Name:	
Class:	

Standardised Competence-Oriented
Written School-Leaving Examination

AHS

12th January 2021

Mathematics

Bundesministerium
Bildung, Wissenschaft
und Forschung

Advice for Completing the Tasks

Dear candidate!

The following booklet contains Part 1 tasks and Part 2 tasks (divided into sub-tasks). The tasks can be completed independently of one another.

Please do all of your working out solely in this booklet and the paper provided to you. Write your name and that of your class on the cover page of the booklet in the spaces provided. Also, write your name and consecutive page numbers on each sheet of paper used. When answering each sub-task, indicate its name/number on your sheet.

In the assessment of your work, everything that is not crossed out will be considered. Your solutions must be marked clearly. If a solution is not clearly marked or if more than one solution is given, the task will be considered to be unsolved.

The use of the official formula booklet that has been approved by the relevant government authority is allowed. Furthermore, the use of electronic device(s) (e.g. graphic display calculators or other appropriate technology) is allowed provided there is no possibility to communicate via internet, Bluetooth, mobile networks, etc. and there is no access to your own data stored on the device.

An explanation of the task types is available in the examination room and can be viewed on request.

Please hand in the task booklet and all used sheets at the end of the examination.

Changing an answer for a task that requires a cross:

1. Fill in the box that contains the cross.
2. Put a cross in the box next to your new answer.

In this instance, the answer "5 + 5 = 9" was originally chosen. The answer was later changed to be "2 + 2 = 4".

1 + 1 = 3	☐
2 + 2 = 4	☒
3 + 3 = 5	☐
4 + 4 = 4	☐
5 + 5 = 9	■

Selecting an item that has been filled in:

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

In this instance, the answer "2 + 2 = 4" was filled in and then selected again.

1 + 1 = 3	☐
2 + 2 = 4	■
3 + 3 = 5	☐
4 + 4 = 4	■
5 + 5 = 9	☐

Assessment

The tasks in Part 1 will be awarded either 0 points or 1 point or 0, ½ or 1 point, respectively. The points that can be reached in each task are listed in the booklet for all Part 1 tasks. Every sub-task in Part 2 will be awarded 0, 1 or 2 points. The tasks marked with an $\boxed{\text{A}}$ will be awarded either 0 points or 1 point.

Two assessment options

1) If you have reached **at least 16** of the 28 points (24 Part 1 points + 4 $\boxed{\text{A}}$ points from Part 2), a grade will be awarded as follows:

Pass	16–23.5 points
Satisfactory	24–32.5 points
Good	33–40.5 points
Very Good	41–48 points

2) If you have reached **fewer than 16** of the 28 points (24 Part 1 points + 4 $\boxed{\text{A}}$ points from Part 2), but have reached a **total of 24 points or more** (from Part 1 and Part 2 tasks), then a "Pass" or "Satisfactory" grade is possible as follows:

Pass	24–28.5 points
Satisfactory	29–35.5 points

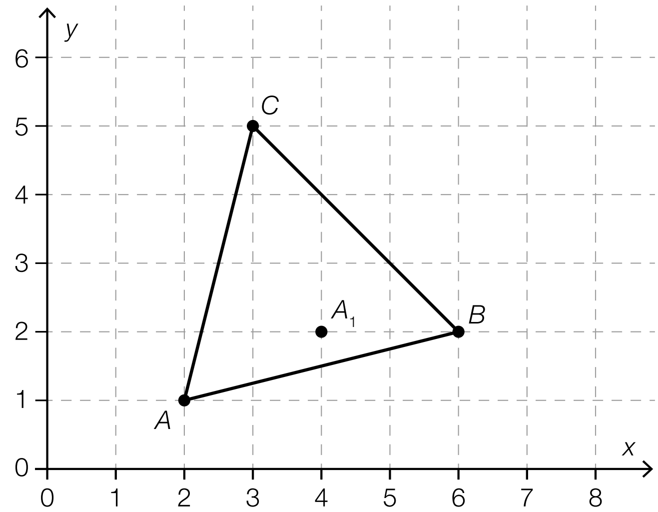
If you have reached fewer than 16 points in Part 1 (including the compensation tasks marked with an $\boxed{\text{A}}$ from Part 2) and if the total is less than 24 points, you will not pass the examination.

Good luck!

Task 1

Translating a Triangle

The diagram below shows a triangle with vertices A, B and C as well as the point A_1. The points shown in bold have integer coordinates.



The triangle is to be translated by the vector $\overrightarrow{AA_1}$ so that the points A, B and C become the points A_1, B_1 and C_1.

Task:

Determine the coordinates of the point C_1.

C_1 = (_______ | _______)

[0/½/1 point]

Task 2

Solution to an Equation

An equation in $x \in \mathbb{R}$ is shown below.

$\sqrt{2 \cdot x - 6} = a$ with $a \in \mathbb{R}_0^+$

Task:

Put a cross next to the interval that contains the solution to the equation given above for all values of $a \in \mathbb{R}_0^+$.

$(-\infty, -3]$	☐
$[3, \infty)$	☐
$[-3, 0)$	☐
$[0, 3)$	☐
$[-6, -3)$	☐
$[3, 6]$	☐

[0/1 point]

Task 3

Cyclists

Alexander's school and Bernhard's school are connected by a 13 km long straight road.
On a particular day, both boys cycle along this road from their respective schools towards each other. They set off at different times and meet each other t hours after Alexander's departure.
Up to the time they meet each other, the following statements hold:

- Alexander cycles at an average speed of 18 km/h.
- Bernhard cycles at an average speed of 24 km/h.

For the context described above, the equation below is formed and solved.

$18 \cdot t + 24 \cdot \left(t - \frac{1}{3}\right) = 13$

$t = \frac{1}{2}$

Task:

Put a cross next to each of the two statements that are true with respect to the equation given above and its solution.

Alexander sets off 10 minutes later than Bernhard.	☐
When they meet, Alexander has been travelling for 30 minutes.	☐
When they meet, Bernhard has been travelling for 20 minutes.	☐
When they meet, Alexander has covered a distance of 9 km.	☐
When they meet, the boys are further away from Bernhard's school than from Alexander's school.	☐

[0/1 point]

Task 4

Quadratic Equation

For $a \in \mathbb{R}\backslash\{0\}$, let $(a \cdot x + 7)^2 = 25$ be a quadratic equation in $x \in \mathbb{R}$.

Task:

Write down all $a \in \mathbb{R}\backslash\{0\}$ for which $x = -4$ is a solution to the quadratic equation shown above.

[0/1 point]

Task 5

Vector Equation of a Line

Let g be a line with vector equation g: $X = A + t \cdot \overrightarrow{AB}$ with $t \in \mathbb{R}$.

Task:

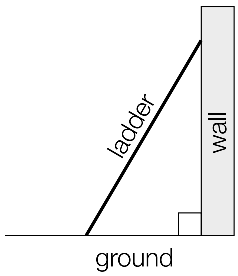
Determine the value of t such that $X = B$ holds.

[0/1 point]

Task 6

Ladder

A ladder is leaning against a vertical wall.
The top of the ladder is at a height of 6 m on the wall, and it makes an angle of 20° with the wall. This situation is depicted in the diagram shown (diagram not to scale).



Task:

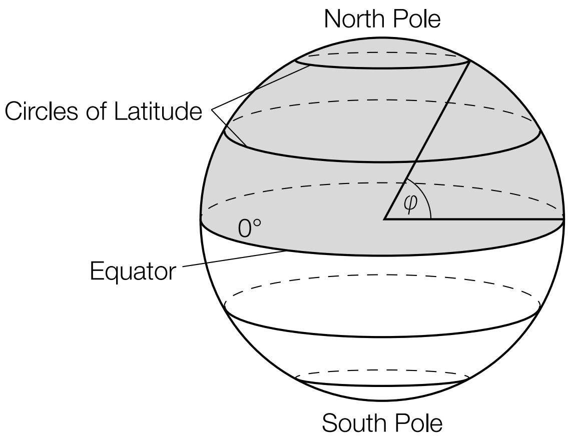
Determine the length of the ladder.

[0/1 point]

Task 7

Latitude

The shape of the Earth is approximately a sphere with a radius of 6370 km.
The diagram below shows the northern hemisphere of the Earth shaded in grey. In the northern hemisphere, the latitude φ is measured from the equator in a northerly direction, where $0° \leq \varphi \leq 90°$.



To determine the radius r (in km) of a circle of latitude (at the latitude φ), the following formula holds:
$r = 6370 \cdot \cos(\varphi)$

Task:

Write down the smallest possible interval W that contains all values of r.

$W = [$ _______ , _______ $]$

[0/1 point]

Task 8

Properties of Functions

Four equations of the real functions f_1 to f_4 (with $a, b \in \mathbb{R}^+$ and $b < 1$) and six lists of properties of functions are shown below.

Task:

Match each of the four equations of functions to the corresponding list (from A to F).

$f_1(x) = a \cdot b^x$	
$f_2(x) = a \cdot x + b$	
$f_3(x) = a \cdot \sin(b \cdot x)$	
$f_4(x) = a \cdot x^3 + b$	

A	– no change in monotonicity – constant gradient – no change in concavity
B	– exactly one local maximum or minimum x_0 – symmetrical about the line $x = x_0$ – at most two zeros
C	– infinitely many local maxima and minima – infinitely many points of inflexion – no asymptote
D	– only defined for $x \in [0, \infty)$ – concave down over its whole domain – no local maxima, local minima or points of inflexion
E	– no local maximum or minimum – exactly one zero – exactly one point of inflexion
F	– no change in monotonicity – the x-axis is an asymptote – no change in concavity

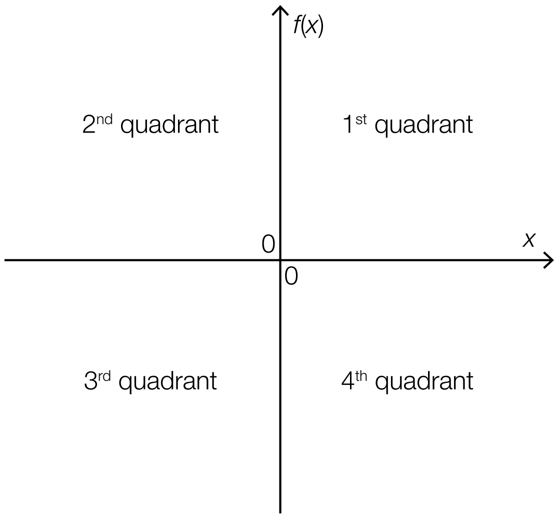
[0/½/1 point]

Task 9

Behaviour of the Graph of a Linear Function

Let f be a linear function with $f(x) = m \cdot x + c$ with $m, c \in \mathbb{R}$ and $c \neq 0$.

The plane is split into four quadrants by the two coordinate axes (see diagram below).



For the graph of f, the following statements hold:

- The graph does not go through the 1st quadrant.
- The graph goes through the 2nd, 3rd and 4th quadrants.

Therefore, certain conditions on m and c must hold.

Task:

Put a cross next to the statement with the correct conditions.

$m < 0$ and $c < 0$	☐
$m < 0$ and $c > 0$	☐
$m > 0$ and $c < 0$	☐
$m > 0$ and $c > 0$	☐
$m = 0$ and $c < 0$	☐
$m = 0$ and $c > 0$	☐

[0/1 point]

Task 10

Polynomial Function

There is a relationship between the degree of a polynomial function and the number of real zeros, local maxima and minima and points of inflexion of the function.

Task:

Complete the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

Every ______①______ polynomial function has ______②______.

①	
4th degree	☐
5th degree	☐
6th degree	☐

②	
at least two distinct local maxima or minima	☐
at least two distinct zeros	☐
at least one point of inflexion	☐

[0/1 point]

Task 11

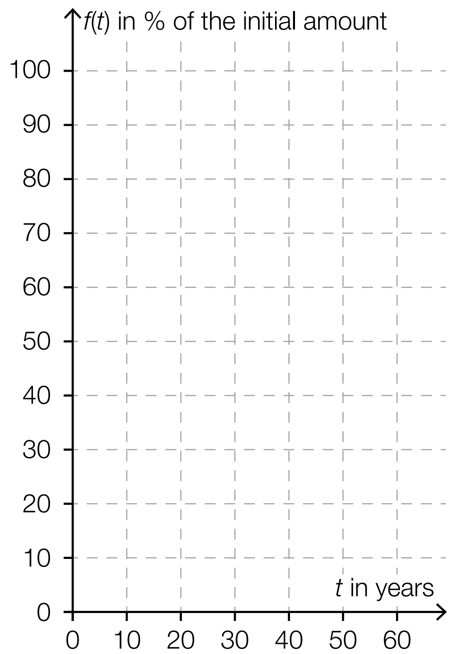
Half-Life

The radioactive isotope ^{137}Cs (caesium) has a half-life of around 30 years.

The function *f* gives the percentage of the initial amount of ^{137}Cs that is still present in terms of the time *t* (*t* in years, *f*(*t*) in % of the initial amount).
The amount of ^{137}Cs that is present at the time $t = 0$ is termed the *initial amount*.

Task:

In the coordinate system shown below, sketch the graph of *f* over the time interval [0, 60].



[0/1 point]

Task 12

Trigonometric Function

Let $f\colon \mathbb{R} \to \mathbb{R}$ be a function with $f(x) = 3 \cdot \cos(x)$. This function is to be represented in the form $x \mapsto a \cdot \sin(x + b)$, where $a, b \in \mathbb{R}$.

Task:

Write down a correct value for each of a and b.

a = ____

b = ____

[0/½/1 point]

Task 13

Measuring a Velocity

The velocity of a moving body in terms of the time t is modelled by a differentiable function v (t in s, $v(t)$ in m/s). The velocity $v(t)$ is measured starting from the time $t = 0$.

The limit $\lim\limits_{t \to 3} \frac{v(t) - v(3)}{t - 3}$ is considered.

Task:

Put a cross next to each of the two statements that correctly describe the limit considered above.

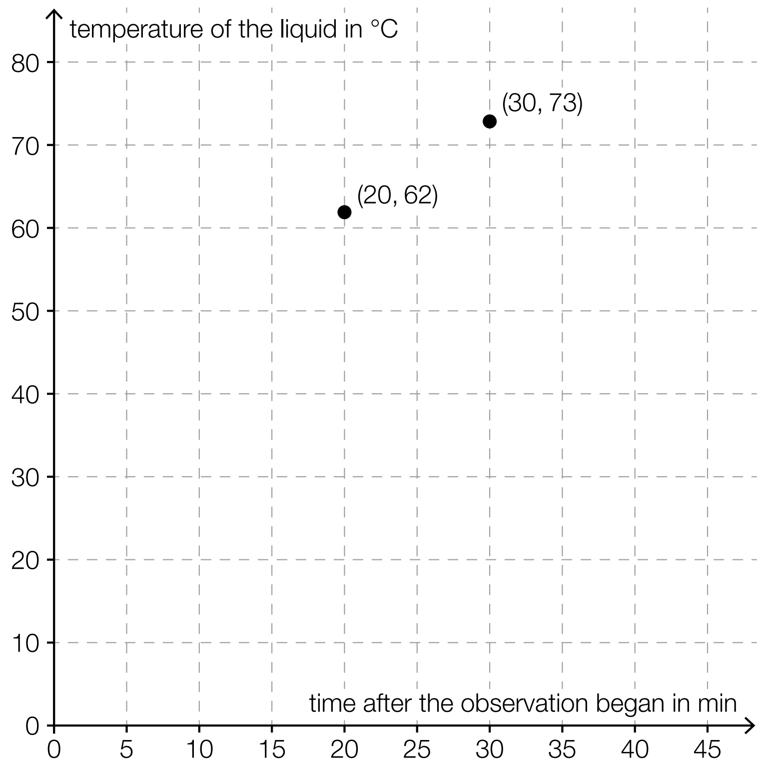
The limit gives the instantaneous rate of change of the velocity of the body 3 seconds after the measurements begin.	☐
The limit gives the average speed of the body in the time period [0, 3].	☐
The limit gives the instantaneous acceleration of the body 3 seconds after the measurements begin.	☐
The limit gives the relative change in the velocity of the body in the time period [0, 3].	☐
The limit gives the distance covered by the body in the first 3 seconds.	☐

[0/1 point]

Task 14

Experiment

During an experiment, the temperature of a particular liquid (in °C) was measured at various points in time.
The diagram below shows the results of the measurements 20 min and 30 min after the observation began.



Task:

Determine the average rate of change of the temperature of the liquid in the time interval [20 min, 30 min].

average rate of change: ____________________ °C/min

[0/1 point]

Task 15

Growth of a Sunflower

The height of a particular sunflower was measured over a number of weeks always at the beginning of each week.

When the measurements began at $t = 0$, the sunflower had a height of $H_0 = 5$ cm.
For each point in time t (with $0 \leq t \leq 5$), H_t gives the height of the sunflower.

The table below shows the (rounded) results of the measurements of the height of the sunflower for the first 5 weeks.

time t (in weeks after the measurements began)	height of the sunflower H_t (in cm)
1	36
2	68
3	98
4	128
5	159

Task:

Complete the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

The absolute weekly increase in the height of the sunflower is _______①_______; the height of the sunflower H_t can therefore be approximated by a difference equation of the form _______②_______.

①	
always smaller than the increase in the previous week	☐
always greater than the increase in the previous week	☐
roughly constant	☐

②	
$H_{t+1} = H_t \cdot (1 + k)$ with $k \in \mathbb{R}$	☐
$H_{t+1} = H_t + k$ with $k \in \mathbb{R}$	☐
$H_{t+1} = H_t + r \cdot (k - H_t)$ with $k, r \in \mathbb{R}$ and $0 < r < 1$	☐

[0/½/1 point]

Task 16

Antiderivatives

Let F be an antiderivative of a polynomial function $f\colon \mathbb{R} \to \mathbb{R}$.

Task:

Two of the functions G_1 to G_5 shown below are also antiderivatives of f for all $c \in \mathbb{R}\setminus\{0\}$.
Put a cross next to each of the two correct functions.

$G_1 = c \cdot F$	☐
$G_2 = c + F$	☐
$G_3 = F - c$	☐
$G_4 = c - F$	☐
$G_5 = \frac{F}{c}$	☐

[0/1 point]

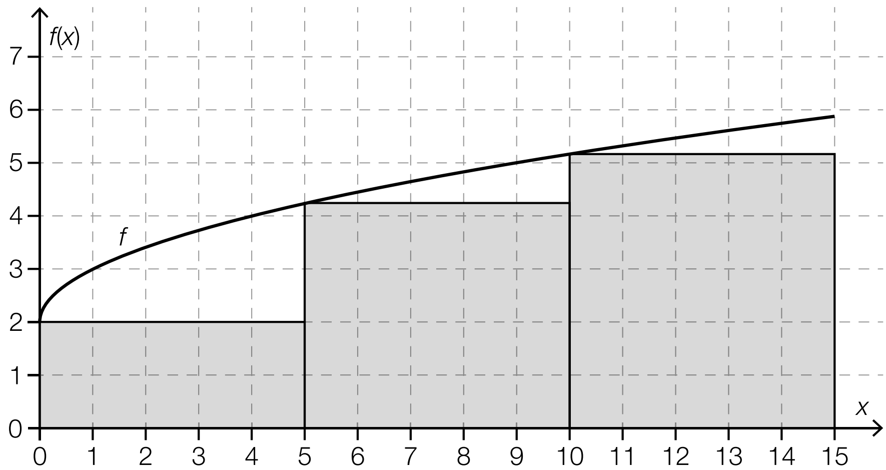
Task 17

Area between a Graph and the x-Axis

Let f: $[0, 15] \to \mathbb{R}^+$ be a power function.
The area A of the region that is bounded by the graph of f, the x-axis and the two lines $x = 0$ and $x = 15$ can be approximated by the expression U shown below.

$U = 5 \cdot (f(0) + f(5) + f(10))$

The diagram below shows the graph of f and the shaded region whose area can be calculated with the expression U.



Task:

Put a cross next to each of the two expressions that produce a better approximation for the area A than the expression U.

$5 \cdot (f(0) + f(5) + f(10) + f(15))$	☐
$2{,}5 \cdot (f(0) + f(2.5) + f(5) + f(7.5) + f(10) + f(12.5))$	☐
$\int_0^{15} f(x)\,\mathrm{d}x$	☐
$f(0) \cdot 15$	☐
$f(15) \cdot 5$	☐

[0/1 point]

Task 18

Work Done by Stretching a Spring

A spring with a spring constant $k = 40$ N/m is stretched from a state of equilibrium $s_0 = 0$ m by $h = 0.08$ m.
The work done by stretching the spring W (in joules) can be calculated using the expression below.
$W = \int_{s_0}^{s_0+h} k \cdot s \, \mathrm{d}s$

Task:

Determine the work done by stretching the spring as described above.

[0/1 point]

Task 19

Box Plot and Statistical Measures

From a box plot, certain statistical measures can be determined.

Task:

Put a cross next to each of the two statistical measures that generally cannot be determined from a box plot.

median	☐
mean	☐
mode	☐
range	☐
maximum	☐

[0/1 point]

Task 20

Estimate

In a particular random experiment, the event E occurs with probability $P(E)$.
In a series of experiments, this random experiment is conducted a times ($a \in \mathbb{N}$ and $a > 1$).
During this series of experiments, the event E occurs b times ($b \in \mathbb{N}$).
An estimate p of the unknown probability $P(E)$ is to be determined.

Task:

Write down a formula in terms of a and b with which p can be calculated.

p = ____

[0/1 point]

Task 21

Probabilities

The random variable X can only take the values 0, 1, 2 and 3.
The following statements hold: $P(X = 1) = 0.1$ and $P(X > 1) = 0.6$.

Task:

Put a cross next to each of the two true statements.

$P(X \leq 2) = 0.3$	☐
$P(X < 2) = 0.4$	☐
$P(X = 0) = 0$	☐
$P(X \geq 0) = 0.9$	☐
$P(X \geq 1) = 0.7$	☐

[0/1 point]

Task 22

Faulty Devices

According to experience, 2.5 % of the devices that are distributed by a particular company are faulty. The binomially distributed random variable X gives the number of faulty devices in a random sample of size n. The expectation value is $E(X) = 20$.

Task:

Determine the size n of the random sample.

$n =$ ____

[0/1 point]

Task 23

Chocolate Figurines

According to experience, 1 % of the chocolate figurines produced in a particular chocolate factory are flawed.
During a particular quality control check, 500 chocolate figurines are selected at random. Each chocolate figurine has the same probability of being flawed (1 %), independent of the other chocolate figurines.

Task:

Determine the probability that at most 2 chocolate figurines are flawed in this quality control check.

[0/1 point]

Task 24

Election Forecast

Before a particular election, 500 people were selected at random and independently from each other, and they participated in a survey. Of these people, 35 % said that they would vote for party A. For the results of the survey, the γ-confidence interval that is symmetrical about this relative proportion is given as [0.315, 0.385] for the unknown proportion of the voters for party A. A normal approximation to the binomial distribution was used to calculate this confidence interval.

Task:

Determine γ.

[0/1 point]

Task 25 (Part 2)

Tea

Worldwide, tea is one of the most frequently consumed beverages.

Task:

a) For the purpose of creating a model, it is assumed that the per capita consumption of tea in Austria increases each year in comparison to the previous year by the same percentage.

Based on this assumption, the function f gives the annual per capita consumption of tea in Austria from 2016 in terms of the time t (t in years, $f(t)$ in litres).

1) $\boxed{\text{A}}$ Write down the type of function of f.

The annual consumption of tea in Austria in the year 2016 was 33 l per capita. The proportion of tea that was prepared using teabags in Austria in the year 2016 was 95 %.

The following assumptions have been made:

- The per capita consumption of tea in Austria has risen each year in comparison to the previous year by 2 % since 2016.
- The proportion of tea that is prepared using teabags each year remains the same.

2) Write down how many litres of tea will be prepared per capita using teabags in Austria in the year 2026 based on the assumptions given above.

b) The largest tea producer worldwide is China. The table below gives the amount of tea produced in China in millions of tonnes for certain years in the time period from 2011 to 2017.

year	2011	2013	2015	2017
amount of tea produced in China in millions of tonnes	1.55	1.85	2.23	2.55

Source: https://de.statista.com/statistik/daten/studie/29847/umfrage/produktion-von-tee-nach-erzeugerlaendern-seit-2006/ [28.08.2018].

The amount of tea produced in China in terms of time t from the year 2011 is to be approximated by a linear function g (t in years from the year 2011, $g(t)$ in millions of tonnes).

1) Using the data from the years 2011 and 2017, write down an equation of the function g.

$g(t) =$ ____

In the years 2013 and 2015, there is a difference between the values of the function g and the values in the table above.

2) Write down in which of the years 2013 and 2015 the magnitude of the absolute difference between the value of the function g and the corresponding value from the table above is larger. For this year, determine the magnitude of the absolute difference.

year: ______________________

magnitude of the absolute difference: ______________________ million tonnes

c) Hot tea cools down when the ambient temperature is lower than the temperature of the tea.

The temperature $T(t)$ of tea with an initial temperature T_0 in constant ambient temperature T_U t minutes after the start of the cooling process can be approximated by the following function.

$T(t) = (T_0 - T_U) \cdot e^{-k \cdot t} + T_U$ with $k \in \mathbb{R}^+$ (t in minutes, T_U in °C, T_0 in °C, $T(t)$ in °C)

A cup of tea with an initial temperature $T_0 = 90$ °C is placed in a room with a constant ambient temperature $T_U = 20$ °C. The tea has cooled to a temperature of 65 °C after 10 minutes.

1) Determine k.

Assume that this value of k can be used to describe both the cooling process of tea that has an initial temperature of 90 °C and the cooling process of another tea that has an initial temperature of 70 °C.

2) Write down the ambient temperature T_U for which both teas cool down to half of their respective initial temperatures (in °C) in the same time.

$T_U =$ ____ °C

Task 26 (Part 2)

Global Warming

The *average global temperature* is the temperature of the whole surface of the Earth in a particular time period under particular conditions.

The development of the average global temperature can be forecast using climate models.

The average global temperatures for a number of years are shown below.

year	1900	1950	1955	1960	1965	1970	1975	1980
average global temperature (in °C)	13.80	13.87	13.89	14.01	13.90	14.02	13.94	14.16

year	1985	1990	1995	2000	2005	2010	2015
average global temperature (in °C)	14.03	14.37	14.37	14.31	14.51	14.55	14.72

The function T models the average global temperature in terms of the time t (t in years from the year 1900, $T(t)$ in °C). The following equation holds:

$T(t) = a \cdot e^{0.008 \cdot t} - 0.03 \cdot t + 11.1 \quad \text{with} \quad a \in \mathbb{R}^+$

Task:

a) For a particular climate model, the value of a is $a = 2.7$.
The function T has a local maximum or minimum when $t = t_0$.

1) $\boxed{\text{A}}$ Determine t_0.

2) Justify mathematically why the average global temperature has been rising increasingly faster from the time t_0 according to this model.

b) Various studies assume that the average global temperature in the year 2100 will be at least 1.5 °C and at most 4.5 °C higher than the average global temperature in the year 2000 (when it was 14.31 °C).

1) Show that the function T with $a = 2.7$ confirms the assumption of these studies for the year 2100.

2) Write down the smallest possible value a_{min} and the largest possible value a_{max} such that the function T confirms these studies.

a_{min} = ________________________________

a_{max} = ________________________________

c) At the UN climate conference in Paris in the year 2015, a new international climate agreement was reached in which the increase in the average global temperature should be limited. According to this agreement, the average global temperature in the year 2100 can be at most 15.3 °C.

In order to fulfil this climate agreement, the average rate of change of the average global temperature from the year 2015 can be at most a particular value k (k in °C per year).

1) Determine k.

It is assumed that the average global temperature from the year 2015 increases linearly and the average rate of change of the average global temperature per year does actually correspond to the value k.

2) Based on this assumption, write down an equation of the linear function M that can be used to model the annual average global temperature (in °C) t years after 2015.

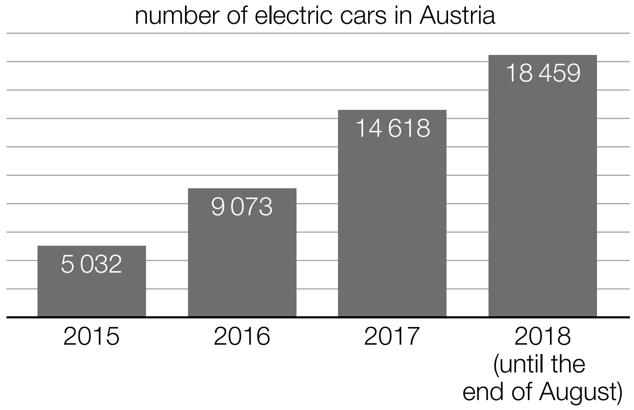
Task 27 (Part 2)

E-Mobility

The number of electric cars has increased in Austria in the last few years. The reasons for this include, among others, technical improvements, such as the increasing capacity of batteries and shorter charging times.
The *Battery capacity* is understood to be the maximum energy E (in kilowatt hours, kWh) that can be stored in the battery of the electric car. This energy is converted into a different form of energy while driving and is replenished when the car is charged.
The *charging time* is understood to be the time that is required to fully charge a (nearly) empty battery.

Task:

a) The diagram below shows the number of electric cars in Austria for the time period from 31st December 2015 to 31st August 2018. For the years 2015 to 2017, the number shown is the value for the end of the respective year; for the year 2018, the number shown is the value up to the end of August.



Data source: Statistik Austria, https://www.statistik.at/web_de/statistiken/energie_umwelt_innovation_mobilitaet/verkehr/strasse/kraftfahrzeuge_-_bestand/index.html [23.03.2020].

The difference equation $B_{n+1} = B_n \cdot a + b$ gives the development of the number of electric cars in Austria starting from the year 2015 for the years 2016 and 2017.
For this equation, the following conditions hold:

- B_0 is the number at the end of the year 2015.
- B_1 is the number at the end of the year 2016.
- B_2 is the number at the end of the year 2017.

1) Write down the values of a and b.

a = ______________________________

b = ______________________________

So that the difference equation given above also holds for the year 2018, the number of electric cars would have had to have risen in the rest of 2018 by a particular number.

2) Determine this number.

b) The battery capacity (in kWh) is the product of the charging capacity (in kW) and the charging time (in h). For example, in order to charge a (nearly) empty battery with a battery capacity of 22 kWh with a charging capacity of 11 kW, a charging time of 2 h is necessary.

The function f describes the charging time $f(P)$ of a battery with a battery capacity of 22 kWh in terms of the charging capacity P (P in kW, $f(P)$ in h).

1) $\boxed{\text{A}}$ Write down $f(P)$.

$f(P) =$ ____

The typical charging capacity of a private charging station lies in the capacity interval [2.3 kW, 3.7 kW].

2) For a battery with a battery capacity of 22 kWh, write down the time interval for the charging time that corresponds to this capacity interval.

c) The following assumptions are made in order to model an electric car driving on a particular test track:
- The electric car drives along the whole test track at constant velocity.
- There is a linear relationship between the energy requirement of this electric car and the corresponding constant velocity.

This electric car has an energy requirement of 12.9 kWh for a constant velocity of 70 km/h on this test track.
The electric car has an energy requirement of 20.9 kWh for a constant velocity of 110 km/h on this test track.

The function E gives the energy requirement $E(v)$ in terms of the velocity v with $50 \leq v \leq 130$ (v in km/h, $E(v)$ in kWh).

1) Write down $E(v)$.

$E(v) =$ ____

The battery of this electric car has a battery capacity of 41 kWh and is fully charged before driving along the test track. After driving along the test track, there are still 30.22 kWh available.

2) Determine the (constant) velocity v_1 with which the electric car drove along the test track.

Task 28 (Part 2)

Muesli Bar

A new muesli bar is about to be launched. The manufacturer of this muesli bar produces 100 000 bars.

The possibility of winning an instant cash prize is displayed on all the wrappers of the muesli bars. The amount won can be read from the inside of the wrapper once it has been opened. The manufacturer of the muesli bar states:

There are

- 9000 instant cash prizes of € 2 each
- 900 instant cash prizes of € 5 each
- 100 instant cash prizes of € 65 each

that will be paid out.

All muesli bars that are produced will be delivered to shops. The distribution of the muesli bars occurs at random.

Task:

a) Once all of the production costs have been taken into consideration, each of the 100000 muesli bars costs on average € 1 to produce.

The sales price of a muesli bar is to be determined such that the manufacturer earns a profit of at least € 80,000 after all the instant cash prizes in the muesli bars have been paid out.

1) Determine the lowest possible sales price p of the muesli bars given these conditions.

2) Write down the percentage by which the lowest possible sales price p can be reduced if the muesli bars are sold without cash prizes and the profit is still at least € 80,000.

b) The random variable X describes the amount of the instant cash prize per muesli bar sold.

1) Determine the expectation value $E(X)$.

A customer buys 4 muesli bars.

2) Determine the probability that the customer wins at least one instant cash prize.

c) From experience it is known that 95 % of the muesli bars meet a minimum weight requirement.

A random sample of 1 000 muesli bars is selected. The binomially distributed random variable Y describes the number of muesli bars in this random sample that meet the minimum weight requirement.

1) $\boxed{\text{A}}$ Determine the standard deviation $\sigma(Y)$ of the random variable Y.

$\sigma(Y) =$ ______________________________

2) Interpret the result of the calculation shown below in the given context.

$P(Y \geq 933) \approx 0.99$