Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

January 2021

Mathematics

Supplementary Examination 2
Examiner's Version

Bundesministerium
Bildung, Wissenschaft
und Forschung

Instructions for the supplementary examination

The following supplementary examination is comprised of five tasks that can be completed independently of one another.

Each task contains two parts: The statement of the task requires the candidate to demonstrate core competencies, and the guiding question that follows it requires the candidate to show their ability to communicate their ideas.

In the following document, the examiner will find the tasks as well as the expected solutions and the answer key.

The preparation time shall be at least 30 minutes and the examination time shall be at most 25 minutes.

Assessment

Each task can be awarded zero, one or two points. There is one point available for each demonstration of core competencies as well as for each guiding question. A maximum of 10 points can be achieved.

The following scale will be used for the grading of the examination:

Grade	Number of points
Pass	4 points for the core competencies + 0 points for the guiding questions 3 points for the core competencies + 1 point for the guiding questions
Satisfactory	5 points for the core competencies + 0 points for the guiding questions 4 points for the core competencies + 1 point for the guiding questions 3 points for the core competencies + 2 points for the guiding questions
Good	5 points for the core competencies + 1 point for the guiding questions 4 points for the core competencies + 2 points for the guiding questions 3 points for the core competencies + 3 points for the guiding questions
Very good	5 points for the core competencies + 2 (or more) points for the guiding questions 4 points for the core competencies + 3 (or more) points for the guiding questions

The examination board will decide on the final grade based on the candidate's performance in the supplementary examination as well as the result of the written examination.

Evaluation grid for the supplementary examination

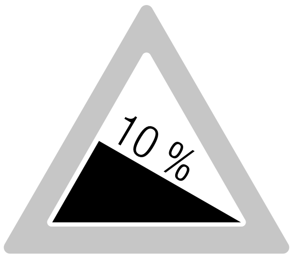
This evaluation grid may be used to assist the examiner in assessing the candidate's performance.

	Point for core competencies reached	Point for the guiding question reached
Task 1		
Task 2		
Task 3		
Task 4		
Task 5		

Task 1

Angle of Depression

The gradient of steeply ascending or steeply descending roads is given as a percentage. The traffic sign shown below states that the height of this road decreases by 10 m for each horizontal distance of 100 m.



Task:

Sonja claims: "If a road has a gradient of 10 %, then the angle of depression of this road is approximately twice as large as a road with a gradient of 5 %."

– Determine both angles of depression.

– Write down whether Sonja's claim is correct or incorrect.

Guiding question:

Martin writes down the following relationship for small angles α:

$\tan(2 \cdot \alpha) \approx 2 \cdot \tan(\alpha)$

– Interpret this expression in the given context.

– Justify why this relationship cannot hold for $\alpha = 45°$.

Solution to Task 1

Angle of Depression

Expected solution to the statement of the task:

$\tan(\alpha_1) = \frac{10}{100} \quad \Rightarrow \quad \alpha_1 = 5.710...° \approx 5.71°$

$\tan(\alpha_2) = \frac{5}{100} \quad \Rightarrow \quad \alpha_2 = 2.862...° \approx 2.86°$

Sonja's claim is correct because $2 \cdot \alpha_2 \approx \alpha_1$ holds.

Answer key:

The point for the core competency is to be awarded if the angles of depression have been calculated correctly and the correctness of Sonja's claim has been recognised.

Expected solution to the guiding question:

possible interpretation:

This expression means that if the angle of depression is doubled then the gradient also approximately doubles.
For $\alpha = 45°$, $\tan(2 \cdot \alpha)$ is not defined.

Answer key:

The point for the guiding question is to be awarded if the expression has been correctly interpreted in the context and it has been justified why this relationship cannot hold for $\alpha = 45°$.

Task 2

Test Tracks

From 1st August 2018 to 29th February 2020, the maximum speed on sections of a motorway in Upper Austria and Lower Austria was increased to 140 km/h for a trial period. The time saved in comparison to the usual permitted maximum speed of 130 km/h (assuming each value is an average speed) is shown in the diagram below.

Image source: https://ooe.orf.at/v2/news/stories/2947525/ [26.09.2019] (adapted).

Task:

– Show by calculation that the value of 33 s given for the time saved on the test track in Upper Austria in the direction of Salzburg is correct.

Guiding question:

Michael drives from Vienna to Salzburg with a constant speed of 140 km/h on both of these test tracks. In total, the time saved is 87 s + 33 s = 2 min.
If a different constant speed v (in km/h) is chosen for both of these test tracks, then the time saved is e.

– Write down an expression that can be used to calculate the corresponding constant speed v (in km/h) in terms of the time saved e in minutes (on the route from Vienna to Salzburg).

$v =$ ____________________

Solution to Task 2

Test Tracks

Expected solution to the statement of the task:

$\left(\frac{16.45}{130} - \frac{16.45}{140}\right) \cdot 3\,600 = 32.5... \approx 33 \quad \Rightarrow \quad$ The given value of 33 s is correct.

Answer key:

The point for the core competency is to be awarded if a correct calculation has been given as a justification.

Expected solution to the guiding question:

possible expression:

$\left(\left(\frac{16.45}{130} - \frac{16.45}{v}\right) + \left(\frac{44}{130} - \frac{44}{v}\right)\right) \cdot 60 = e$

$\Rightarrow \quad v = \frac{-36\,270}{10 \cdot e - 279}$

Answer key:

The point for the guiding question is to be awarded if a correct expression has been given.

Task 3

Polynomial Functions

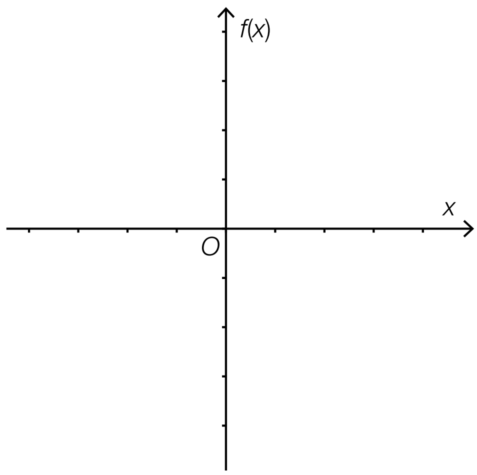
The number of zeros, local maxima and minima and points of inflexion is dependent, among other things, on the degree of a polynomial function.

Task:

– In the coordinate system shown below, sketch the graph of a polynomial function *f* such that exactly one zero and exactly three local maxima or minima are shown.

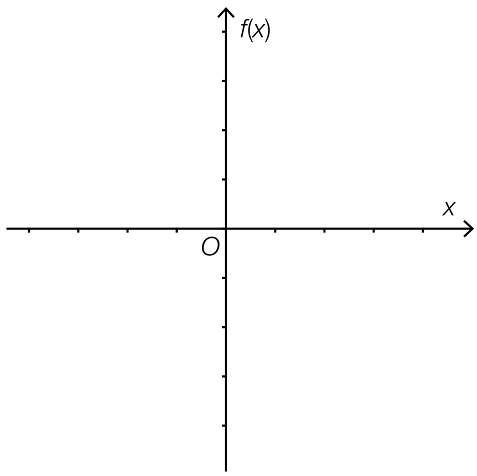
All polynomial functions that fulfil the condition given above are at least of degree *n*.

– Write down *n*.



Guiding question:

– Write down how the number of zeros for the given section of the graph you sketched above changes through vertical translation of the graph and how the equations of these polynomial functions (with different numbers of zeros) differ from each other.
– Sketch the graph of a fourth degree polynomial function *f* that has the smallest possible number of zeros, local maxima and minima and points of inflexion.

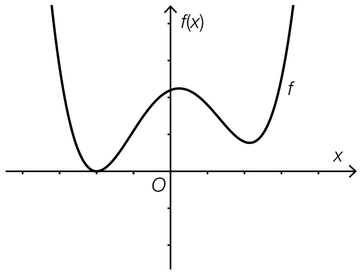


Solution to Task 3

Polynomial Functions

Expected solution to the statement of the task:

possible graph:



$n = 4$

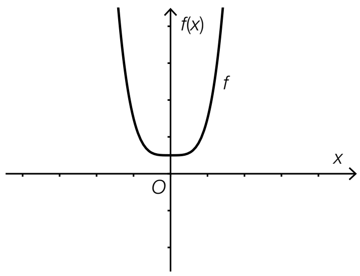
Answer key:

The point for the core competency is to be awarded if a possible graph has been sketched correctly and the correct value of n has been given.

Expected solution to the guiding question:

By translating the sketched graph in the vertical direction, the graph could have 0, 1, 2, 3, or 4 zeros.
The equations of these polynomial functions only differ by a constant value.

possible graph:



The graph shown has no zeros, only one local maximum or minimum and no points of inflexion.

Answer key:

The point for the guiding question is to be awarded if the number of zeros and the difference between the equations of the functions have been given correctly and a possible graph has been sketched correctly.

Task 4

Cooling of a Liquid

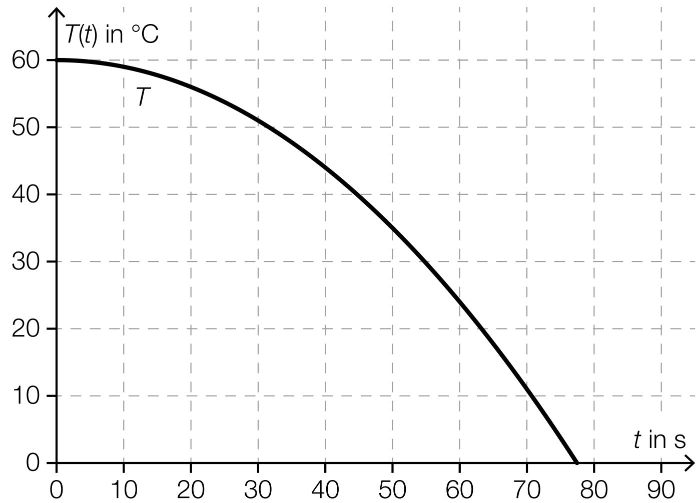
The temperature T of a cooling liquid can be approximated in terms of the time t by the function $T(t) = 60 - 0.01 \cdot t^2$ (t in s, $T(t)$ in °C).

Task:

– Write down the average rate of change of the temperature in the interval [30, 70] and interpret the result in the given context.

Guiding question:

– Sketch the average rate of change calculated above graphically (using the diagram shown below).
– Explain how the point t_1 on the graph of T for which the instantaneous rate of change is equal to the average rate of change calculated above can be determined. Write down the value of t_1.



Solution to Task 4

Cooling of a Liquid

Expected solution to the statement of the task:

average rate of change: $\frac{T(70) - T(30)}{70 - 30} = \frac{11 - 51}{40} = -1$

possible interpretation:

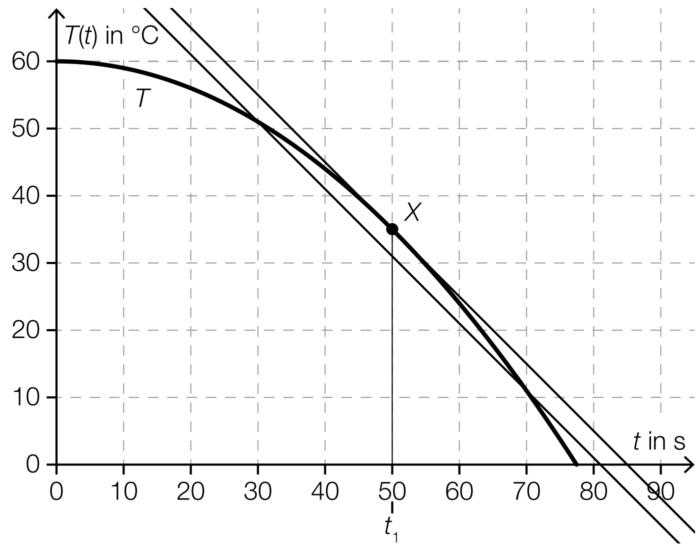
In the time interval [30, 70], the temperature reduces by an average of 1 °C per second.

Answer key:

The point for the core competency is to be awarded if the correct average rate of change and a correct interpretation have been given.

Expected solution to the guiding question:

The average rate of change calculated above is equal to the gradient of the secant function of T over the interval [30, 70].



The point t_1 can be determined graphically by finding the point X on T for which the gradient of the tangent is equal to the gradient of the secant line (i.e. these lines must be parallel).

Calculation of t_1: $T'(t_1) = -0.02 \cdot t_1 = -1 \quad \Rightarrow \quad t_1 = 50$

Answer key:

The point for the guiding question is to be awarded if the average rate of change has been correctly identified as the gradient of the secant function, a method for determining t_1 has been correctly explained, and the correct value for t_1 has been calculated.

Task 5

Normally Approximated Random Variable

The normal approximation of a binomially distributed random variable X results in a random variable Y with an expectation value μ and a standard deviation σ.

Task:

– Describe and determine the probabilities given below.
  - $P(Y < \mu - \sigma)$
  - $P(\mu - 2 \cdot \sigma \leq Y \leq \mu + 2 \cdot \sigma)$

Guiding question:

– Draw the probabilities determined in the task above graphically (as areas under the graph of an appropriate function) and explain the shape of the graph of the function by referring to local maxima/minima and the symmetry of the graph.

Solution to Task 5

Normally Approximated Random Variable

Expected solution to the statement of the task:

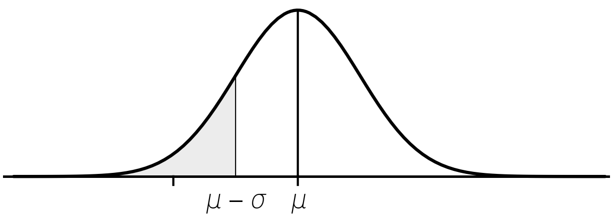
- The expression describes the probability of the random variable taking a value that is less than one standard deviation below the expectation value.
  $P(Y < \mu - \sigma) \approx 0.159$
- The expression describes the probability of the random variable taking a value that is at most two standard deviations above or below the expectation value.
  $P(\mu - 2 \cdot \sigma \leq Y \leq \mu + 2 \cdot \sigma) \approx 0.954$

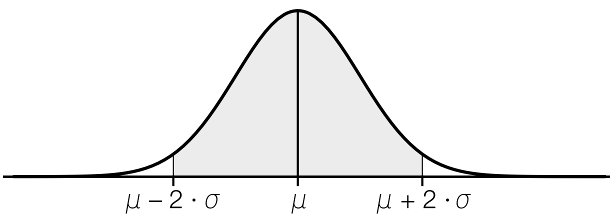
Answer key:

The point for the core competency is to be awarded if both of the probabilities have been determined and described correctly.

Expected solution to the guiding question:

density of Y:





The graph (the Gaussian bell curve) is symmetrical about the expectation value and has a local maximum at this value.

Answer key:

The point for the guiding question is to be awarded if both of the probabilities have been represented correctly on a graph and the shape of the graph has been explained correctly.