Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

Main Examination Session 2021

Mathematics

Supplementary Examination 6
Examiner's Version

Bundesministerium
Bildung, Wissenschaft
und Forschung

Instructions for the standardised implementation of the supplementary examination

The following supplementary examination booklet contains five tasks that can be completed independently of one another. Each task comprises two sub-tasks: the “task” and the “guiding question”.

The preparation time is to be at least 30 minutes; the examination time is at most 25 minutes.

The use of the official formula booklet that has been approved by the relevant government authority for use in the standardised school-leaving examination in mathematics is allowed. Furthermore, the use of electronic devices (e. g. graphic display calculators or other appropriate technology) is allowed provided there is no possibility to communicate (e. g. via the internet, intranet, Bluetooth, mobile networks etc.) and there is no access to an individual’s data stored on the device.

After the examination, all materials (tasks, extra sheets of paper etc.) from all candidates are to be collected in. The examination materials (tasks, extra sheets of paper, digital materials etc.) may only be made public after the time period allocated for the examination has passed.

Evaluation grid for the supplementary examination

The evaluation grid below may be used to assist in assessing the candidates’ performances.

	Candidate 1		Candidate 2		Candidate 3		Candidate 4		Candidate 5	
Task 1										
Task 2										
Task 3										
Task 4										
Task 5										
Total										

Explanatory notes on assessment

Each task can be awarded zero, one or two points. There is one point available for each task for the demonstration of a core competency and one point available for each guiding question. A maximum of ten points can be achieved.

Assessment scale for the supplementary examination

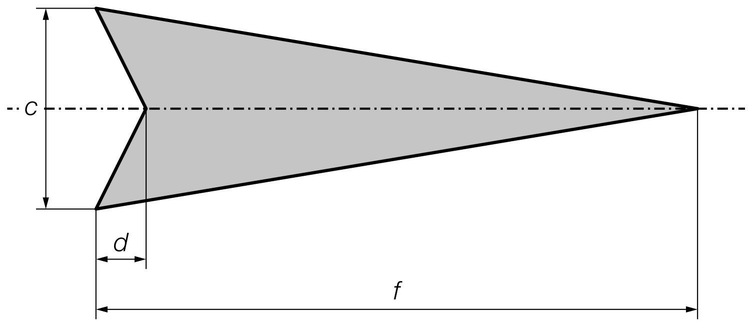
Grade	Number of points achieved (core competencies + guiding questions)
Very good	7 – 10
Good	6
Satisfactory	5
Pass	4

Task 1

Arrow

Task:

The diagram below shows a model of an arrow in 2-dimensional space. The dotted line is the axis of symmetry of this arrow.

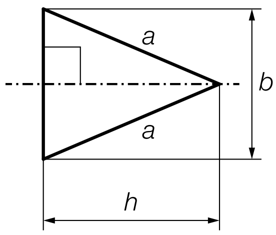


– Using c, d and f, write down a formula that can be used to calculate the area A of the area shaded in grey.

$A =$ ____

Guiding question:

The diagram below shows a model of the tip of an arrow. The dotted line is the axis of symmetry of the tip of the arrow.



The isosceles triangle shown in the diagram above has a base $b = 6$ cm, side length a cm and height $h = 7$ cm. The length of the base is to be kept the same, but the area of the triangle is to be increased by 20 %.

– Determine the side lengths of the triangle after the area has been increased.

Solution to Task 1

Arrow

Expected solution to the statement of the task:

$$A = \frac{1}{2} \cdot (c \cdot f - c \cdot d)$$

Answer key:

The point for the core competency is to be awarded if the formula has been given correctly.

Expected solution to the guiding question:

$$A_{\text{before}} = \frac{6 \cdot 7}{2} = 21$$
$$A_{\text{after}} = 21 \cdot 1.2 = 25.2$$
$$A_{\text{after}} = \frac{h_{\text{after}} \cdot 6}{2} \quad \Rightarrow \quad h_{\text{after}} = 8.4 \text{ cm}$$
$$a_{\text{after}} = \sqrt{8.4^2 + 3^2} = 8.91...$$

The sides of the triangle have a length of around 8.9 cm after increasing the area.

Answer key:

The point for the guiding question is to be awarded if the length has been calculated correctly.

Task 2

Mountain Railway

The station at the bottom of a mountain railway is at an altitude of 1 000 m. The horizontal distance between the station at the bottom of the mountain and the station at the top of the mountain is 2 500 m. In this task, the railway line is modelled as a straight line and has a constant gradient of 41 %.

Task:

– Determine the angle of elevation of the railway line.

– Determine the altitude of the station at the top of the mountain.

Guiding question:

The duration of the journey from the station at the bottom of the mountain to the station at the top of the mountain is 5 min.

The function p with $p(h) = 1\,000 \cdot e^{-0.000126 \cdot h}$ can be used to approximate the air pressure at an altitude of h (h in m, $p(h)$ in mbar).

– Determine the average absolute change in air pressure per minute over the course of a journey with this mountain railway from the station at the bottom of the mountain to the station of the top of the mountain.

Solution to Task 2

Mountain Railway

Expected solution to the statement of the task:

α ... angle of elevation of the mountain railway
x ... difference between the altitude of the station at the top of the mountain and the altitude of the station at the bottom of the mountain

$\tan(\alpha) = 0.41 \quad \Rightarrow \quad \alpha = 22.29...°$

$0.41 = \frac{x}{2\,500} \quad \Rightarrow \quad x = 1\,025$

The station at the top of the mountain is at an altitude of 2025 m.

Answer key:

The point for the core competency is to be awarded if the angle of elevation and the altitude of the station at the top of the mountain have been calculated correctly.

Expected solution to the guiding question:

$\frac{p(2\,025) - p(1\,000)}{5} = -21.3...$

The average absolute change in the air pressure is around 21 mbar/min.

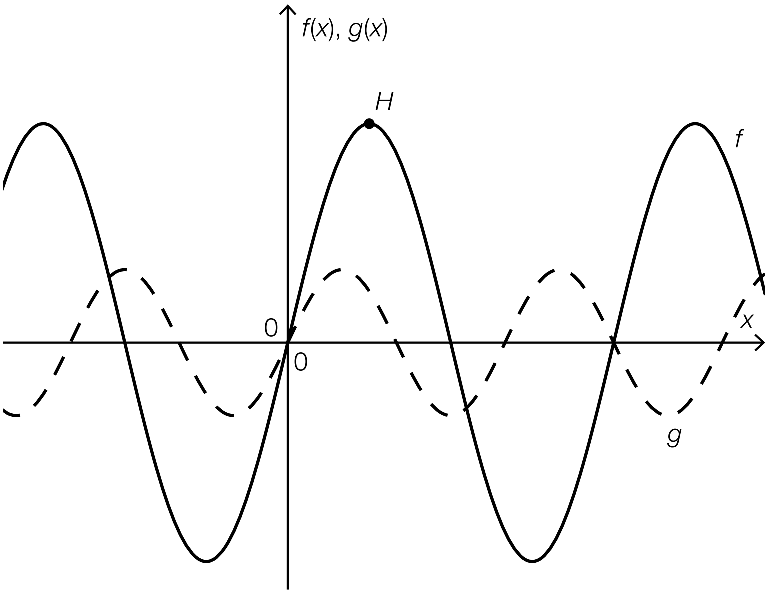
Answer key:

The point for the guiding question is to be awarded if the average absolute change in air pressure has been calculated correctly. The unit “mbar/min” does not need to be given.

Task 3

Trigonometric Functions

The diagram below shows the graphs of the functions *f* and *g* with $f(x) = a \cdot \sin(b \cdot x)$ and $g(x) = c \cdot \sin(d \cdot x)$ with $a, b, c, d \in \mathbb{R}^+$.



Task:

– Complete each of the gaps below with the appropriate symbol "<", ">" or "=" and justify your answers.

a _____ *c*

b _____ *d*

Guiding question:

The maximum point of the graph of *f* labelled *H* in the diagram above has coordinates $H = \left(\frac{\pi}{4}, 3\right)$.

– Determine *a* and *b*.

Solution to Task 3

Trigonometric Functions

Expected solution to the statement of the task:

$a > c$
The function f has a larger maximum value than the function g.

$b < d$
The function f has a larger period length than the function g.

Answer key:

The point for the core competency is to be awarded if the correct symbols have been used and correct justifications (that may also be given using the terms “amplitude” and “frequency”) have been given.

Expected solution to the guiding question:

$a = 3$

$b = \frac{2 \cdot \pi}{\frac{\pi}{4} \cdot 4} = 2$

Answer key:

The point for the guiding question is to be awarded if a and b have been determined correctly.

Task 4

Drag Race

Jan and Tom are participating in a drag race. They set off at the same time when $t = 0$. The velocities of their vehicles in the first few seconds can be described by the two functions v_J and v_T.

t ... time in s
$v_J(t)$ ... velocity of Jan's vehicle at time t in m/s
$v_T(t)$ ... velocity of Tom's vehicle at time t in m/s

Task:

For the time-velocity function v_J, the following relationship holds:

$v_J(t) = 0.6 \cdot t^2 \cdot e^{-0.09 \cdot t}$

– Determine the acceleration of Jan's vehicle when $t = 10$.

Guiding question:

At time t_1, Tom's vehicle is ahead of Jan's vehicle. The distance between the vehicles at time t_1 is d metres.

– Using v_J and v_T, write down a formula that can be used to calculate d.

d = ____

Solution to Task 4

Drag Race

Expected solution to the statement of the task:

$v_J'(10) = 2.68...$

The acceleration is around 2.7 m/s^2.

Answer key:

The point for the core competency is to be awarded if the acceleration has been determined correctly.

Expected solution to the guiding question:

$$d = \int_0^{t_1} v_T(t)\,dt - \int_0^{t_1} v_J(t)\,dt$$

Answer key:

The point for the guiding question is to be awarded if the formula has been given correctly.

Task 5

Balls

Task:

A box with 30 balls contains 14 red and 16 yellow balls.

Maria takes 2 balls out of the box at random and without replacement.

– Determine the probability that Maria removes 2 balls of the same colour from the box.

Guiding question:

Another box contains 3 white balls and 1 green ball.
Eva takes balls out of the box at random until she removes the green ball.

The random variable X gives the number of balls removed. If X takes the value 2, this means that the first ball is white and the second ball is green.

– Determine the expectation value of X.

Solution to Task 5

Balls

Expected solution to the statement of the task:

$\frac{14}{30} \cdot \frac{13}{29} + \frac{16}{30} \cdot \frac{15}{29} = \frac{211}{435} = 0.4850... \approx 48.5\ \%$

Answer key:

The point for the core competency is to be awarded if the probability has been determined correctly.

Expected solution to the guiding question:

expectation value: $1 \cdot P(X = 1) + 2 \cdot P(X = 2) + 3 \cdot P(X = 3) + 4 \cdot P(X = 4)$

$= 1 \cdot \frac{1}{4} + 2 \cdot \frac{3}{4} \cdot \frac{1}{3} + 3 \cdot \frac{3}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} + 4 \cdot \frac{3}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} = 2.5$

Answer key:

The point for the guiding question is to be awarded if the expectation value has been calculated correctly.