
- The number of inhabitants in the country is 10 % higher in 2015 than in 2000.

Answer key:

The point for the core competencies is to be given if both equations have been interpreted correctly.

Expected solution to the guiding question:

The number of inhabitants in the country has increased on average by 100 000 inhabitants per year from 2000 to 2015.

The number of inhabitants in 2015 was 16.5 million.

Possible method:

Over the 15 years, the number of inhabitants increased by 1.5 million.

As the equation $\frac{B(2015) - B(2000)}{B(2000)} = 0.1$ corresponds to an increase of 10 %, $B(2000) = 15$ million must hold.

$B(2015) = B(2000) + 1.5 = 16.5$

Answer key:

The point for the guiding question is to be awarded if the equation has been correctly interpreted within the given context, the number of inhabitants in the year 2015 has been determined correctly, and a correct method has been explained.

Task 5

Discount Dice

A shop has a game that customers can play to win discounts. The aim of the game is to roll the highest possible number with a (fair) dice. (A dice is considered to be “fair” if the probability of the dice showing any of its faces after being thrown is equal for all six faces.)
If a person rolls a number from 1 to 5, they win a percentage discount equal to the number shown. If a person rolls a six on their first roll, they are allowed to roll again. The sum of the numbers from both rolls corresponds to the percentage discount the customer receives.

Task:

Determine the probability, P, that a customer gets a 10 % discount.
Explain your reasoning.

Guiding question:

The random variable X describes the percentage discount that a customer can receive.

Write down all of the possible values of the random variable X and their corresponding probabilities.

Determine the expectation value $E(X)$ of the random variable X and explain the meaning of the value obtained in the given context.

Solution to Task 5

Discount Dice

Expected solution to the statement of the task:

A customer receives a 10 % discount if they roll a six on their first roll and a four on their second roll.

$P = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} \approx 0.0278 = 2.78\ \%$

Answer key:

The point for the core competencies is to be given if the probability has been given correctly and a correct method has been explained.

Expected solution to the guiding question:

Values of the random variable:

1, 2, 3, 4, 5 each with a probability of $\frac{1}{6} \approx 0.1667 = 16.67\ \%$

7, 8, 9, 10, 11, 12 each with a probability of $\frac{1}{36} \approx 0.0278 = 2.78\ \%$

$E(X) = (1 + 2 + 3 + 4 + 5) \cdot \frac{1}{6} + (7 + 8 + 9 + 10 + 11 + 12) \cdot \frac{1}{36} \approx 4.08$

On average, a discount of 4 % is to be expected.

Answer key:

The point for the guiding question is to be awarded if the possible values of the random variable and the corresponding probabilities have been given correctly. Also, the expectation value must have been both determined and explained correctly.

Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

October 2018

Mathematics

Supplementary Examination 1
Examiner’s Version

Bundesministerium
Bildung, Wissenschaft
und Forschung

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Task 1

Price

The net price of a product is N euros. The gross price is the sum of the net price and m % Value Added Tax (calculated using the net price).
The sale price V is obtained by subtracting a discount of r % of the gross price from the gross price.

Task:

Write down a formula for the sale price V in terms of N, m and r.

V = ____

Guiding question:

Write down whether the same sale price V would be obtained if a discount of r % of the net price were subtracted from the net price first, before m % of the resulting price were added on to this result. Justify your decision.

Furthermore, write down the size of the discount that would have to be applied to a product that has 20 % Value Added Tax so that the net price were equal to the sale price.

Solution to Task 1

Price

Expected solution to the statement of the task:

Possible formula:

$$V = N \cdot \left(1 + \frac{m}{100}\right) \cdot \left(1 - \frac{r}{100}\right)$$

Answer key:

The point for the core competencies is to be given if a correct expression has been given. Equivalent terms are also to be considered correct.

Expected solution to the guiding question:

Possible justification:

The same sale price V is obtained if the discount is applied first and then the VAT is added due to the commutative rule of multiplication:

$$N \cdot \left(1 + \frac{m}{100}\right) \cdot \left(1 - \frac{r}{100}\right) = N \cdot \left(1 - \frac{r}{100}\right) \cdot \left(1 + \frac{m}{100}\right)$$

$$V = V \cdot \left(1 + \frac{20}{100}\right) \cdot \left(1 - \frac{r}{100}\right) \quad \Rightarrow \quad r = 16.\dot{6}$$

A discount of around 17 % has to be applied.

Answer key:

The point for the guiding question is to be awarded if the alternative calculation of V has been both given and explained correctly. Furthermore, the percentage has to have been given correctly.
Tolerance interval: [16 %, 17 %]

Task 2

Iodine-131

The isotope iodine-131 is radioactive.
After t days, the remaining amount $N(t)$ of iodine-131 decreases approximately exponentially. The amount of iodine-131 at time $t = 0$ is given by N_0.

Task:

After four days, 30 % of the original amount of iodine-131 has decayed. Determine the percentage of the amount of iodine-131 that decays per day and explain your method.

Guiding question:

Determine the general (without using concrete values) relative change of an exponential function of the form $N(t) = N_0 \cdot a^t$ in the time periods $[0, t_1]$ and $[t_0, t_0 + t_1]$ (where $t_0, t_1 \in \mathbb{R}^+$).

Interpret the results (drawing on a characteristic property of exponential functions).

Solution to Task 2

Iodine-131

Expected solution to the statement of the task:

After four days, 70 % of the original amount is still left.
$N(4) = 0.7 \cdot N_0 = N_0 \cdot a^4 \quad \Rightarrow \quad a \approx 0.915$

Thus, around 91.5 % of the original amount of isotope iodine-131 is left after one day.
Therefore, around 8.5 % of the original amount of iodine-131 decays per day.

Answer key:

The point for the core competencies is to be given if a correct percentage has been calculated and a correct method has been explained.
Tolerance interval: [8 %, 9 %]

Expected solution to the guiding question:

$$\frac{N(t_1) - N(0)}{N(0)} = \frac{N_0 \cdot a^{t_1} - N_0}{N_0} = a^{t_1} - 1$$

$$\frac{N(t_0 + t_1) - N(t_0)}{N(t_0)} = \frac{N_0 \cdot a^{t_0 + t_1} - N_0 \cdot a^{t_0}}{N_0 \cdot a^{t_0}} = a^{t_1} - 1$$

Possible interpretation:
The relative change of the value of the function is constant across time periods of equal length.

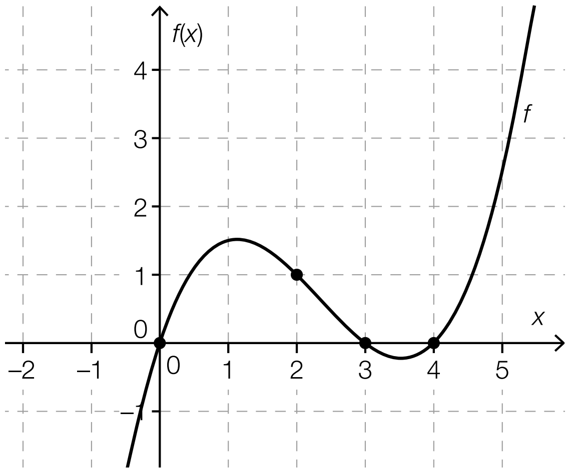
Answer key:

The point for the guiding question is to be awarded if the relative change for both intervals has been determined correctly and a correct interpretation has been given.

Task 3

Function and the Antiderivative

The diagram below shows a section of the graph of a third degree polynomial function, f. The points shown in bold have integer coordinates.



Task:

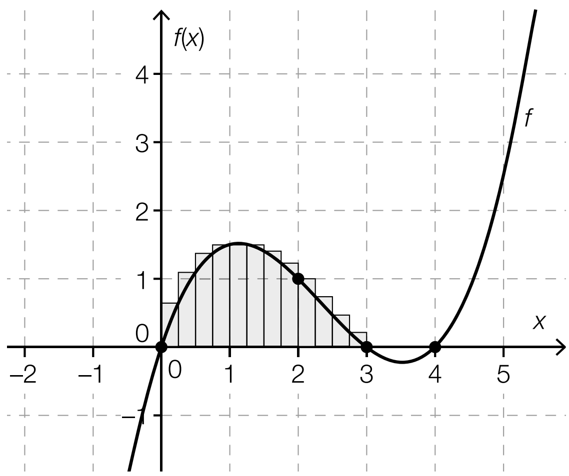
Write down the number and positions of any maxima, minima and points of inflexion of an antiderivative, F, of f.

Guiding question:

The graph of the function f encloses two finite areas bounded by the x-axis.

Write down two formulae, one using f and one using F, that could be used to calculate the total area of these finite areas.

In the diagram shown, the larger area has been divided into 12 rectangles of equal width. The sum of these rectangular areas (the upper sum) can be used to approximate the value of the area of the larger area.
Write down the value of the upper sum and determine by how much this value differs from the actual area of this region by using an adequate model function f.



Solution to Task 3

Function and the Antiderivative

Expected solution to the statement of the task:

The antiderivative F has three maxima/minima.
These are at the points where $x = 0$, $x = 3$ and $x = 4$.
The antiderivative F has two points of inflexion.
These are at the points where $x \approx 1.1$ and $x \approx 3.5$.

Answer key:

The point for the core competencies is to be given if the number and positions of the maxima, minima and points of inflexion of F have been given correctly.
Tolerance intervals for the points of inflexion: [1, 1.3] and [3.4, 3.7]

Expected solution to the guiding question:

Possible formulae:
$A = \int_0^3 f(x)\,\mathrm{d}x - \int_3^4 f(x)\,\mathrm{d}x$
$A = [F(3) - F(0)] - [F(4) - F(3)]$

$O_s \approx 3.17$
$f(x) = 0.25 \cdot x^3 - 1.75 \cdot x^2 + 3 \cdot x$
$A \approx 2.81$
$O_s - A \approx 0.36$

Answer key:

The point for the guiding question is to be awarded if correct formulae for both f and F for calculating the value of the area have been given and a correct value of the upper sum has been given, and the difference to the actual value has been given correctly.
Tolerance interval for the deviation: [0.3, 0.4]

Task 4

Vertical Throw

The height of a body thrown at time $t = 0$ can be described by a function h where $h(t) = 50 + 60 \cdot t - 5 \cdot t^2$ ($h(t)$ in metres, t in seconds).

Task:

Determine the equation of the first derivative of h and write down the meaning of this function in the context of the movement of the body using the correct units.

Determine the maximum $E = (t_1, h(t_1))$ of the function h and interpret the meaning of both coordinates, t_1 and $h(t_1)$, in the given context.

Guiding question:

Determine the average rate of change of the function h in the time period $[0, t_1]$ and interpret the result in the context of the movement of the body.

The Mean Value Theorem of differentiation says that for a function f, under certain conditions, in an interval $[a, b]$ there exists at least one point $x_0 \in (a, b)$ such that $f'(x_0) = \frac{f(b) - f(a)}{b - a}$ holds.

Interpret this statement for the function h in the given context for the time period $[0, t_1]$.

Determine the point t_0 as described in the Mean Value Theorem for the function h in the time period $[0, t_1]$.

Solution to Task 4

Vertical Throw

Expected solution to the statement of the task:

$h'(t) = 60 - 10 \cdot t$
Meaning: $h'(t)$ gives the instantaneous velocity of the body (at time t) in m/s.

$h'(t) = 0 \quad \Rightarrow \quad t_1 = 6,\; h(6) = 230 \quad \Rightarrow \quad E = (6,230)$
Meaning: After 6 seconds, the body has reached a (maximum) height of 230 metres.

Answer key:

The point for the core competencies is to be given if the derivative and its meaning (using the correct units) have been given correctly, the maximum point has been determined correctly and the meaning of both coordinates has been explained correctly.

Expected solution to the guiding question:

$$\frac{h(6) - h(0)}{6 - 0} = \frac{230 - 50}{6} = 30$$
Meaning: The average velocity (or speed) of the body in the time period [0 s, 6 s] is 30 m/s.

Possible interpretation:
In the time period [0, 6] there is at least one point in time at which the instantaneous velocity (or speed) of the body is equal to the average speed for the time period [0, 6].

Determining t_0:
$h'(t_0) = 30 \quad \Rightarrow \quad 60 - 10 \cdot t_0 = 30 \quad \Rightarrow \quad t_0 = 3$ s

Answer key:

The point for the guiding question is to be awarded if the average rate of change has been calculated correctly, a correct interpretation of the Mean Value Theorem has been given in the context, and the point t_0 has been determined correctly.

Task 5

Statistical Values

An ordered list of data with values $a_1, a_2, \dots, a_n$ ($n \in \mathbb{N}$, $n > 3$) is given.

Task:

For each of the statements given below, write down if it is true or false and in each case justify your decision.

Statement 1: The median is definitely a value that appears in the ordered list of data.

Statement 2: For the median m of the list of data, the formula $m = \frac{a_1 + a_n}{2}$ holds.

Statement 3: For the mean $\bar{x}$ of the list of data, the formula $n \cdot \bar{x} = a_1 + a_2 + \dots + a_n$ holds.

Guiding question:

Write down an ordered list of data that contains 11 values and satisfies the following conditions:
The mean, the median and the range have the value 10.
Explain your reasoning.

Explain how you could add two values to your list of data such that neither the median nor the mean change, but the range increases by 6.

Solution to Task 5

Statistical Values

Expected solution to the statement of the task:

Statements 1 and 2 are false. This can be demonstrated by a counterexample:

$a_1 = 1 \quad a_2 = 2 \quad a_3 = 3 \quad a_4 = 6$

$m = 2.5$ and thus does not belong to the list of data.

$\frac{a_1 + a_4}{2} = 3.5$ and is thus not the median.

Statement 3 is true as by definition for the mean $\bar{x} = \frac{a_1 + a_2 + ... + a_n}{n}$ holds.

Answer key:

The point for the core competencies is to be given if for each of the three statements the decision about whether the statement is true has been made correctly and a correct justification has been given.

Expected solution to the guiding question:

Possible lists of data:

5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15
5, 5, 7, 8, 9, 10, 11, 12, 13, 15, 15
5, 6, 7, 8, 10, 10, 10, 12, 13, 14, 15
4, 5, 7, 8, 9, 10, 12, 13, 14, 14, 14

Possible method:

The middle (sixth) value of the ordered list has to be 10.
The difference between the largest and the smallest numbers has to be 10.
The sum of the 11 values has to be 110.

A value that is 3 smaller than the smallest value and a second value that is 3 larger than the largest value should be added.

Answer key:

The point for the guiding question is to be awarded if a correct ordered list of data with 11 values has been given, a correct method has been explained and the additional numbers have been correctly explained.

Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

January 2019

Mathematics

Supplementary Examination 1
Examiner’s Version

Bundesministerium
Bildung, Wissenschaft
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Task 1

Three Vectors in $\mathbb{R}^3$

Below, you will see three vectors.

$$\vec{a} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}, \; \vec{b} = \begin{pmatrix} 2 \\ 1 \\ b_z \end{pmatrix}, \; \vec{c} = \begin{pmatrix} -3 \\ c_y \\ 5 \end{pmatrix}$$

Task:

Determine the components b_z and c_y so that the vectors $\vec{b}$ and $\vec{c}$ are both perpendicular to $\vec{a}$.

Show that the vectors $\vec{b}$ and $\vec{c}$ are also perpendicular to one another for the components you have calculated. Show your method.

Guiding question:

For each of the lines g, h and i, determine a vector equation such that each of the conditions below is fulfilled.

I: The line g has the vector $\vec{a}$ as its direction vector and goes through the origin.

II: The line h has the vector $\vec{b}$ as its direction vector and crosses the line g at exactly one point.

III: The line i is parallel to the line h and skew to the line g (i. e. it does not cross the line g).

Explain your method and show that i is skew to g.

Solution of Task 1

Three Vectors in $\mathbb{R}^3$

Expected solution of the statement of the task:

A pair of vectors, neither of which is the zero vector, is perpendicular if and only if their scalar product is zero.

$\vec{a} \cdot \vec{b} = 0 \;\Rightarrow\; 2 - 2 + 3 \cdot b_z = 0 \;\Rightarrow\; b_z = 0$

$\vec{a} \cdot \vec{c} = 0 \;\Rightarrow\; -3 - 2 \cdot c_y + 15 = 0 \;\Rightarrow\; c_y = 6$

Then $\vec{b} \cdot \vec{c} = -6 + 6 + 0 = 0$ also holds.

Answer key:

The point for the core competencies is to be awarded if both components have been determined correctly and it has been shown that vectors $\vec{b}$ and $\vec{c}$ are also perpendicular. A correct method also needs to have been given.

Expected solution of the guiding question:

Possible solution:

$$g\colon X = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + s \cdot \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \qquad h\colon X = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + t \cdot \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \qquad i\colon X = \begin{pmatrix} 0 \\ 0 \\ 7 \end{pmatrix} + u \cdot \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

It needs to be shown that *g* and *i* do not cross:

$s = 2 \cdot u$
$-2 \cdot s = u$
$3 \cdot s = 7$

This system of equations in the variables *s* and *u* has no solution.

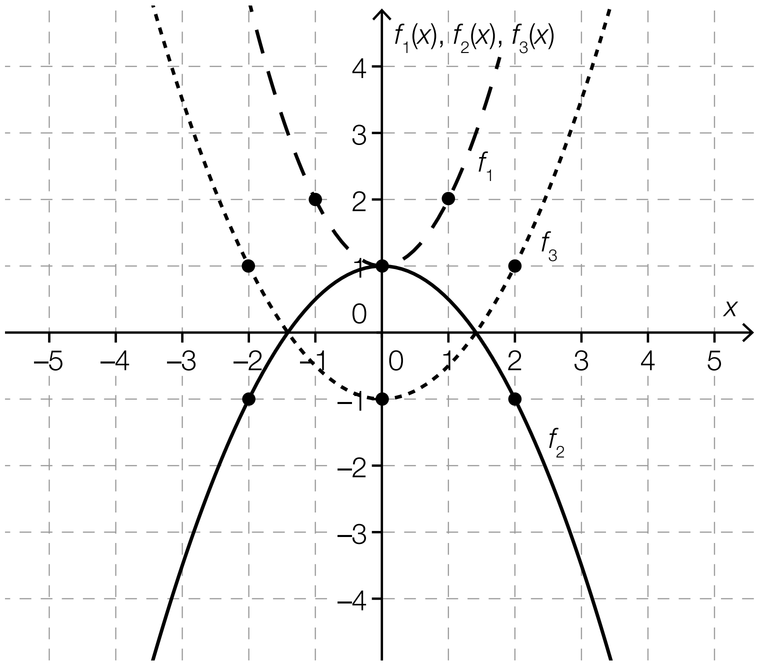
Answer key:

The point for the guiding question is to be awarded if three vector equations have been given that fulfil the conditions, a correct method has been used, and it has been shown that the lines *g* and *i* are skew.

Task 2

Functions

In the coordinate system below, three graphs of functions of the form $x \mapsto a \cdot x^2 + b$ are shown. The points marked in bold have integer coordinates.



Task:

Determine an equation of the function f_2.

Guiding question:

Explain the influence of the parameters a and b on the behaviour of the graph of a general function f where $f(x) = a \cdot x^2 + b$ and $a \neq 0$.

By comparing the parameters of the three functions f_1, f_2 and f_3, show how your explanation applies to concrete examples.

Solution of Task 2

Functions

Expected solution of the statement of the task:

If $f_2(x) = a \cdot x^2 + b$ and $f_2(0) = 1$ and, for example, $f_2(2) = -1$, we have $f_2(x) = -0.5 \cdot x^2 + 1$.

Answer key:

The point for the core competencies is to be awarded if an equation of the function f_2 has been determined correctly.

Expected solution of the guiding question:

The parameter b determines the intercept with the vertical axis.
The parameter a determines the concavity of the parabola.
For $a > 0$: The parabola is concave up; the larger a is, the “steeper” the graph of f is.
For $a < 0$: The parabola is concave down; the smaller a is, the “steeper” the graph of f is.

For f_1 and f_2, the parameter b is the same and has the value $b = 1$.
The parameter a is positive for f_1 and negative for f_2.

For f_3, $b = -1$ and a is positive (and has the same absolute value as in f_2).

Answer key:

The point for the guiding question is to be awarded if the general influence of the parameters a and b has been explained correctly and the differences in the parameters of the functions f_1, f_2 and f_3 have been correctly described.

Task 3

Wild Pigs

According to a newspaper article, the population of wild pigs in Bavaria in the year 2013 increased sharply, even though so many wild pigs had never been shot before. In the hunting season 2012/13, 66 000 wild pigs were shot. In the hunting season 2011/12, only 42 300 wild pigs had been shot.

Task:

Determine the absolute and relative increase of wild pig shootings in Bavaria from the hunting season of 2011/12 to that of 2012/13.

Guiding question:

State the type of functional relationship between time and the number of wild pig shootings that describes a constant annual rate of increase in shootings and corresponds to the relative change in shooting numbers in Bavaria calculated above.

Determine an equation for the function W that describes the number of wild pig shootings in Bavaria as a function of time, t (measured in years), where $W(0)$ gives the number of shootings in the 2012/13 season.

Using this equation, determine the number of wild pig shootings in the hunting season 2022/23. Estimate whether or not it is realistic for the number of wild pig shootings to develop in accordance with this function over a long period of time.

Solution of Task 3

Wild Pigs

Expected solution of the statement of the task:

Absolute increase: 23 700 shootings

Relative increase: $\frac{66\,000 - 42\,300}{42\,300} \approx 0.56$

The number of shootings increased by around 56 %.

Answer key:

The point for the core competencies is to be awarded if both values for the increase have been given correctly.

Expected solution of the guiding question:

The situation can be modelled by an exponential function:

$W(t) = 66\,000 \cdot 1.56^t$
$W(10) \approx 5.6$ million

It is not realistic, as this exponential function is strictly monotonically increasing, but the number of wild pig shootings cannot become infinitely large.

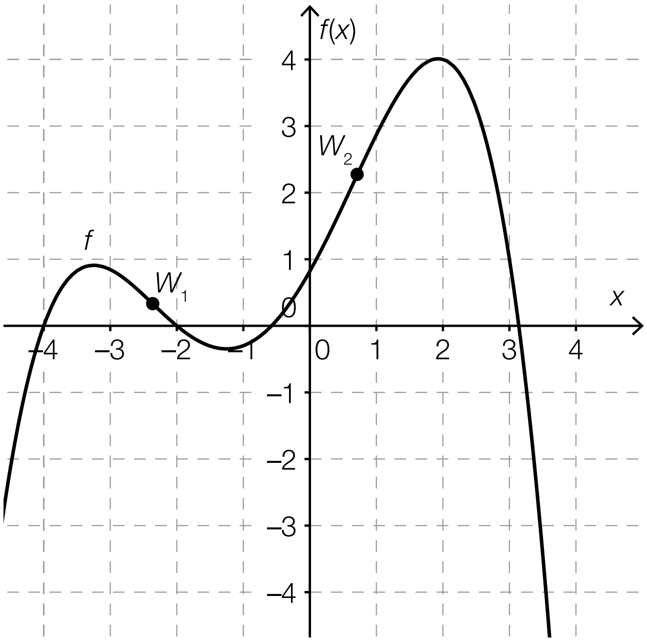
Answer key:

The point for the guiding question is to be awarded if a correct equation of the function has been given, the value for the 2022/23 season has been correctly determined, and a correct explanation has been provided.

Task 4

Derivatives and Antiderivatives

The diagram below shows a section of a graph of a fourth degree polynomial function, f, with the points of inflexion W_1 and W_2.



Task:

Determine whether the following statements are true or false. For each statement, justify your answer.

Statement 1: For all $x \in [-1, 1]$, $f'(x) > 0$.

Statement 2: There exists an $x \in [0, 1]$ for which $f'(x) = 0$.

Statement 3: For all $x \in [-4, -2]$, $f''(x) < 0$.

Statement 4: There exists an $x \in [1, 3]$ for which $f''(x) = 0$.

Guiding question:

Determine the intervals in the range $[-4, 3]$ for which an antiderivative of f is strictly monotonically increasing and explain your answer.

Solution of Task 4

Derivatives and Antiderivatives

Expected solution of the statement of the task:

Statement 1: The statement is true, as f is strictly monotonically increasing in this interval.
Statement 2: The statement is false, as f has no local maximum or minimum in this interval (and has no saddle point).
Statement 3: The statement is false, as the concavity of f changes in this interval.
Statement 4: The statement is false, as f has no point of inflexion in this interval.

Answer key:

The point for the core competencies is to be awarded if for each statement it has been correctly identified whether the statement is true or false and the decision has been justified correctly.

Expected solution of the guiding question:

An antiderivative of f is strictly monotonically increasing in the intervals $[-4, -2]$ and $[-0.5, 3]$ as the function f has non-negative values in these intervals.

Answer key:

The point for the guiding question is to be awarded if both intervals of an antiderivative of f have been given correctly. Different interval notations (open or half-open) as well as correct formal or verbal descriptions are all to be accepted as correct.
For the boundaries of the intervals, deviations of ± 0.2 are to be accepted.

Task 5

Medication

According to the information from a pharmaceutical company, only 2 % of people who consume a particular medication experience mild side effects.

The medication is consumed by 50 people.

As a simplification, it should be assumed in the following that the number of people who experience mild side effects is binomially distributed.

Task:

Determine how many people can be expected to experience mild side effects.

Determine the probability that more than two people experience mild side effects.

Guiding question:

Determine the lowest number n ($n \in \mathbb{N}$) of people that would have to take the medication such that there is a probability of at least 90 % that at least one person experiences mild side effects. Explain your method.

Solution of Task 5

Medication

Expected solution to the statement of the task:

Expectation value $E = n \cdot p = 1$

$P(X > 2) \approx 0.078 = 7.8\ \%$

Answer key:

The point for the core competencies is to be awarded if the expectation value and the probability have been given correctly.
Tolerance interval: [0.07, 0.08] or [7 %, 8 %]

Expected solution to the guiding question:

Possible method:

$P(X \geq 1) \geq 0.9 \quad \Rightarrow \quad 1 - P(X = 0) \geq 0.9$

$1 - 0.98^n \geq 0.9 \quad \Rightarrow \quad 0.98^n \leq 0.1 \quad \Rightarrow \quad n \geq 113.97\ldots$

$n = 114$

Answer key:

The point for the guiding question is to be awarded if the lowest number of people has been given correctly and the method has been explained correctly.

Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

January 2018

Mathematics

Supplementary Examination 1
Examiner's Version

BMB
Bundesministerium
für Bildung

Instructions for the supplementary examination

The following supplementary examination is comprised of five tasks that can be completed independently of one another.

Each task contains two parts: The statement of the task requires the candidate to demonstrate core competencies, and the guiding question that follows it requires the candidate to show their ability to communicate their ideas.

In the following document, the examiner will find the tasks as well as the expected solutions and the answer key.

The preparation time is to be at least 30 minutes and the examination time is to be at most 25 minutes.

Assessment

Each task can be awarded zero, one or two points. There is one point available for each demonstration of core competencies as well as for each guiding question. A maximum of 10 points can be achieved.

For the grading of the examination the following scale should be used:

Grade	Minimum number of points
Pass	4 points for the core competencies + 0 points for the guiding questions 3 points for the core competencies + 1 point for the guiding questions
Satisfactory	5 points for the core competencies + 0 points for the guiding questions 4 points for the core competencies + 1 point for the guiding questions 3 points for the core competencies + 2 points for the guiding questions
Good	5 points for the core competencies + 1 point for the guiding questions 4 points for the core competencies + 2 points for the guiding questions 3 points for the core competencies + 3 points for the guiding questions
Very good	5 points for the core competencies + 2 points for the guiding questions 4 points for the core competencies + 3 points for the guiding questions

The examination board will decide on the final grade based on the candidate's performance in the supplementary examination as well as the result of the written examination.

Evaluation grid for the supplementary examination

This evaluation grid may be used to assist the examiner in assessing the candidate's performance.

	Point for core competencies reached	Point for the guiding question reached
Task 1		
Task 2		
Task 3		
Task 4		
Task 5		

Task 1

Lines

The vector equation of a line g as well as the equations of three further lines g_1, g_2, g_3 are given below.

$g: X = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + s \cdot \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ where $s \in \mathbb{R}$

$g_1: 3 \cdot x + y = 9$

$g_2: y = -3 \cdot x + 10$

$g_3: x - 3 \cdot y = -7$

Task:

Determine which of the lines g_1, g_2, g_3 are perpendicular to the line g and justify your answer.

Guiding question:

Determine which of the four lines are identical and justify your answer.

Determine how the values a_1 and b_2 (where $a_1, b_2 \in \mathbb{R}$) of the line $h: X = \begin{pmatrix} a_1 \\ 3 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ b_2 \end{pmatrix}$ where $t \in \mathbb{R}$ should be chosen so that g and h intersect at exactly one point.
Justify your answer.

Solution of Task 1

Lines

Expected solution of the statement of the task:

The lines g_1 and g_2 are perpendicular to g.

Possible justification:
The direction vector of g, $\vec{v} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$, is also a normal vector of the lines g_1 and g_2.

Answer key:

The point for the core competencies is to be awarded if it has been stated that only g_1 and g_2 are perpendicular to g and a correct justification has been given.

Justifications involving the scalar product or using sketches are also acceptable.

Expected solution of the guiding question:

The lines g and g_3 are identical.

Possible justification:
Both lines have the direction vector $\vec{v} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and go through the point $P = (2,3)$.

For the lines g and h to have exactly one point of intersection, their gradients must be different; therefore, $b_2 \neq \frac{1}{3}$ must hold. As a_1 only determines the position of the point of intersection, not its existence, any real number can be chosen for a_1.

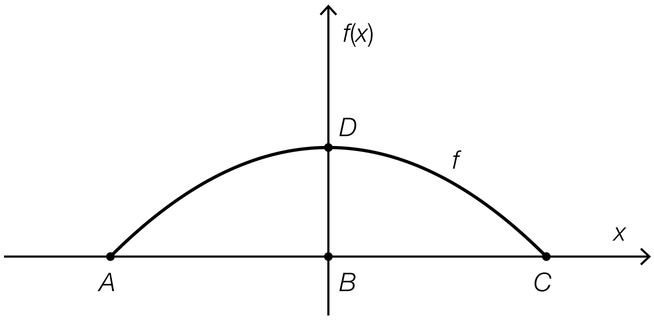
Answer key:

The point for the guiding question is to be awarded if it has been stated that g_3 and g are identical, and this has been justified correctly.
Furthermore, the conditions for the values of a_1 and b_2 must be given correctly with a correct justification.
If specific correct values are given for a_1 and b_2, then the point is to be awarded.

Task 2

Arc of a Bridge

The arc of a bridge is shown in the diagram below. The line segment AC with midpoint B has a length of 40 metres. The maximum height of the arc of the bridge, BD, is 10 metres.



Task:

Find the equation of the function f, where $f(x) = a \cdot x^2 + b$ ($a, b \in \mathbb{R}$), that can be used to model the curve of the arc of the bridge. Explain your approach.

Guiding question:

In order for larger vehicles to also pass under the bridge, the height BD must be increased. Explain whether the parameters a and b of the function f, where $f(x) = a \cdot x^2 + b$ ($a, b \in \mathbb{R}$), should be changed to be greater than, less than or equal to their current values if the distance AC is to remain unchanged.

If the point A is taken to be at the origin, the function g, where $g(x) = c \cdot x^2 + d \cdot x + e$ ($c, d, e \in \mathbb{R}$), should be used to model the situation.

Using the symbols “<”, “>” or “=”, complete the statements below about c, d and e so that the statements are true for the function g.

c ___ 0; d ___ 0; e ___ 0

Solution of Task 2

Arc of a Bridge

Expected solution of the statement of the task:

Possible approach:
$D = (0,10) \quad \Rightarrow \quad b = 10$

The root of f is at $x = 20$ (and -20).
$f(20) = 0 \quad \Rightarrow \quad 0 = 400 \cdot a + 10 \quad \Rightarrow \quad a = -\frac{1}{40} = -0.025$

$f(x) = -0.025 \cdot x^2 + 10$

Answer key:

The point for the core competencies is to be awarded if a correct equation of the function has been determined and a correct approach has been explained.

Expected solution of the guiding question:

If the height BD is to be increased, then the parameter b needs to be made bigger. So that the zeros remain unchanged, the parameter a needs to be made smaller.

$c < 0; \quad d > 0; \quad e = 0$

Answer key:

The point for the guiding question is to be awarded if the explanation of how both parameters should be changed is correct and the correct symbols have been given in the appropriate locations.

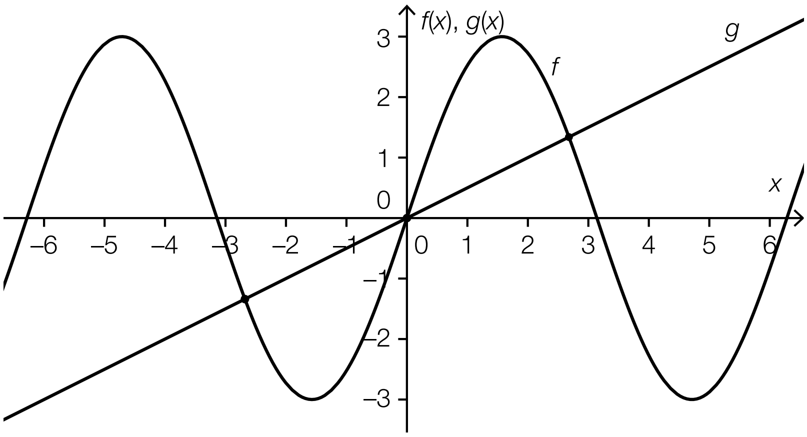
Task 3

Functions

The equations and graphs of the functions f and g are given below:

$f(x) = 3 \cdot \sin(x)$

$g(x) = \frac{x}{2}$



Task:

Determine the value of x_1 in the interval $[0, \pi]$ such that $f'(x_1) = g'(x_1)$ holds and explain how this point can be determined graphically.

Guiding question:

The equation $f(x) = g(x)$ has three solutions for x at a, 0 and c, where $a < 0 < c$.

On the diagram above, represent the value of the expression $\int_0^c (f(x) - g(x))\,dx$ graphically.

Determine the value of the expression $\int_a^c (f(x) - g(x))\,dx$.

Solution of Task 3

Functions

Expected solution of the statement of the task:

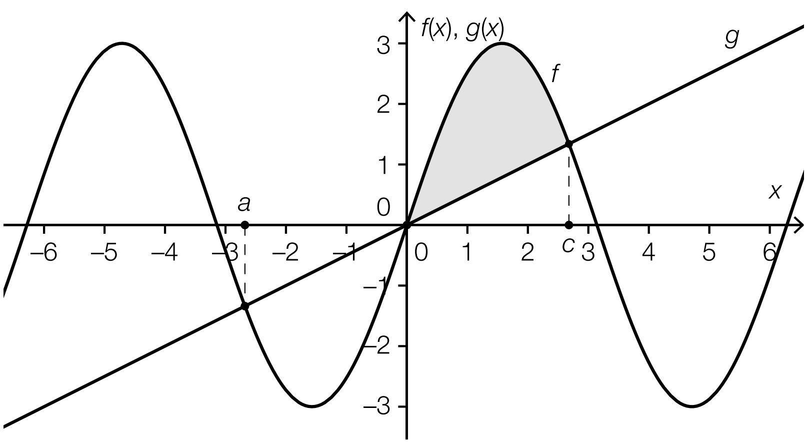
$3 \cdot \cos(x_1) = \frac{1}{2} \quad \Rightarrow \quad x_1 \approx 1.4$

The value of x_1 gives the location of the point (in the interval $[0, \pi]$) at which the gradient of the line is the same as the gradient of the tangent to the graph of f. If the line g is translated so that it becomes a tangent to the graph of f in the interval $[0, \pi]$, then the value is given by the x-coordinate of the point where the two functions meet.

Answer key:

The point for the core competencies is to be awarded if x_1 has been calculated correctly and a method for obtaining the value graphically has been explained correctly.

Expected solution of the guiding question:



The value of the expression $\int_a^c (f(x) - g(x))\,\mathrm{d}x$ is zero.

Answer key:

The point for the guiding question is to be awarded if the correct area has been identified and the value of the integral $\int_a^c (f(x) - g(x))\,\mathrm{d}x$ has been given correctly.

Task 4

Reaction Times

A test subject determines their reaction time (in s) using an online test that they take ten times. The subject obtains the values given below:

0.38 s; 0.27 s; 0.30 s; 0.34 s; 0.25 s; 0.39 s; 0.28 s; 0.24 s; 0.33 s; 0.32 s

Task:

Determine the mean $\bar{t}$ and the standard deviation s of the ten results given above.

Determine what percentage of the reaction times given lie in the interval $[\bar{t} - s, \bar{t} + s]$.

Guiding question:

The test subject carries out the test two more times and obtains the results t_{11} and t_{12} where $t_{11} \neq t_{12}$. The mean that is calculated using all twelve times is referred to as $\bar{t}_{\text{new}}$, and the resulting standard deviation is referred to as s_{new}.

State which conditions the times t_{11} and t_{12} have to fulfil such that $\bar{t}_{\text{new}} = \bar{t}$ and $s_{\text{new}} < s$ hold.

Solution of Task 4

Reaction Times

Expected solution of the statement of the task:

$\bar{t} = 0.31$
$s \approx 0.05$
$[\bar{t} - s, \bar{t} + s] \approx [0.26, 0.36]$

Six of the reaction times lie within the given interval, which corresponds to 60 %.

Answer key:

The point for the core competencies is to be awarded if the mean, the standard deviation and the percentage have been given correctly.

Tolerance intervals:
for the standard deviation: [0.048, 0.052]
for the lower bound of the interval: [0.258, 0.262]
for the upper bound of the interval: [0.358, 0.362]

Expected solution of the guiding question:

The new values must be symmetrical about the mean $\bar{t}$ i.e. $\frac{t_{11} + t_{12}}{2} = \bar{t}$ must hold.

If the new values lie in the interval $(\bar{t} - s, \bar{t} + s)$ then $s_{\text{new}} < s$.

Answer key:

The point for the guiding question is to be awarded if the correct conditions for the times t_{11} and t_{12} have been given. Answers that state that the values t_{11} and t_{12} have to be close to the mean can be accepted as a condition for $s_{\text{new}} < s$.

Task 5

Raffle

Among 100 raffle tickets, there are 30 winning tickets. Of these 30 tickets, 25 result in winnings of € 10 each and five result in winnings of € 100 each.

Task:

Three tickets are selected at random from the 100 tickets.
Find the probability that no winning tickets are selected and explain your method.

Guiding question:

A person receives a randomly selected ticket from these 100 raffle tickets as a present. Determine the expectation value for this person's winnings.

Another person receives two randomly selected tickets from these 100 raffle tickets as a present. Find an expression that could be used to calculate the probability that this person wins € 110 and explain your approach.

Solution of Task 5

Raffle

Expected solution of the statement of the task:

If no winning ticket is selected, this means that the three tickets have been taken from the 70 losing tickets without replacement.

$\frac{70}{100} \cdot \frac{69}{99} \cdot \frac{68}{98} \approx 0.3385 = 33.85\ \%$

Answer key:

The point for the core competencies is to be awarded if the probability has been given correctly and a correct method has been shown.
Tolerance interval for the probability: [33%, 34%]

Expected solution of the guiding question:

$\frac{25}{100} \cdot 10 + \frac{5}{100} \cdot 100 = 7.5$

The expectation value for the winnings is € 7.50.

Winnings of € 110 means that out of the two tickets, one ticket wins € 10 and one wins € 100.

A possible expression for calculating the probability: $2 \cdot \frac{25 \cdot 5}{100 \cdot 99}$

Answer key:

The point for the guiding question is to be awarded if the expectation value for the winnings and the correct expression have been given and a correct method has been explained. Equivalent expressions are to be accepted.

Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

May 2017

Mathematics

Supplementary Examination 6
Examiner's Version

BMB
Bundesministerium
für Bildung

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The examination board will decide on the final grade based on the candidate's performance in the supplementary examination as well as the result of the written examination.

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Task 1		
Task 2		
Task 3		
Task 4		
Task 5		

Task 1

Equivalent Transformations

Below, you will see two equations that hold for $x \in \mathbb{R}$:

- $3 - \frac{2x}{5} = -1$
- $\frac{3x}{5} + 1 = x - 3$

Task:

Determine whether these two equations are equivalent.

If the equations are equivalent, show a series of possible equivalent transformations that transform the first equation into the second.
If the equations are not equivalent, justify why this is the case.

Guiding question:

With specific reference to the example given below, explain why the rearrangement shown does not give rise to an equivalent equation. The equation is defined over the set of all real numbers.

$(x-2)^2 = 25 \quad | \ \sqrt{\ }$
$x - 2 = 5$

Solution of Task 1

Equivalent Transformations

Expected solution of the statement of the task:

The two equations are equivalent.

Possible equivalent transformations:
By subtracting the number 2, the equation $1 - \frac{2x}{5} = -3$ is obtained.
Then, by adding x, the equation becomes $1 + \frac{3x}{5} = -3 + x$, which is the same as the second equation.

Answer key:

The point for the core competencies is to be awarded if the equations are determined to be equivalent and possible equivalent transformations have been given correctly.

Expected solution of the guiding question:

The first equation has solutions -3 and 7; however, the second equation has only one solution, $x = 7$. Therefore, the two equations do not have the same set of solutions and are not equivalent.

Answer key:

The point for the guiding question is to be awarded if it has been correctly explained why the two equations are not equivalent.

Task 2

Cooling

At time $t_0 = 0$, a container with hot water is put outside where the ambient temperature is 0 °C. The temperature of the water, $T(t)$ (in ºC), is dependent on the time t (in minutes) and can be described by the function T where $T(t) = 90 \cdot e^{-0.2 \cdot t}$.

Task:

Determine the half-life of this cooling process and explain the significance of this result within the context given.

Guiding question:

Show that the instantaneous rate of change of the temperature of the water, $T'(t)$, is directly proportional to the instantaneous temperature of the water at time t. Determine the constant of proportionality, k.

k = ____

Explain the meaning of the absolute value T' in the context of the cooling process.

Solution of Task 2

Cooling

Expected solution of the statement of the task:

$45 = 90 \cdot e^{-0{,}2 \cdot t} \;\Rightarrow\; t \approx 3.5$

The temperature of the water has reduced to half of its starting temperature (from 90 ºC to 45 ºC) after approximately 3.5 minutes.

Answer key:

The point for the core competencies is to be awarded if the half-life has been correctly determined and correctly interpreted within the context of the question.

Expected solution of the guiding question:

$T'(t) = 90 \cdot (-0.2) \cdot e^{-0.2 \cdot t} = -0.2 \cdot T(t)$
$k = -0.2$

The absolute value of T' gives the speed of the cooling process.

Answer key:

The point for the guiding question is to be awarded if the directly proportional relationship has been shown and the constant of proportionality has been correctly determined ($k = -5$ is also to be accepted as a correct answer, as $T(t) = -5 \cdot T'(t)$). Also, the meaning of T' needs to have been given correctly.

Task 3

Crude Oil Price

In December 2015, the price of crude oil tended to fall daily. The price of crude oil is given by the barrel in US dollars. One barrel contains 159 litres.

On the 1st December 2015 at 12:00 noon, the crude oil price was 41.70 US dollars per barrel. On the 11th December 2015 at 12:00 noon, the price was 37.94 US dollars per barrel.

Task:

Determine the absolute and relative (percentage) change of the crude oil price per barrel for the given time period.

Guiding question:

Determine the average rate of change of the crude oil price per litre for the given time period (in days) and interpret your result in the given context.

Determine the price of 1 litre of crude oil on the 16th December 2015 if the crude oil price from the 11th December 2015 had continued to develop with the same average rate of change per day.

Solution of Task 3

Crude Oil Price

Expected solution of the statement of the task:

Absolute change: –3.76 US dollars per barrel
Relative change: –9 % or –0.09

Answer key:

The point for the core competencies is to be awarded if both values are given correctly. Positive values (3.76 US dollars and 9 %) are also to be accepted if the candidate explains verbally that these values represent a reduction.

Expected solution of the guiding question:

Possible solution:

Average rate of change: $\dfrac{\frac{37.94}{159} - \frac{41.7}{159}}{10} \approx -0.00236$

The crude oil price per litre reduced by an average of approximately 0.00236 US dollars per day in this time period.

The price per litre on 16.12.2015 if the development had continued:
$\frac{37.94}{159} - 5 \cdot 0.00236 \approx 0.2268$
The price per litre would have been approximately 0.2268 US dollars.

Answer key:

The point for the guiding question is to be awarded if the average rate of change of the crude oil price per litre has been given and interpreted correctly. Also, the crude oil price per litre on the 16.12.2015 needs to have been correctly given.
Tolerated range for the average rate of change: [–0.0024, –0.002]
Tolerated range for the price per litre: [0.22, 0.23]

Task 4

Integral

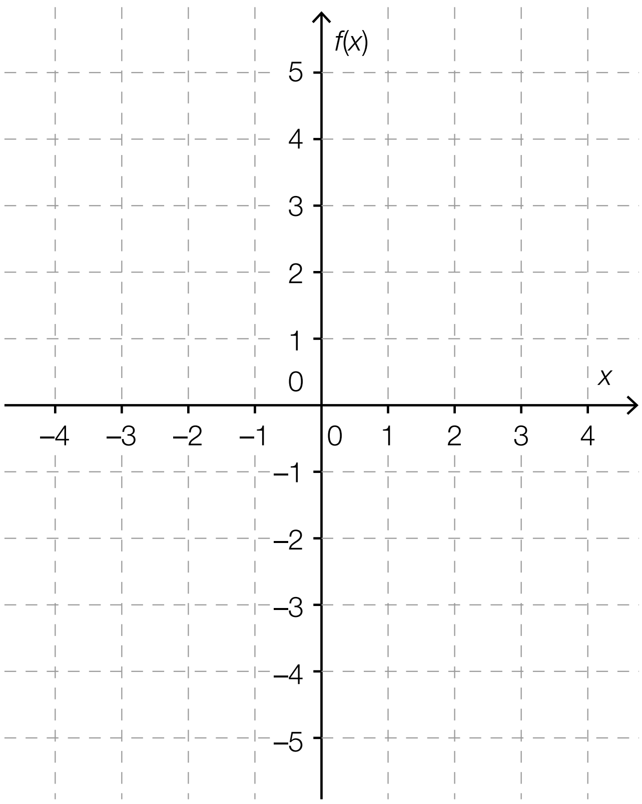
Let f be a linear function where $f(x) = -2 \cdot x + 2$.

Task:

Find the equation of the antiderivative of the function f, F, for which $F(2) = 1$ holds. Show your method.

Guiding question:

Find the value of the definite integral $\int_0^3 f(x)\,\mathrm{d}x$. Show your method.
Draw the graph of the function f in the coordinate system given below and explain why, in this case, the value of the definite integral does not correspond with the area enclosed by the graph of the function and the x-axis in the range [0, 3].



Solution of Task 4

Integral

Expected solution of the statement of the task:

Possible approach:

The equation $F(x) = -x^2 + 2 \cdot x + c$ holds for all antiderivatives.
As $F(2) = 1$, $-2^2 + 2 \cdot 2 + c = 1 \;\Rightarrow\; c = 1$.
Hence: $F(x) = -x^2 + 2 \cdot x + 1$.

Answer key:

The point for the core competencies is to be awarded if a correct equation for F has been given and a correct method has been explained.

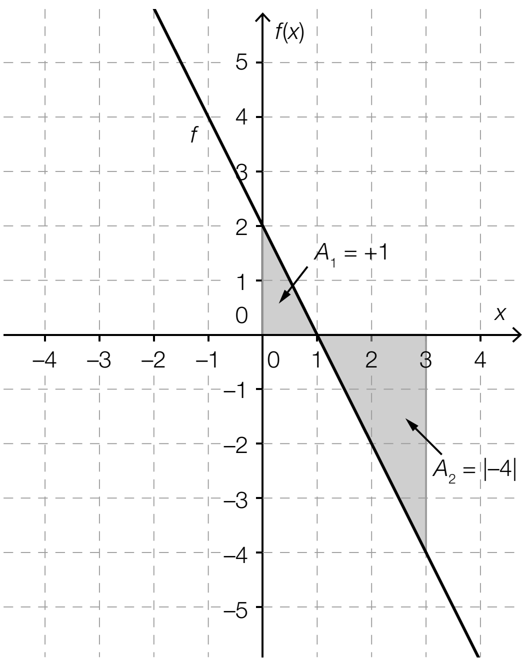
Expected solution of the guiding question:

Possible methods:
$\int_0^3 f(x)\,\mathrm{d}x = \int_0^3 (-2 \cdot x + 2)\,\mathrm{d}x = (-x^2 + 2 \cdot x)\Big|_0^3 = -3$ and/or $F(3) - F(0) = -2 - 1 = -3$

Possible explanation: When calculating areas, it needs to be considered that the integral of the function for regions that lie underneath the x-axis is negative. Therefore, the calculation has to be carried out in sections corresponding to the positions of the regions.

$\int_0^3 f(x)\,\mathrm{d}x = 1 + (-4) = -3$

Area:
$A_1 + A_2 = 1 + |-4| = 5$



Answer key:

The point for the guiding question is to be awarded if the value of the integral has been determined correctly, a correct method has been used, the graph has been drawn correctly, and a correct explanation has been given.

Task 5

Cone

A cone that is thrown can either land on its curved surface or on its base.

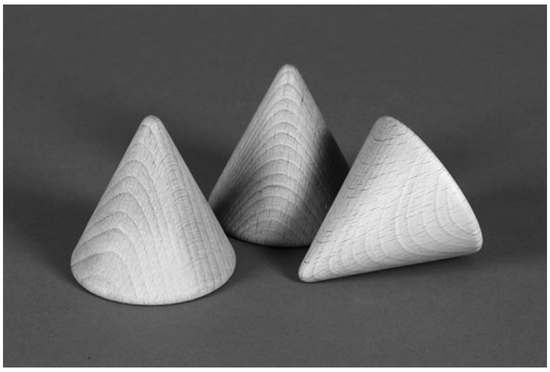


Image source: http://www.holzbausteine.at/images/Spitzkegel60.jpg [28.04.2016].

Task:

Throwing a cone of this kind can be seen as a random experiment. At first, the cone is thrown 50 times. In 12 of the throws, the cone lands on its base.

Felix carries out the following calculation:

$\left(\frac{12}{50}\right)^2 = \frac{144}{2{,}500} = 0.0576 = 5.76\ \%$

Interpret the result in the given context.

Guiding question:

Selin says that actually, the probability of the cone landing on its base is unknown.

Suggest an argument Selin can use to support her statement and how the experiment would need to be changed to calculate this probability as accurately as possible.

Solution of Task 5

Cone

Expected solution of the statement of the task:

The probability of the cone landing on its base twice in two throws is 5.76 % (given that the probability of the cone landing on its base is $\frac{12}{50}$).

Answer key:

The point for the core competencies is to be awarded if a correct interpretation in the given context has been supplied.

Expected solution of the guiding question:

The relative frequency only gives an estimate for the probability.
If an experiment is only conducted 50 times, then the probability calculated by the relative frequency is a very inaccurate approximation.
The experiment would have to be conducted much more often to be able to obtain a more accurate estimate for the probability.

Answer key:

The point for the guiding question is to be awarded if an answer corresponding to one of the suggested solutions has been given.

Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

June 2016

Mathematics

Supplementary Examination 9
Examiner's Version

BM | BF Bundesministerium für Bildung und Frauen

Bundesinstitut bifie

Instructions for the supplementary examination

The following supplementary examination is comprised of five tasks that can be completed independently of one another.

Each task contains two parts: The statement of the task requires the candidate to demonstrate core competencies, and the guiding question that follows it requires the candidate to show their ability to communicate their ideas.

In the following document, the examiner will find the tasks as well as the expected solutions and the answer key.

The preparation time is to be at least 30 minutes and the examination time is to be at most 25 minutes.

Assessment

Each task can be awarded zero, one or two points. There is one point available for each demonstration of core competencies as well as for each guiding question. A maximum of 10 points can be achieved.
For the grading of the examination the following scale should be used:

Grade	Minimum number of points
Pass	4 points for the core competencies + 0 points for the guiding questions 3 points for the core competencies + 1 point for the guiding questions
Satisfactory	5 points for the core competencies + 0 points for the guiding questions 4 points for the core competencies + 1 point for the guiding questions 3 points for the core competencies + 2 points for the guiding questions
Good	5 points for the core competencies + 1 point for the guiding questions 4 points for the core competencies + 2 points for the guiding questions 3 points for the core competencies + 3 points for the guiding questions
Very good	5 points for the core competencies + 2 points for the guiding questions 4 points for the core competencies + 3 points for the guiding questions

The examination board will decide on the final grade based on the candidate's performance in the supplementary examination as well as the result of the written examination.

Evaluation grid for the supplementary examination

This evaluation grid may be used to assist the examiner in assessing the candidate's performance.

	Point for core competencies reached	Point for the guiding question reached
Task 1		
Task 2		
Task 3		
Task 4		
Task 5		

Task 1

Lines in $\mathbb{R}^3$

Let g and h be two lines in $\mathbb{R}^3$.

The line g goes through the point $P = (3,1,5)$ and is parallel to the y-axis.

Task:

Determine a vector equation of the line g.

Explain why it is not possible to find a point Q, where $Q = (1,y_Q,z_Q)$, such that the point Q lies on the line g.

Guiding question:

Describe all possible relative positions of two lines in $\mathbb{R}^3$.

The line h is described by the vector equation $X = \begin{pmatrix} x_h \\ 1 \\ 3 \end{pmatrix} + s \cdot \begin{pmatrix} 2 \\ y_h \\ 1 \end{pmatrix}$ where $s, x_h, y_h \in \mathbb{R}$.

Is it possible to determine values for x_h and y_h such that the two lines g and h are perpendicular to one another and meet at point P?
If not, justify by means of calculation why this is not possible.
If so, determine the relevant values of x_h and y_h.

Solution of Task 1

Lines in $\mathbb{R}^3$

Expected solution of the statement of the task:

Possible vector equation:

$$g: X = \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix} + t \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \text{ with } t \in \mathbb{R}$$

Possible justification:
For all points on the line g, $x = 3$; $y = 1 + t$, and $z = 5$ (with $t \in \mathbb{R}$) hold.
⇒ Therefore, a point with $x = 1$ cannot lie on the line g.

Answer key:

The point for the core competencies is to be awarded if a correct vector equation of the line g has been given and a correct justification has been provided.

Expected solution of the guiding question:

Two lines in $\mathbb{R}^3$ can be identical, parallel, intersecting or skew.

It is possible to determine values for x_h and y_h such that the two conditions are fulfilled:

$x_h = -1$ (from the value of the parameter $s = 2$)

$y_h = 0$ (from $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = 0$)

Answer key:

The point for the guiding question is to be awarded if all the different possible relative positions have been given and the values of x_h and y_h have been given correctly.

Task 2

Quadratic Function

Let f be a function with $f(x) = r \cdot x^2 + s$ where $r, s \in \mathbb{R}, r \neq 0$.

Task:

Explain the effects that changes of the values of the parameters r and s have on the behaviour of the graph of f.

Guiding question:

The graph of the function f goes through the two points $B = (a,b)$ and $E = (0,e)$, where $a \neq 0$.

Determine expressions for r and s in terms of the coordinate values a, b, and e.

State the value of b for which the function f is not a quadratic function.

Solution of Task 2

Quadratic Function

Expected solution of the statement of the task:

For $r > 0$, the graph of the function f is a parabola that opens upwards; for $r < 0$, the graph of the function f is a parabola that opens downwards. The bigger the absolute value of r is, the "steeper" the graph of f is.

The change ofvalue of the parameter s determines the extent of the vertical translation of the parabola along the y-axis.

or:

The vertex of the parabola is at the point $(0,s)$.

or:

$(0,s)$ is the point where the graph crosses the vertical axis.

Answer key:

The point for the core competencies is to be awarded if the effects the changes of the values of the parameters r and s have on the behaviour of the graph of f have been explained comprehensibly and correctly.

Expected solution of the guiding question:

As B and E lie on the graph of f and E is the vertex of the parabola, we have:

$s = e$

$b = r \cdot a^2 + e \;\Rightarrow\; r = \dfrac{b - e}{a^2}$

For $b = e$, the function f is not a quadratic function.

Answer key:

The point for the guiding question is to be awarded if the values of the parameters r and s have been given correctly and it has been justified comprehensibly and correctly that $b = e$, so that f is not a quadratic function.

Task 3

Spring Force

If a spring is stretched, then the force required to stretch the spring is directly proportional to the amount by which the spring is stretched. The function F describes the force required in terms of the amount by which the spring is stretched, x.

Therefore: $F(x) = k \cdot x$.

The distance x is measured in metres (m) and $F(x)$ is measured in Newtons (N). The constant k is known as the spring constant and describes the "strength" of a spring.

Task:

Sketch a possible graph of F and label k on your sketch.

Guiding question:

Write down an expression in terms of k that could be used to calculate the work required to stretch the spring by a distance of x_0.

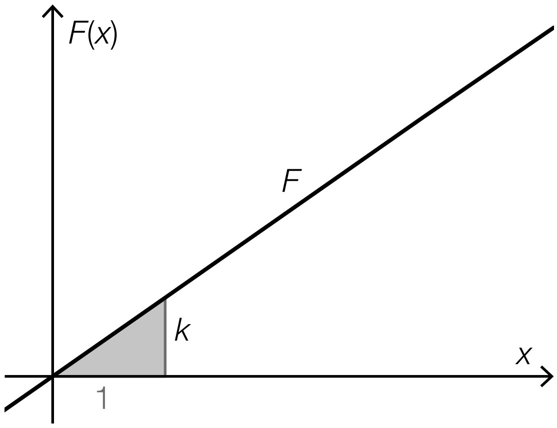
Explain how the work done changes if the spring is stretched by the length $2 \cdot x_0$.

Solution of Task 3

Spring Force

Expected solution of the statement of the task:

Possible sketch:



Answer key:

The point for the core competencies is to be awarded if an appropriate sketch of a homogenous linear function has been drawn and *k* has been labelled correctly.

Expected solution of the guiding question:

$$W = \int_0^{x_0} k \cdot x \, dx = \frac{k \cdot x_0^{\,2}}{2}$$

For the case where the spring is stretched by a length that is twice this amount, the following calculation holds:

$$W = \int_0^{2x_0} k \cdot x \, dx = \frac{k \cdot (2x_0)^2}{2}$$

The work done increases fourfold.

Answer key:

The point for the guiding question is to be awarded if an expression for calculating the work done has been given correctly and the change in work done has been described correctly.
Equivalent expressions are to be marked as correct.

Task 4

Marginal Costs

For a business, the cost function, K, where $K(x) = 4 \cdot x^3 - 60 \cdot x^2 + 400 \cdot x + 1\,000$, gives the cost of manufacturing x units of a product. The production costs, $K(x)$, are given in monetary units (GE), and the number of units manufactured is given in production units (ME).

The marginal costs (in GE/ME) are the additional costs accrued from increasing the number of units manufactured by 1 ME.

Task:

The approximate calculation of the marginal costs for a particular production amount, x_0, can be approximated by the first derivative $K'(x_0)$.

Using the derivative of the function K, determine the marginal costs for a production volume of 15 ME.

Guiding question:

Determine the difference in GE between the approximated marginal costs for a production volume of 15 ME and the actual increase in cost when the production volume is increased from 15 ME to 16 ME.

The first derivative K' is strictly monotonically increasing from $x = 5$ ME. Explain the implications of this statement for the production costs if the production volume increases.

Solution of Task 4

Marginal Costs

Expected solution of the statement of the task:

$K'(x) = 12 \cdot x^2 - 120 \cdot x + 400$
$K'(15) = 1\,300$

Answer key:

The point for the core competencies is to be awarded if $K'(15)$ has been calculated correctly.

Expected solution of the guiding question:

$K'(15) = 1\,300$

$K(16) - K(15) = 8\,424 - 7\,000 = 1\,424$

The value of the approximated marginal costs differs for $x_0 = 15$ ME from the actual cost increase for an extra 1 ME by 124 GE.

This means that the increase in costs upwards from the production volume $x = 5$ is progressive (i. e. the costs increase more and more the higher the production volume is).

Answer key:

The point for the guiding question is to be awarded if the difference has been calculated correctly and the meaning has been interpreted comprehensibly and correctly.

Task 5

Discrete Random Variable

For a discrete random variable, X, the following table shows all of its possible values as well as their corresponding probabilities. The parameter n is a natural number with $n \neq 0$.

k	1	4	7	10	15
$P(X = k)$	0.2	$\frac{2}{n}$	$\frac{6}{n}$	0.1	0.3

Task:

Determine the value of the parameter n and the expectation value of the random variable X. Explain your method.

Guiding question:

The standard deviation σ for the random variable described above has a value of $\sigma = 5.2$.

In the table, change the given probabilities $P(X = k)$ for at least two values of k such that the standard deviation becomes smaller and the table still shows a valid probability distribution. The values of the random variable (in the first row of the table) should remain unchanged. Explain your method.

Solution of Task 5

Discrete Random Variable

Expected solution of the statement of the task:

As the sum of all probabilities is 1, this means that $\frac{8}{n} = 0.4$. Therefore, $n = 20$.

$E(X) = 1 \cdot 0.2 + 4 \cdot 0.1 + 7 \cdot 0.3 + 10 \cdot 0.1 + 15 \cdot 0.3 = 8.2$

Answer key:

The point for the core competencies is to be awarded if both n and $E(X)$ have been calculated correctly and the method has been explained correctly.

Expected solution of the guiding question:

Possible method:
The table must be changed in such a way that both demands – a smaller standard deviation and a valid probability distribution i. e. the sum of the probabilities adds up to 1 – are fulfilled.
A sensible strategy is to reduce the probabilities of the events at the edges of the distribution and to increase the probabilities of the events in the middle.

Possible example:

k	1	4	7	10	15
$P(X = k)$	0.1	0.2	0.3	0.3	0.1

Answer key:

The point for the guiding question is to be awarded if a correct strategy for reducing the standard deviation has been given and an acceptable probability distribution remains. A calculation of the new standard deviation does not need to be given.