Name:	
Class:	

Standardised Competence-Oriented
Written School-Leaving Examination

AHS

Main Examination Session 2021

Mathematics

Bundesministerium
Bildung, Wissenschaft
und Forschung

Advice for Completing the Tasks

Dear candidate,

The following booklet contains Part 1 tasks and Part 2 tasks (divided into sub-tasks). The tasks can be completed independently of one another.

Please do all of your working out solely in this booklet and on the paper provided to you. Write your name and that of your class on the cover page of the booklet in the spaces provided. Please also write your name on any separate sheet of paper used and number these pages consecutively. When answering each sub-task, write the name/number of the sub-task (e. g. 25a1) on your sheet.

In the assessment of your work, everything that is not crossed out will be considered. Your solutions must be marked clearly. If a solution is not clearly marked or if more than one solution is given, the task will be considered to be unsolved.

The use of the official formula booklet for this examination that has been approved by the relevant government authority is permitted. Furthermore, the use of electronic device(s) (e. g. graphic display calculators or other appropriate technology) is allowed provided there is no possibility of communicating via the internet, Bluetooth, mobile networks etc. and there is no access to your own data stored on the device.

An explanation of the task types is available in the examination room and can be viewed on request.

Please hand in this booklet and all worksheets you have used at the end of the examination.

Changing an answer for a task that requires a cross:

1. Fill in the box that contains the cross.
2. Put a cross in the box next to your new answer.

In this instance, the answer “5 + 5 = 9” was originally chosen. The answer was later changed to be “2 + 2 = 4”.

1 + 1 = 3	☐
2 + 2 = 4	☒
3 + 3 = 5	☐
4 + 4 = 4	☐
5 + 5 = 9	■
6 + 6 = 10	☐

Selecting an item that has been filled in:

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

In this instance, the answer “2 + 2 = 4” was filled in and then selected again.

1 + 1 = 3	☐
2 + 2 = 4	(■)
3 + 3 = 5	☐
4 + 4 = 4	■
5 + 5 = 9	☐
6 + 6 = 10	☐

Assessment

Each task in Part 1 and each sub-task in Part 2 will be awarded either 0 points or 1 point or 0, ½ or 1 point respectively. The points that can be achieved in each (sub-)task are shown in the booklet.

Grading System

points awarded	grade
32–36 points	very good
27–31.5 points	good
22–26.5 points	satisfactory
17–21.5 points	pass
0–16.5 points	fail

Best-of Assessment: A best-of assessment approach will be applied to tasks 26, 27 and 28. Of these three Part 2 tasks, the task with the lowest point score will not be included in the total point score.

Good luck!

Task 1

Rational Numbers

Statements about rational numbers are shown below.

Task:

Put a cross next to each of the two correct statements. *[2 out of 5]*

The statement $a + b \geq 0$ holds for all rational numbers a and b.	☐
For each rational number a, there exists a rational number b such that the statement $a + b = 0$ holds.	☐
There are rational numbers a and b for which $a \cdot b < b$ holds.	☐
If exactly one of the two rational numbers a and b, $b \neq 0$, is positive, then the quotient $\frac{a}{b}$ is always positive.	☐
If at least one of the two rational numbers a and b is negative, then the product $a \cdot b$ is always negative.	☐

[0/1 p.]

Task 2

Item of Clothing

The price of a particular item of clothing was € 49.90 at the end of the year 2017. At this point in time, it was 17.8 % more expensive than it had been at the beginning of the year 2017.

Task:

Determine the price increase in euros of the item of clothing over the course of the year 2017.

[0/1 p.]

Task 3

School Sport Week

For a school sport week, a school books x rooms with four beds and y rooms with six beds at a youth hostel. All of the rooms that are booked will be fully occupied.
The booking can be described by the system of equations shown below.

I: $4 \cdot x + 6 \cdot y = 56$
II: $x + y = 12$

Task:

Put a cross next to each of the two correct statements. *[2 out of 5]*

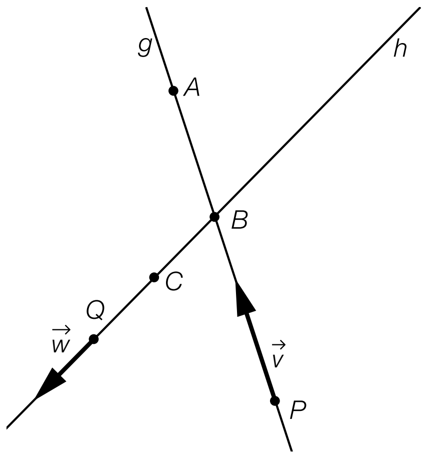
Exactly 4 rooms with four beds and exactly 6 rooms with six beds are booked.	☐
Fewer rooms with four beds are booked than rooms with six beds.	☐
Exactly 12 rooms are booked.	☐
Beds for exactly 56 people are booked.	☐
Exactly 10 rooms are booked.	☐

[0/1 p.]

Task 4

Vector Equations of Lines

The diagram below shows the two lines g and h. Three points are shown on each of the lines: $A, B, P \in g$ and $B, C, Q \in h$.
In addition to this, a direction vector for each of the lines is shown.



Task:

Put a cross next to each of the two statements for which $s, t \in \mathbb{R}$ with $s \neq 0$ and $t \neq 0$ can be chosen such that the corresponding statement is true. *[2 out of 5]*

$A = C + s \cdot \vec{v} + t \cdot \vec{w}$	☐
$B = C + s \cdot \vec{v}$	☐
$B = Q + t \cdot \vec{w}$	☐
$A = P + s \cdot \vec{v} + t \cdot \vec{w}$	☐
$C = P + t \cdot \vec{w}$	☐

[0/1 p.]

Task 5

Square

A square has vertices A, B, C and D. The vertex $C = (5, -3)$ and the point of intersection of the diagonals $M = (3, 1)$ are given. The vertices A, B, C and D of the square are labelled in an anti-clockwise direction.

Task:

Determine the coordinates of the vertices A and B.

A = ____

B = ____

[0/½/1 p.]

Task 6

Ramp

A ramp with a (sloped) length of d metres has been designed to overcome a vertical rise of h metres ($d > 0$, $h > 0$). The angle of elevation of the ramp is given by α.

Task:

Put a cross next to each of the two equations that correctly describe the situation. *[2 out of 5]*

$d = \frac{h}{\sin(\alpha)}$	☐
$d = h \cdot \cos(\alpha)$	☐
$d = \frac{h}{\cos(90° - \alpha)}$	☐
$d = h \cdot \sin(90° - \alpha)$	☐
$d = h \cdot \tan(\alpha)$	☐

[0/1 p.]

Task 7

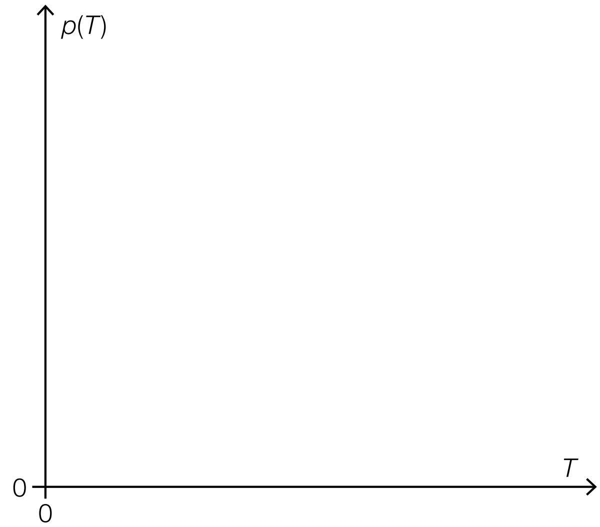
Ideal Gas

The equation $p \cdot V = n \cdot R \cdot T$ models the relationship between the pressure p, the volume V, the amount of substance n and the absolute temperature T of an ideal gas, whereby R is a constant ($V, n, R \in \mathbb{R}^+$ and $p, T \in \mathbb{R}_0^+$).

The function p models the pressure $p(T)$ in terms of the temperature T when the other quantities given in the equation remain constant.

Task:

In the coordinate system shown below, sketch the graph of one such function p.



[0/1 p.]

Task 8

Types of Functions

Four types of functions as well as six tables of values of the functions f_1 to f_6, which each correspond to a particular type of function, are shown below. The values of the function f_1 are rounded to two decimal places.

Task:

Match each of the four types of function shown below to the corresponding table of values (from A to F).

linear function	
quadratic function	
exponential function	
sine function	

A

x	$f_1(x)$
–2	–0.91
–1	–0.84
0	0
1	0.84
2	0.91

B

x	$f_2(x)$
–2	8
–1	2
0	0
1	2
2	8

C

x	$f_3(x)$
–2	–7
–1	–1
0	0
1	1
2	9

D

x	$f_4(x)$
–2	0.25
–1	0.5
0	1
1	2
2	4

E

x	$f_5(x)$
–2	–3
–1	–1
0	1
1	3
2	5

F

x	$f_6(x)$
–2	–0.5
–1	–1
0	undefined
1	1
2	0.5

[0/½/1 p.]

Task 9

Direct Proportion

The graph of a linear function $f: \mathbb{R} \to \mathbb{R}$ with $f(x) = m \cdot x + c$ with $m, c \in \mathbb{R}$ goes through the points $A = (x_A, 6)$ and $B = (12, 16)$.

Task:

Determine the coordinate x_A of the point A such that the function f describes a directly proportional relationship.

x_A = ___________________________________

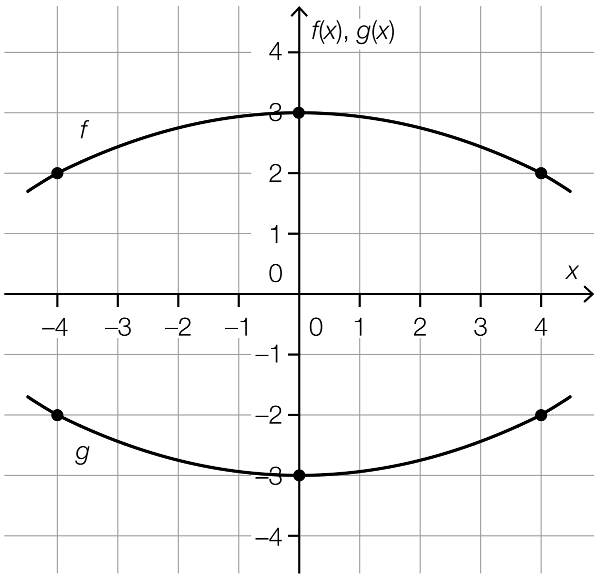
[0/1 p.]

Task 10

Quadratic Functions

The diagram below shows the graphs of the two real functions f and g. The following statements hold:

$f(x) = a \cdot x^2 + b$ with $a, b \in \mathbb{R}$
$g(x) = c \cdot x^2 + d$ with $c, d \in \mathbb{R}$



The points shown in bold have integer coordinates.

Task:

Put a cross next to each of the two correct statements. *[2 out of 5]*

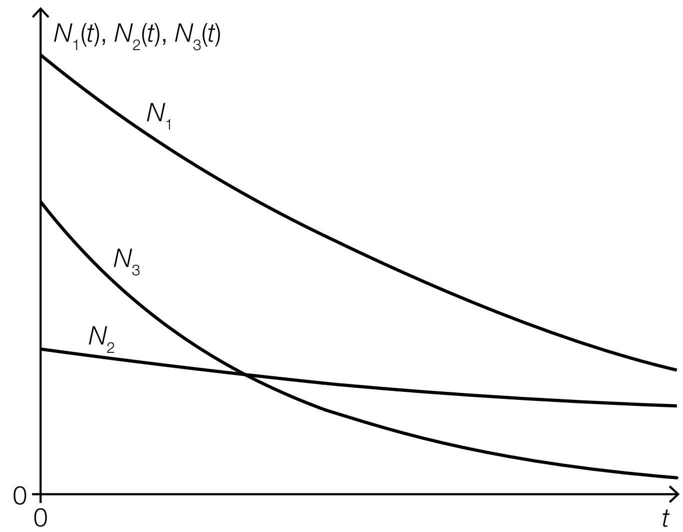
$d = f(0)$	☐
$b = d$	☐
$a = -c$	☐
$-f(x) = g(x)$ for all $x \in \mathbb{R}$	☐
$f(2) = g(2)$	☐

[0/1 p.]

Task 11

Half-Lives of Decay Processes

The three exponential functions N_1, N_2 and N_3 each describe a decay process with the corresponding half-lives τ_1, τ_2 and τ_3.
Sections of the graphs of these three functions are shown below.



Task:

Write down the half-lives τ_1, τ_2 and τ_3 in increasing order of magnitude. Start with the shortest half-life.

_______ < _______ < _______

[0/1 p.]

Task 12

Equation of a Function

The following information is known about a real function $f: \mathbb{R} \rightarrow \mathbb{R}^+$:

- $f(1) = 3$
- For all real numbers x, $f(x + 1)$ is 50 % greater than $f(x)$.

Task:

Write down the equation of one such function f.

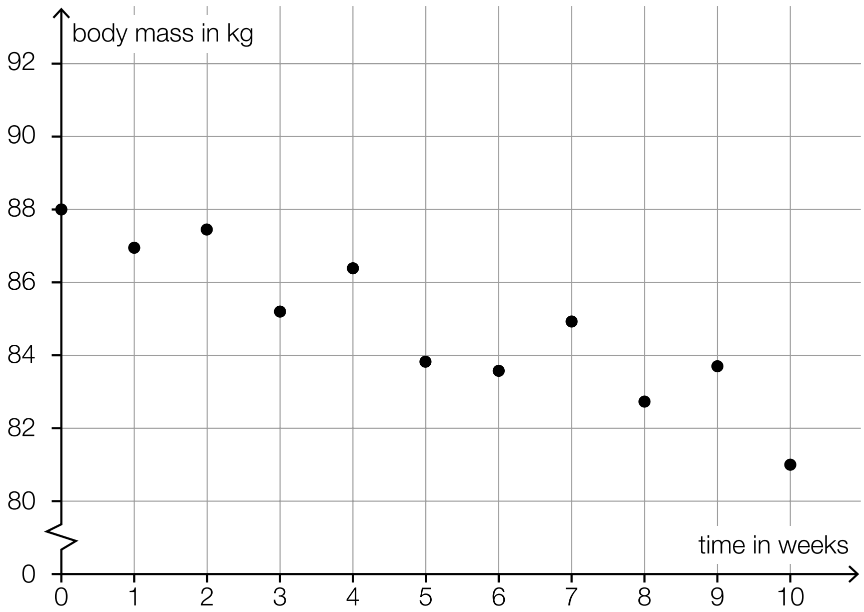
$f(x) =$ ____

[0/1 p.]

Task 13

Diet

Hannes went on a diet that lasted ten weeks. At the start of each week and at the end of the diet, he recorded his body mass (in kg). These values are shown in the diagram below.



Task:

Write down the absolute change (in kg) and the relative change (in %) of Hannes's body mass from the beginning to the end of the diet.

absolute change: ____ kg

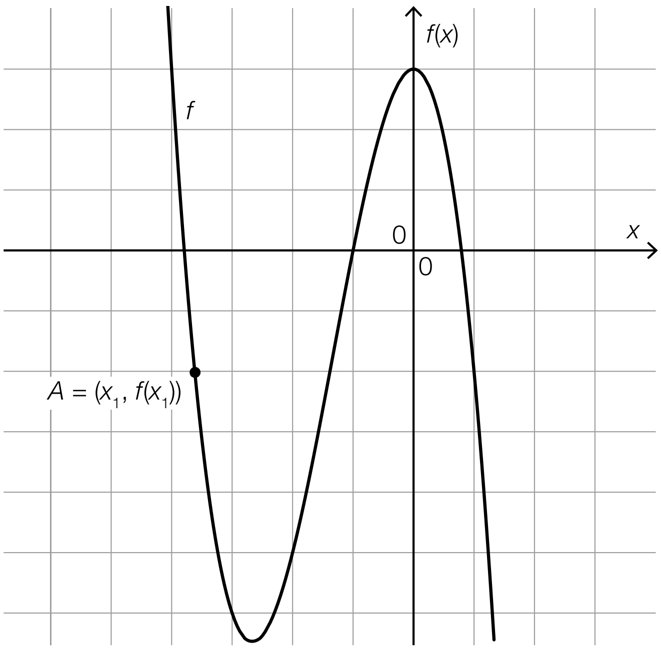
relative change: ____ %

[0/½/1 p.]

Task 14

Rates of Change of a Polynomial Function

The diagram below shows the graph of a polynomial function f and the point $A = (x_1, f(x_1))$ on the graph of f.



For an x_2 on the diagram above with $x_2 > x_1$, the following conditions hold:

- The differential quotient of f at x_2 is negative.
- The difference quotient of f in the interval $[x_1, x_2]$ is zero.

Task:

In the diagram above, label the point $P = (x_2, f(x_2))$ for which both of the conditions given above are fulfilled.

[0/1 p.]

Task 15

Carp

The number of carp in a pond should be restricted to 800 carp. For modelling purposes, it is assumed that the number of carp increases each year by 7 % of the difference from the maximum number of 800 carp.

The number of carp after n years is given by $F(n)$ and $F(0) = 500$ holds.

Task:

Put a cross next to the difference equation that correctly describes the development of the number of carp. *[1 out of 6]*

$F(n + 1) = F(n) + 0.07 \cdot (800 - F(n))$	☐
$F(n) = F(n + 1) + 0.07 \cdot (800 - F(n + 1))$	☐
$F(n + 1) = F(n) + 1.07 \cdot (800 - F(n))$	☐
$F(n + 1) = F(n) + 0.07 \cdot (F(n) - 800)$	☐
$F(n + 1) = 800 - 0.07 \cdot F(n)$	☐
$F(n) = 800 - 0.07 \cdot F(n + 1)$	☐

[0/1 p.]

Task 16

Definite Integral

The function F is an antiderivative of the polynomial function f.

Task:

Put a cross next to the expression that definitely corresponds to $\int_2^5 f(x)\,dx$. *[1 out of 6]*

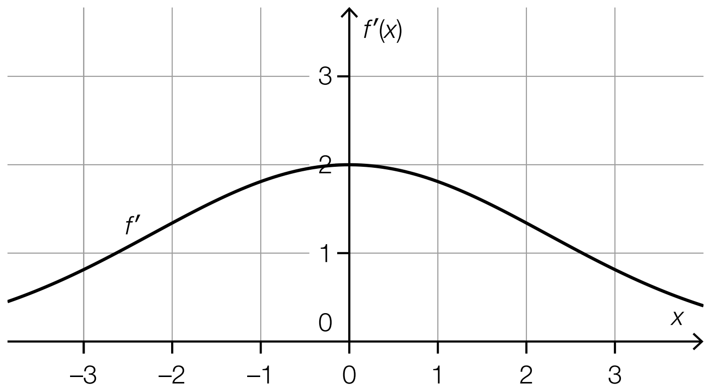
$\frac{F(5) - F(2)}{5 - 2}$	☐
$\frac{F(5) - F(2)}{F(2)}$	☐
$F(5) - F(2)$	☐
$F(5) + F(2)$	☐
$\frac{F(2) + F(5)}{2}$	☐
$\frac{F(5)}{F(2)}$	☐

[0/1 p.]

Task 17

Properties of a Function

The diagram below shows the graph of the first derivative f' of a polynomial function f.



Task:

Put a cross next to each of the two statements that are definitely true about the function f.

[2 out of 5]

In the interval [–3, 3] the function f is strictly monotonically increasing.	☐
The graph of f is symmetrical about the vertical axis in the interval [–3, 3].	☐
The function f has at least one point of inflexion in the interval [–3, 3].	☐
In the interval [–3, 3] all values of the function f are positive.	☐
The function f has at least one local maximum or minimum in the interval [–3, 3].	☐

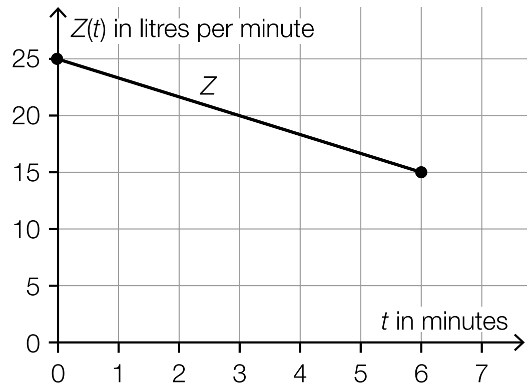
[0/1 p.]

Task 18

Water Inflow

A container is filled with water for 6 minutes.
The inflow rate gives the number of litres of water that flow into the container per minute.
The inflow rate $Z(t)$ decreases linearly in terms of the time t.

The diagram below shows the graph of the function Z (t in minutes, $Z(t)$ in litres per minute). The points shown in bold have integer coordinates.



Task:

Determine the number of litres of water that flow into this container in these 6 minutes.

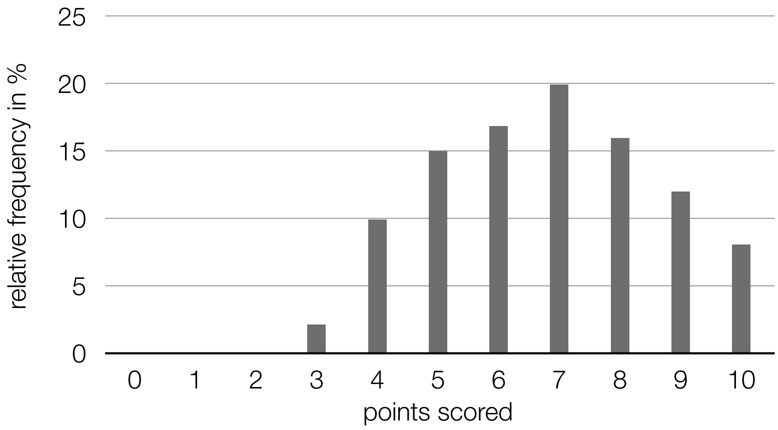
____ litres

[0/1 p.]

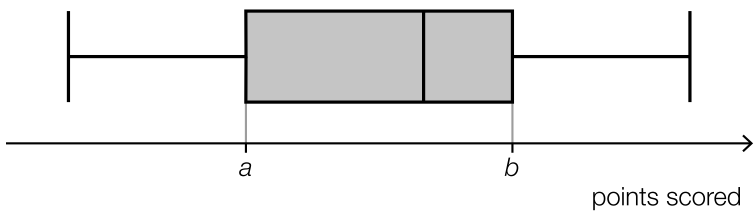
Task 19

Entrance Exam

For a particular entrance exam, the maximum score is 10 points. The bar chart shown below gives the relative frequencies of the points scored as a percentage.



The points scored in the entrance exam are shown in the box plot below.



Task:

Determine a and b.

a = ______________________________

b = ______________________________

[0/½/1 p.]

Task 20

Salaries

Seven people work at a small company. Their monthly salaries are given as follows:
€ 1500, € 2300, € 1500, € 1400, € 4500, € 2200, € 1300.

One more person is to be employed, but the median salary value will not change.

Task:

Based on the information given above, write down the largest possible salary of this additional person.

[0/1 p.]

Task 21

Tossing a Coin

After being tossed, a coin shows either “heads” or “tails”. The probability of the coin showing “heads” is exactly the same as the probability of the coin showing “tails” for each toss. The results of the tosses are independent of each other.

In a random experiment, the coin is tossed 4 times.

Task:

Determine the probability of “heads” occurring more often than “tails” in this random experiment.

[0/1 p.]

Task 22

Probabilities of a Random Variable

A particular random variable X can only take the value –4, the value 0 or the value 2.

For the probabilities, the following statements hold:

$P(X = -4) = 0.3$
$P(X = 0) = a$
$P(X = 2) = b$

a and b are positive real numbers.

The expectation value of X is zero i.e. $E(X) = 0$.

Task:

Write down the values of a and b.

a = ____

b = ____

[0/½/1 p.]

Task 23

Smoking Behaviour

According to a study, 34 % of all smokers want to quit smoking.

Task:

Interpret the expression shown below in the given context.

$\binom{200}{57} \cdot 0.34^{57} \cdot 0.66^{143}$

[0/1 p.]

Task 24

Corked Wine

The flavour of wine can be impaired by a particular substance that can get into the wine from the cork. If this happens, the wine is said to be “corked”.

In a winery, all wine bottles from a particular vintage are sealed with corks from the same production batch. During a later check of 200 wine bottles from this vintage, it is found that the wine in 12 of these bottles is corked.

The relative proportion of corked wine bottles from a sample is given by h.

Task:

For this winery and this vintage, write down a 95 % confidence interval that is symmetrical about h for the unknown relative proportion of wine bottles in which the wine is corked.

[0/1 p.]

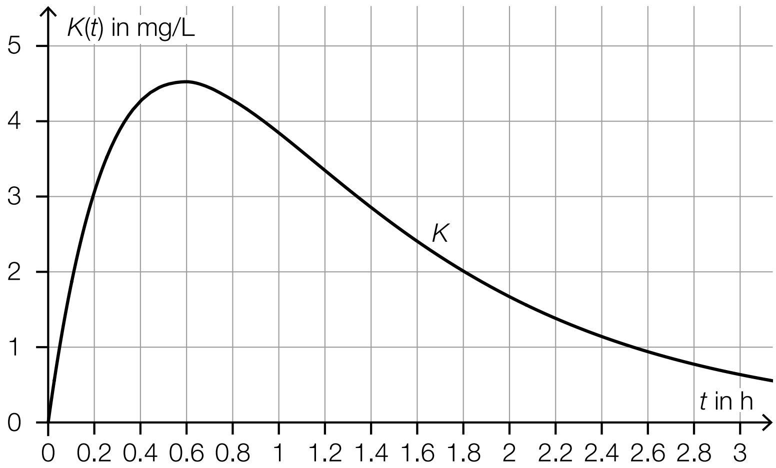
Continue overleaf

Task 25 (Part 2)

Caffeine

Task:

a) Lea drinks a cup of coffee. The diagram below shows the graph of the function K, which models the concentration $K(t)$ of caffeine in Lea's blood in terms of the time t after Lea drinks the coffee (t in h, $K(t)$ in mg/L).



1) Using the diagram above, determine the time in minutes after drinking the coffee at which the concentration of caffeine in the blood is at a maximum.

_______________________________________ min *[0/1 p.]*

2) Complete the gaps in the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement. *[0/½/1 p.]*

The function K has _______①_______ in the interval (0, 0.8) and in this interval the sign of the _______②_______ changes.

①	
a point of inflexion	☐
a maximum or minimum	☐
a zero	☐

②	
concavity	☐
gradient	☐
values of the function	☐

b) The solubility of caffeine in water gives the maximum number of grams of caffeine per litre (g/L) that can be dissolved. The solubility is dependent on the temperature. It can be approximated by the function f.

$f(T) = 6.42 \cdot e^{0.05 \cdot T}$ with $0 \leq T \leq 90$

T … temperature in °C
$f(T)$ … solubility of caffeine in water at the temperature T in g/L

A person claims:
"If the temperature increases by 10 °C, then the solubility of caffeine in water increases by around 1.65 times."

1) Verify by calculation whether the claim is true. *[0/1 p.]*

The following equation is formed:
$2 \cdot 6.42 = 6.42 \cdot e^{0.05 \cdot T}$

2) Interpret the solution to this equation in the given context. *[0/1 p.]*

Task 26 (Part 2, Best-of Assessment)

CO_2 and Climate Protection

In recent decades, the CO_2 concentration of the Earth's atmosphere has increased due to vehicle traffic, among other factors.

Task:

a) For each car that runs on petrol, it is assumed that 2.32 kg of CO_2 is emitted per litre of petrol used.

Car *A* travels a distance of *s* km with an average petrol consumption of 7.9 litres per 100 km.

In order to offset these CO_2 emissions, *b* trees are to be planted. It is assumed that each of these trees captures 500 kg of CO_2 over its whole lifetime.

1) Write down the number *b* of trees to be planted in terms of *s*.

$b =$ ______________________________ *[0/1 p.]*

Car *B* travels a distance of 15000 km. In order to offset these CO_2 emissions, 5 trees are planted.

2) Determine the average petrol consumption (in litres per 100 km) of car *B* on this journey. *[0/1 p.]*

b) Alongside CO_2, other gases also contribute to global warming. The emissions of these gases are converted into a so-called *CO_2 equivalent*.

The table below shows information about the population (in millions) for some EU countries in the year 2015 and the corresponding CO_2 equivalents (in tonnes per person).

	population in millions	CO_2 equivalent in tonnes per person
Belgium	11.2	11.9
France	66.4	6.8
Italy	60.8	7.0
Luxembourg	0.6	18.5
The Netherlands	16.9	12.3

Data sources: https://ec.europa.eu/eurostat/statistics-explained/index.php?title=Population_and_population_change_statistics/de&oldid=320539 [24.07.2020],
https://de.wikipedia.org/wiki/Liste_der_Länder_nach_Treibhausgas-Emissionen [24.07.2020].

1) Determine the average CO_2 equivalent $\overline{e}$ (in tonnes per person) for the whole of the part of the EU represented in the table above.

$\overline{e}$ = ____ tonnes per person *[0/1 p.]*

Lukas is only aware of the values for the CO_2 equivalents for the individual countries given in the table above but not the corresponding population values. He calculates the mean $\overline{x}$ of the CO_2 equivalents: $\overline{x} = 11.3$.

2) Without using the value of $\overline{e}$ calculated above, explain why $\overline{x}$ must be greater than $\overline{e}$.

[0/1 p.]

Task 27 (Part 2, Best-of Assessment)

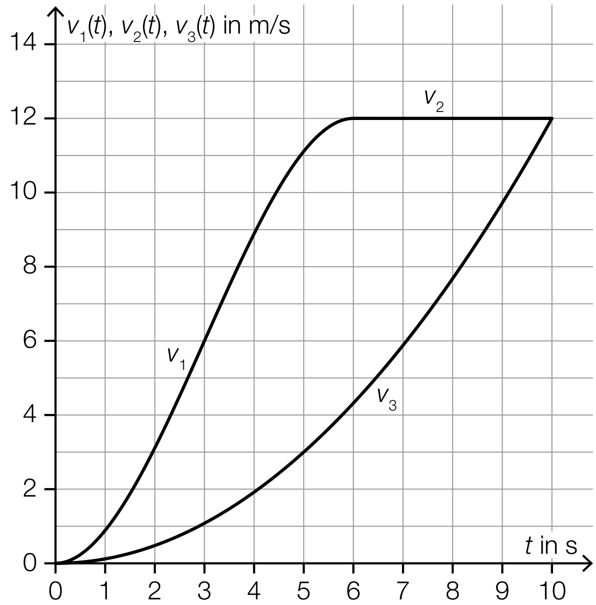
Velocity-Time Graph

The velocities of 2 cars (car *A* and car *B*) are modelled as functions in terms of time. The velocity-time graph shown below shows the corresponding graphs. The time t is given in seconds and the velocities are given in m/s.

Car *A* and car *B* set off from standing at time $t = 0$. They both have a velocity of 12 m/s at time $t = 10$.

Car *A* moves for $t \in [0, 6]$ at a velocity of $v_1(t)$ and for $t \in [6, 10]$ at a constant velocity of $v_2(t)$.

Car *B* moves for $t \in [0, 10]$ at a velocity of $v_3(t) = 0.12 \cdot t^2$.



Task:

a) In the time interval [0, 6], car *A* covers a distance of 36 m.
In the time interval $[0, t_1]$ with $6 \leq t_1 \leq 10$, car *A* covers a distance of length d (d in m).

1) Write d in terms of t_1.

$d =$ ____ *[0/1 p.]*

In the time interval [0, 10] car *A* covers a longer distance than car *B*.

2) Determine the length in metres by which this distance is longer. *[0/1 p.]*

b) For car *A* the following conditions hold:

- At time $t = 6$, the velocity is 12 m/s.
- At time $t = 0$, the acceleration is 0 m/s².
- At time $t = 3$, the acceleration is at its maximum value.

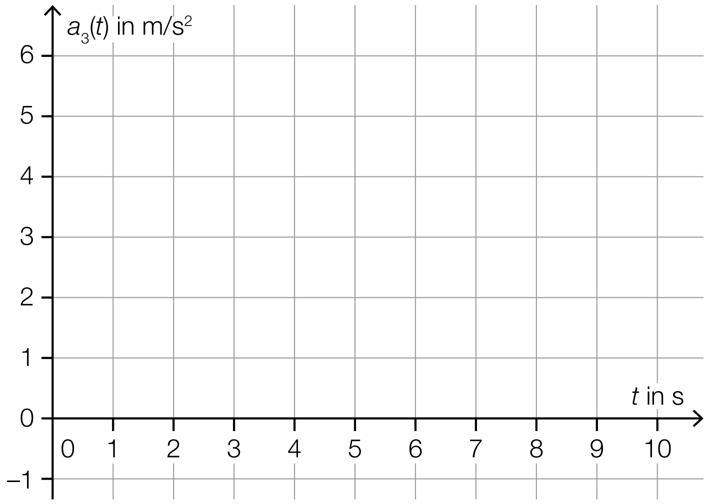
The following statement holds for the function $v_1: [0, 6] \to \mathbb{R}$:

$v_1(t) = p \cdot t^3 + q \cdot t^2 + r \cdot t$ for all $t \in [0, 6]$ with $p, q, r \in \mathbb{R}$

1) Write down a system of 3 equations with which the coefficients *p*, *q* and *r* can be calculated. *[0/½/1 p.]*

c) The acceleration of car *B* in the time interval [0, 10] is described in terms of the time *t* by the function a_3 (*t* in s, $a_3(t)$ in m/s²).

1) In the coordinate system given below, draw the graph of the acceleration function a_3. *[0/1 p.]*

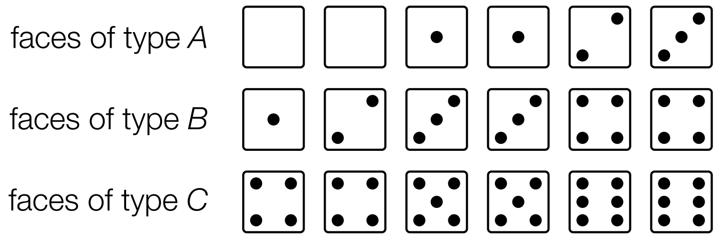


Task 28 (Part 2, Best-of Assessment)

Dice Game

In a dice game, different dice, each with 6 faces, are used. For all of the dice used, the probability of any of the faces occurring is the same for all faces. The results of each throw are independent of each other.

Three types of dice, *A*, *B* and *C*, are used. The faces of these dice are shown in the diagram below.



Task:

a) A player throws one type *B* dice and one type *C* dice simultaneously one time.

1) Determine the probability that the sum of the numbers shown on the faces is 8. *[0/1 p.]*

b) The random variables X_A, X_B and X_C give the number that occurs after being thrown for a type *A* dice, a type *B* dice and a type *C* dice respectively. One of these three random variables has an integer expectation value.

1) Write down this integer expectation value. *[0/1 p.]*

Both of the other random variables have the same standard deviation.

2) Determine this standard deviation. *[0/1 p.]*

c) A type *C* dice is thrown *n* times. The random variable Y_n gives the number of throws for which an odd number is shown on the face in these *n* throws ($n \in \mathbb{N}$). The expectation value of Y_n is given by μ_n and the standard deviation is given by σ_n.

1) Write down μ_n and σ_n in terms of *n*.

μ_n = ____

σ_n = ____ *[0/½/1 p.]*