

Name:

Class:

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

January 2021

Mathematics

Supplementary Examination 2
Candidate's Version

Instructions for the supplementary examination

Dear candidate,

The following supplementary examination is comprised of five tasks that can be completed independently of one another.

Each task contains two parts: The statement of the task requires you to demonstrate core competencies, and the guiding question that follows it requires you to demonstrate your ability to communicate your ideas.

You will be given preparation time of at least 30 minutes, and the examination will last at the most 25 minutes.

Assessment

Each task can be awarded zero, one or two points. There is one point available for each demonstration of core competencies as well as for each guiding question. A maximum of 10 points can be achieved.

For the grading of the examination the following scale will be used:

Grade	Number of points
Pass	4 points for the core competencies + 0 points for the guiding questions 3 points for the core competencies + 1 point for the guiding questions
Satisfactory	5 points for the core competencies + 0 points for the guiding questions 4 points for the core competencies + 1 point for the guiding questions 3 points for the core competencies + 2 points for the guiding questions
Good	5 points for the core competencies + 1 point for the guiding questions 4 points for the core competencies + 2 points for the guiding questions 3 points for the core competencies + 3 points for the guiding questions
Very good	5 points for the core competencies + 2 (or more) points for the guiding questions 4 points for the core competencies + 3 (or more) points for the guiding questions

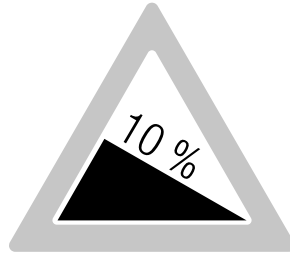
The examination board will decide on the final grade based on your performance in the supplementary examination as well as the result of the written examination.

Good Luck!

Task 1

Angle of Depression

The gradient of steeply ascending or steeply descending roads is given as a percentage. The traffic sign shown below states that the height of this road decreases by 10 m for each horizontal distance of 100 m.



Task:

Sonja claims: “If a road has a gradient of 10 %, then the angle of depression of this road is approximately twice as large as a road with a gradient of 5 %.”

- Determine both angles of depression.
- Write down whether Sonja’s claim is correct or incorrect.

Guiding question:

Martin writes down the following relationship for small angles α :

$$\tan(2 \cdot \alpha) \approx 2 \cdot \tan(\alpha)$$

- Interpret this expression in the given context.
- Justify why this relationship cannot hold for $\alpha = 45^\circ$.

Task 2

Test Tracks

From 1st August 2018 to 29th February 2020, the maximum speed on sections of a motorway in Upper Austria and Lower Austria was increased to 140 km/h for a trial period. The time saved in comparison to the usual permitted maximum speed of 130 km/h (assuming each value is an average speed) is shown in the diagram below.

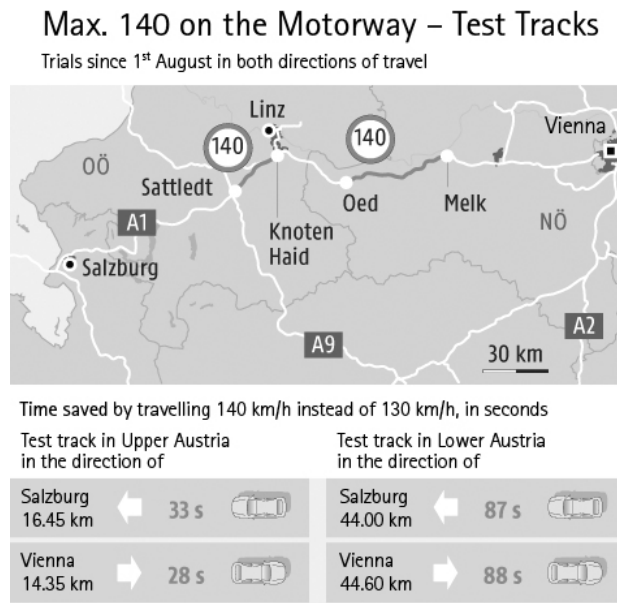


Image source: <https://ooe.orf.at/v2/news/stories/2947525/> [26.09.2019] (adapted).

Task:

- Show by calculation that the value of 33 s given for the time saved on the test track in Upper Austria in the direction of Salzburg is correct.

Guiding question:

Michael drives from Vienna to Salzburg with a constant speed of 140 km/h on both of these test tracks. In total, the time saved is $87 \text{ s} + 33 \text{ s} = 2 \text{ min}$.

If a different constant speed v (in km/h) is chosen for both of these test tracks, then the time saved is e .

- Write down an expression that can be used to calculate the corresponding constant speed v (in km/h) in terms of the time saved e in minutes (on the route from Vienna to Salzburg).

$$v = \underline{\hspace{10em}}$$

Task 3

Polynomial Functions

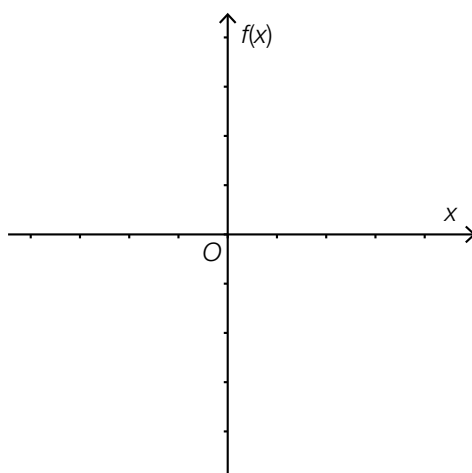
The number of zeros, local maxima and minima and points of inflexion is dependent, among other things, on the degree of a polynomial function.

Task:

- In the coordinate system shown below, sketch the graph of a polynomial function f such that exactly one zero and exactly three local maxima or minima are shown.

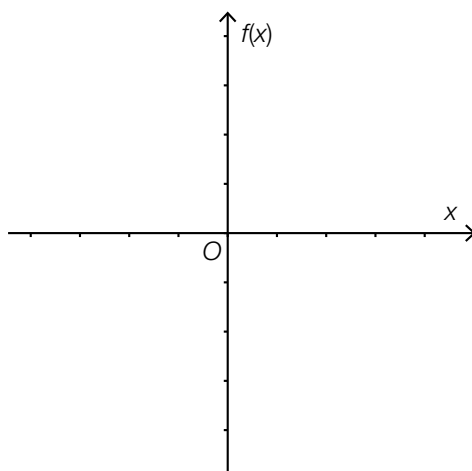
All polynomial functions that fulfil the condition given above are at least of degree n .

- Write down n .



Guiding question:

- Write down how the number of zeros for the given section of the graph you sketched above changes through vertical translation of the graph and how the equations of these polynomial functions (with different numbers of zeros) differ from each other.
- Sketch the graph of a fourth degree polynomial function f that has the smallest possible number of zeros, local maxima and minima and points of inflexion.



Task 4

Cooling of a Liquid

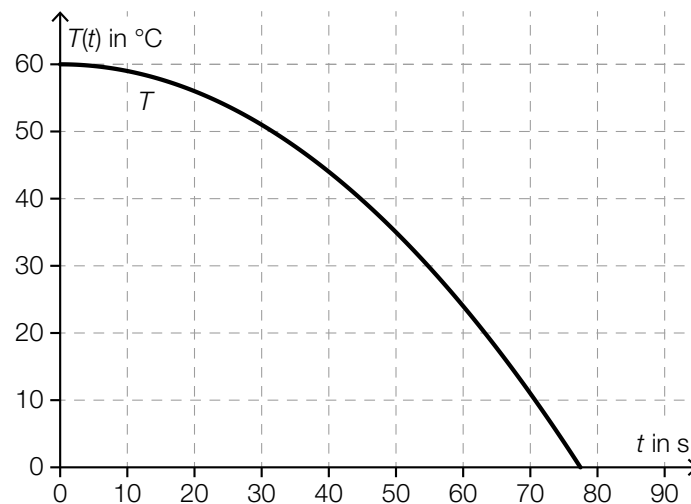
The temperature T of a cooling liquid can be approximated in terms of the time t by the function $T(t) = 60 - 0.01 \cdot t^2$ (t in s, $T(t)$ in $^{\circ}\text{C}$).

Task:

- Write down the average rate of change of the temperature in the interval $[30, 70]$ and interpret the result in the given context.

Guiding question:

- Sketch the average rate of change calculated above graphically (using the diagram shown below).
- Explain how the point t_1 on the graph of T for which the instantaneous rate of change is equal to the average rate of change calculated above can be determined. Write down the value of t_1 .



Task 5

Normally Approximated Random Variable

The normal approximation of a binomially distributed random variable X results in a random variable Y with an expectation value μ and a standard deviation σ .

Task:

– Describe and determine the probabilities given below.

- $P(Y < \mu - \sigma)$
- $P(\mu - 2 \cdot \sigma \leq Y \leq \mu + 2 \cdot \sigma)$

Guiding question:

– Draw the probabilities determined in the task above graphically (as areas under the graph of an appropriate function) and explain the shape of the graph of the function by referring to local maxima/minima and the symmetry of the graph.