Name:	
Class:	

Standardised Competence-Oriented
Written School-Leaving Examination

AHS

17th September 2021

Mathematics

Bundesministerium
Bildung, Wissenschaft
und Forschung

Advice for Completing the Tasks

Dear candidate,

The following booklet contains Part 1 tasks and Part 2 tasks (divided into sub-tasks). The tasks can be completed independently of one another.

Please do all of your working out solely in this booklet and on the paper provided to you. Write your name and that of your class on the cover page of the booklet in the spaces provided. Please also write your name on any separate sheet of paper used and number these pages consecutively. When answering each sub-task, write the name/number of the sub-task (e. g. 25a1) on your sheet.

In the assessment of your work, everything that is not crossed out will be considered. Your solutions must be marked clearly. If a solution is not clearly marked or if more than one solution is given, the task will be considered to be unsolved.

The use of the official formula booklet for this examination that has been approved by the relevant government authority is permitted. Furthermore, the use of electronic device(s) (e. g. graphic display calculators or other appropriate technology) is allowed provided there is no possibility of communicating via the internet, Bluetooth, mobile networks etc. and there is no access to your own data stored on the device.

An explanation of the task types is available in the examination room and can be viewed on request.

Please hand in this booklet and all worksheets you have used at the end of the examination.

Changing an answer for a task that requires a cross:

1. Fill in the box that contains the cross.
2. Put a cross in the box next to your new answer.

In this instance, the answer "5 + 5 = 9" was originally chosen. The answer was later changed to be "2 + 2 = 4".

1 + 1 = 3	☐
2 + 2 = 4	☒
3 + 3 = 5	☐
4 + 4 = 4	☐
5 + 5 = 9	■
6 + 6 = 10	☐

Selecting an item that has been filled in:

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

In this instance, the answer "2 + 2 = 4" was filled in and then selected again.

1 + 1 = 3	☐
2 + 2 = 4	■
3 + 3 = 5	☐
4 + 4 = 4	■
5 + 5 = 9	☐
6 + 6 = 10	☐

Assessment

Each task in Part 1 and each sub-task in Part 2 will be awarded either 0 points or 1 point or 0, ½ or 1 point respectively. The points that can be achieved in each (sub-)task are shown in the booklet.

Grading System

points awarded	grade
32–36 points	very good
27–31.5 points	good
22–26.5 points	satisfactory
17–21.5 points	pass
0–16.5 points	fail

Best-of Assessment: A best-of assessment approach will be applied to tasks 26, 27 and 28. Of these three Part 2 tasks, the task with the lowest point score will not be included in the total point score.

Good luck!

Task 1

Difference between Two Natural Numbers

The following statement holds for two natural numbers n and m: $n \neq m$.
For the difference $n - m$ to be a natural number, a particular mathematical relationship between n and m must hold.

Task:

Write down this mathematical relationship.

[0/1 p.]

Task 2

Quadratic Equation

Let $x^2 - 6 \cdot x + c = 0$ with $c \in \mathbb{R}$ be a quadratic equation.

Task:

Determine all $c \in \mathbb{R}$ such that the equation has no real solution.

[0/1 p.]

Task 3

Height

The components of the vector K_1 give the heights of the children of a particular school class (in cm) at the beginning of a school year.
The components of the vector K_2 give the heights of these children (in cm) n months later ($n \in \mathbb{N}\backslash\{0\}$). (The heights given in both vectors K_1 and K_2 are ordered alphabetically by the names of the children.)

Task:

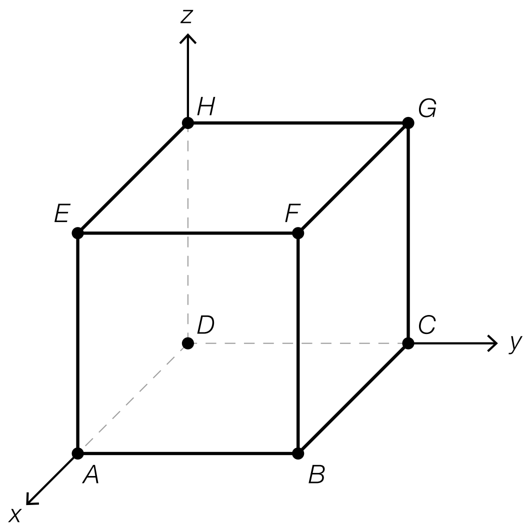
Interpret the vector $\frac{1}{n} \cdot (K_2 - K_1)$ in the given context.

[0/1 p.]

Task 4

Cube and Vector

The diagram below shows a cube whose base *ABCD* lies in the *xy*-plane.



Two vertices of this cube constitute a particular vector that goes in the direction of the vector $\vec{v} = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$.

Task:

Put a cross next to this vector. *[1 out of 6]*

$\overrightarrow{EC}$	☐
$\overrightarrow{FD}$	☐
$\overrightarrow{GA}$	☐
$\overrightarrow{GD}$	☐
$\overrightarrow{HA}$	☐
$\overrightarrow{HB}$	☐

[0/1 p.]

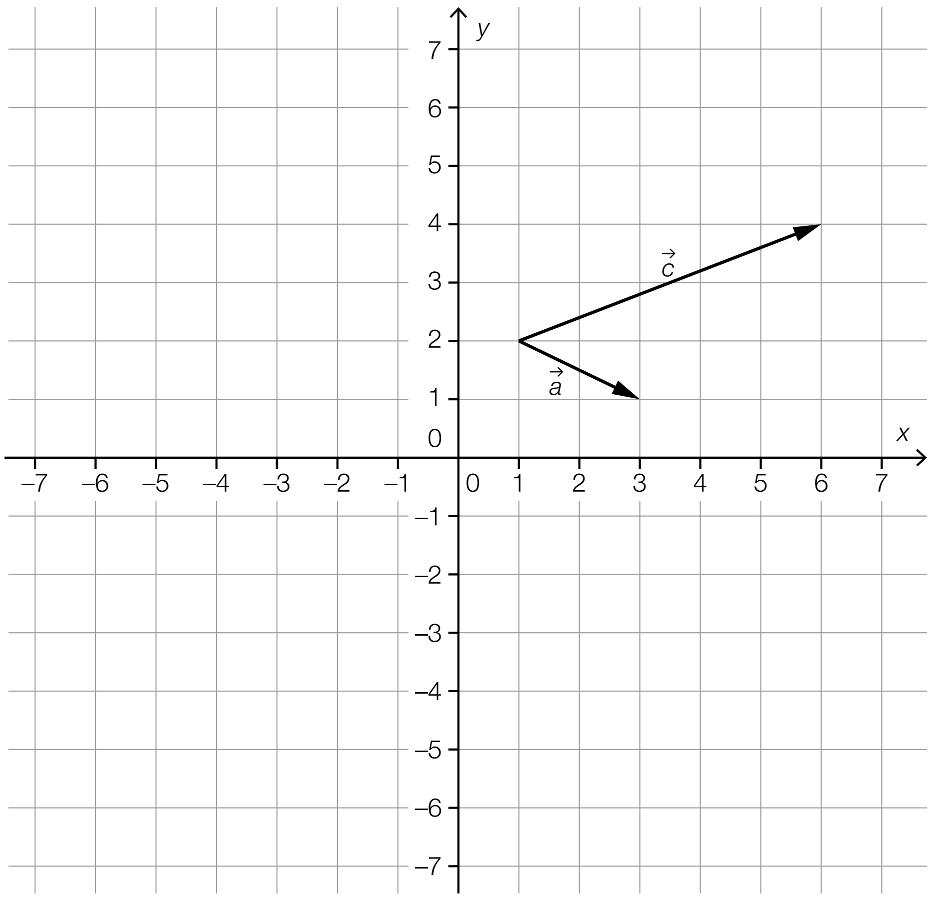
Task 5

Vectors

The vectors $\vec{a}$ and $\vec{c}$ are shown in the coordinate system below.
The following statement holds: $\vec{c} = 2 \cdot \vec{a} + \vec{b}$.

Task:

Draw the vector $\vec{b}$ into the coordinate system below.



[0/1 p.]

Task 6

Angles and Sides of Right-Angled Triangles

For particular right-angled triangles, the following statements hold:

The angles α, β and γ are opposite the sides a, b and c in this order.
The angles are measured in degrees and the side lengths are measured in centimetres.
Furthermore, $\cos(\alpha) = \frac{3}{5}$ and $\cos(\gamma) = 0$ hold.

Task:

Put a cross next to each of the two statements that are true for every one of these triangles.
[2 out of 5]

$c = 5$ cm	☐
$\beta < 90°$	☐
$\sin(\beta) = \frac{3}{5}$	☐
$a < b < c$	☐
$\tan(\alpha) = 0.75$	☐

[0/1 p.]

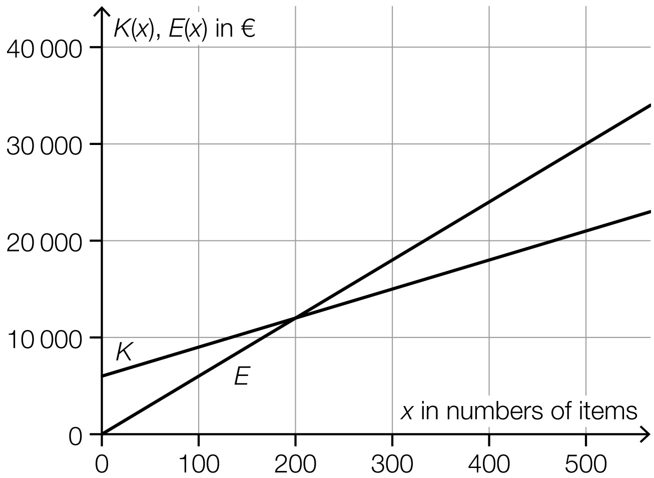
Task 7

Shirts

A company produces and sells shirts.

The linear function K describes the costs $K(x)$ in euros in terms of the number x of items produced.
The linear function E describes the revenue $E(x)$ in euros in terms of the number x of items sold.

The diagram below shows the graph of the function K and the graph of the function E.



The point of intersection of K and E has coordinates $(200, 12\,000)$ and $K(0) = 6\,000$ holds.

Task:

Put a cross next to each of the two true statements. *[2 out of 5]*

The sales price of a shirt is € 60.	☐
The production cost of a shirt is € 25.	☐
If the company produces and sells 400 shirts, a profit of € 6 000 will be made.	☐
There are no fixed costs in the production.	☐
If the company produces and sells fewer than 200 shirts, a profit will be made.	☐

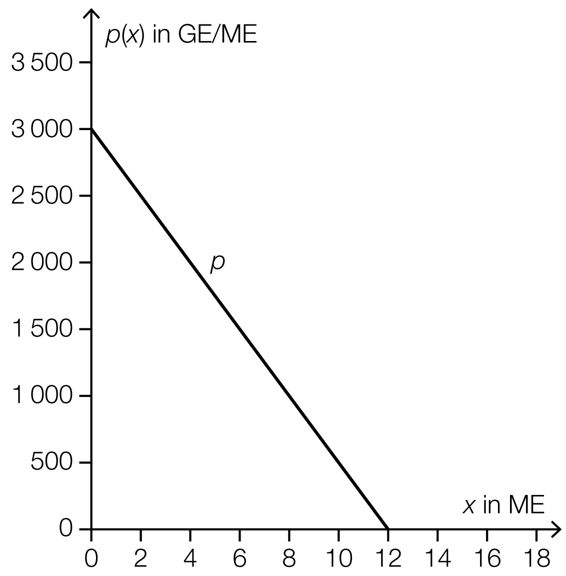
[0/1 p.]

Task 8

Revenue Function

For a particular product, the relationship between the amount in demand x and the demand price $p(x)$ can be modelled by the linear function p shown below.

x ... amount in demand in units of quantity, $0 \leq x \leq 12$
$p(x)$ ... demand price for the amount x in monetary units per unit of quantity (GE/ME)



For the revenue function E, the following statement holds: $E(x) = p(x) \cdot x$.

Task:

Write down an equation of the function E.

$E(x) =$ ____

[0/1 p.]

Task 9

Expansion of a Bridge

The length of a particular bridge is dependent on its temperature.
At a temperature of the bridge of –14 °C, it is 300 m long.
When the bridge warms up by 25 °C, it expands by 0.1 m.

The linear function l models the length of the bridge in terms of its temperature T. Each temperature $T \in [-20\text{ °C}, 40\text{ °C}]$ is associated with the length of the bridge $l(T)$ (T in °C, $l(T)$ in m).

Task:

Write down an equation of the function l.

$l(T) =$ ____

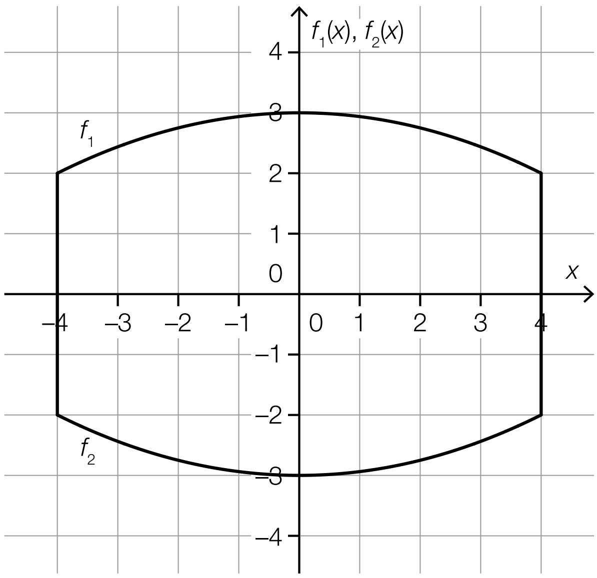
[0/1 p.]

Task 10

Two Quadratic Functions

A particular cross-section is bounded by the graphs of the quadratic functions f_1 and f_2 as well as the lines $x = -4$ and $x = 4$.
The following statements hold:
$f_1\colon [-4, 4] \to \mathbb{R}, x \mapsto a \cdot x^2 + b$ with $a, b \in \mathbb{R}$
$f_2\colon [-4, 4] \to \mathbb{R}, x \mapsto c \cdot x^2 + d$ with $c, d \in \mathbb{R}$

The situation is represented in the diagram below.



Task:

Complete the statements (1) and (2) below by choosing from the symbols ”<”, “=” or “>” so that each sentence becomes a correct statement.

(1) a ______ c

(2) b ______ d

[0/½/1 p.]

Task 11

Medication

The pain-relieving active ingredient of a medication reduces in the body of a particular patient approximately exponentially. The amount of active ingredient decreases by 8 % per hour.
At time $t = 0$ the amount of active ingredient is 700 micrograms.

Task:

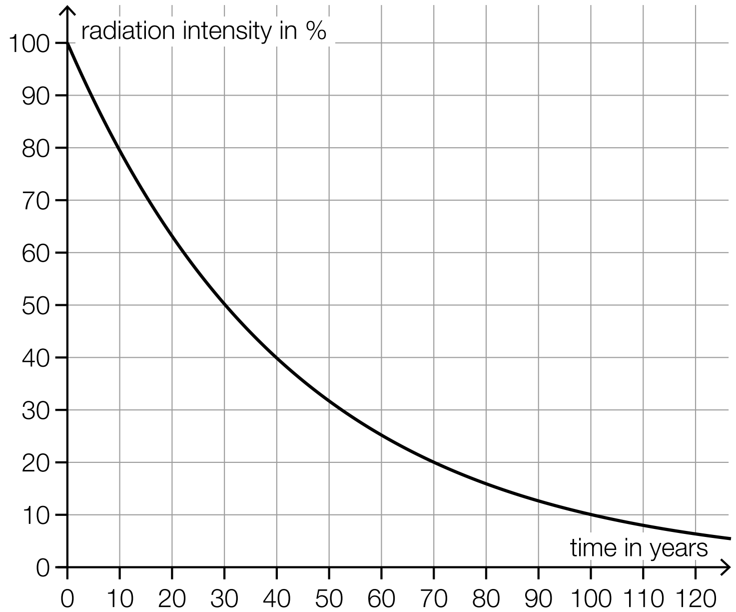
Determine after which period of time (in h) the active ingredient in the body of the patient has reduced to 100 micrograms.

[0/1 p.]

Task 12

Half-Life

The diagram below shows a model of the development of the radiation intensity of a particular radioactive substance in terms of time.



Task:

Write down the half-life T of the radioactive intensity of this radioactive substance.

T = ____________________ years

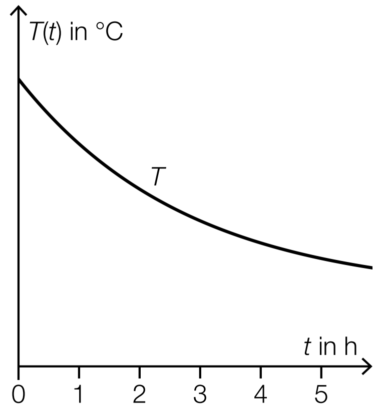
[0/1 p.]

Task 13

Cooling

The differentiable function T assigns the time $t \geq 0$ to the temperature $T(t)$ of a body (t in h, $T(t)$ in °C).

The diagram below shows the graph of this function T.



The following statement holds: $T'(1) = -15$.

Task:

Put a cross next to each of the two true statements. *[2 out of 5]*

At the time $t = 2$, the instantaneous rate of change of the temperature of the body is smaller than –15 °C/h.	☐
The temperature of the body one hour after the start of the cooling process is 15 °C lower than at the time $t = 0$.	☐
At the time $t = 1$, the instantaneous rate of change of the temperature of the body is –15 °C/h.	☐
The following statement holds: $\frac{T(3) - T(1)}{2} > -15$.	☐
Over the course of the first hour, the average speed of cooling of the body is 15 °C/h.	☐

[0/1 p.]

Task 14

Difference Equation

Let $x_{n+1} = 1.2 \cdot x_n - 2$ with $n \in \mathbb{N}$ be a difference equation with the starting value $x_0 \in \mathbb{R}$.

Task:

Write down a formula that could be used to calculate x_2 in terms of x_0.

[0/1 p.]

Task 15

Derivative and Antiderivative

The polynomial function f has the derivative f' and the antiderivative F.

Task:

Put a cross next to each of the two statements that are true in all cases. *[2 out of 5]*

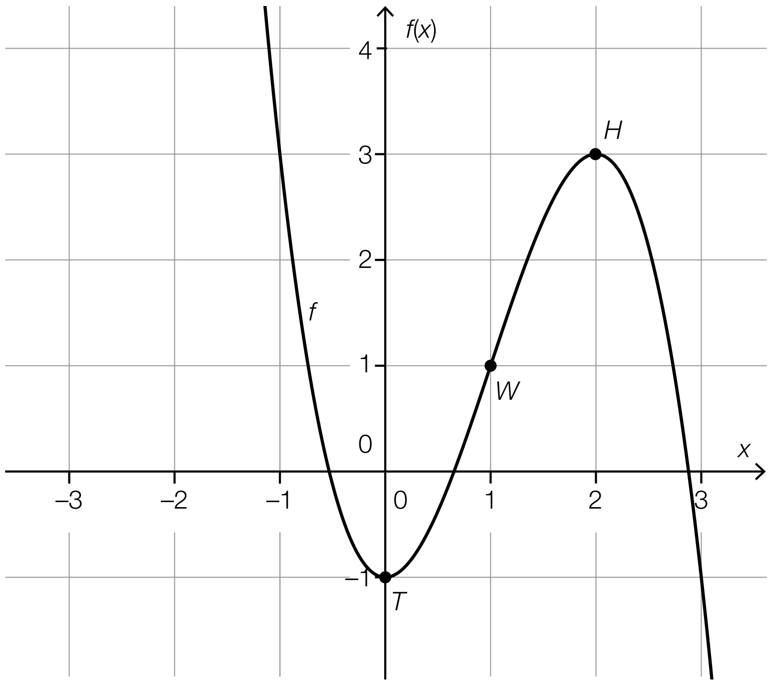
The expression $F(a)$ gives the gradient of f at the point $(a, f(a))$ for all $a \in \mathbb{R}$.	☐
The antiderivative F is unique. There exist no other antiderivatives of f.	☐
The derivative f' is unique. There exist no other derivatives of f.	☐
The expression $F'(0)$ gives the gradient of the function f at the point $(0, f(0))$.	☐
The following statement holds: $F'(a) = f(a)$ for all $a \in \mathbb{R}$.	☐

[0/1 p.]

Task 16

Derivatives

The graph of the 3rd degree polynomial function f is shown below. The points marked in bold (the minimum T, the point of inflexion W and the maximum H) have integer coordinates.



Various statements about the 1st and 2nd derivatives of f are shown below.

Task:

Put a cross next to each of the two true statements. *[2 out of 5]*

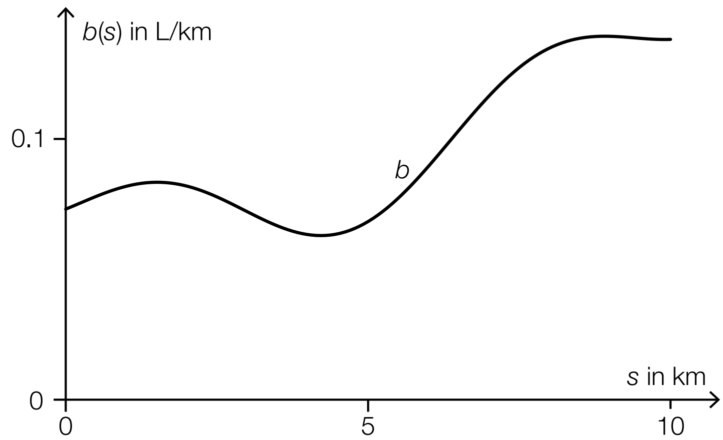
$f'(0) > 0$	☐
$f''(0) > 0$	☐
$f'(1) > 0$	☐
$f'(2) > 0$	☐
$f''(2) > 0$	☐

[0/1 p.]

Task 17

Petrol Consumption on a Journey on a Country Road

Maria drives her car on a country road for a distance of 10 km.
The function b gives the instantaneous petrol consumption $b(s)$ (in L/km) in terms of the distance covered s (in km) from the start of the journey (see diagram below).



The expression V has units L/km and can be calculated using the formula shown below.

$V = \frac{1}{10} \cdot \int_0^{10} b(s)\,\mathrm{d}s$

Task:

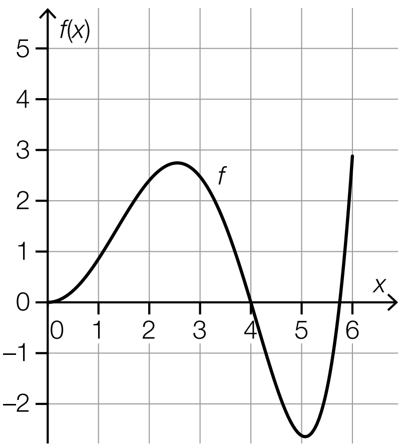
Interpret V in the given context.

[0/1 p.]

Task 18

Statements about Definite Integrals

The diagram below shows the graph of the function *f* in the interval [0, 6].



Various statements about definite integrals of the function *f* are shown below.

Task:

Put a cross next to each of the two true statements. *[2 out of 5]*

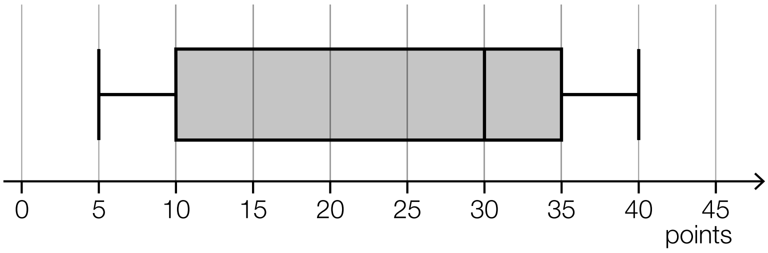
$\int_0^4 f(x)\,\mathrm{d}x > \int_0^5 f(x)\,\mathrm{d}x$	☐
$\int_3^4 f(x)\,\mathrm{d}x > \int_4^5 f(x)\,\mathrm{d}x$	☐
$\int_0^6 f(x)\,\mathrm{d}x > \int_0^4 f(x)\,\mathrm{d}x$	☐
$\int_0^4 f(x)\,\mathrm{d}x = 0$	☐
$\int_4^6 f(x)\,\mathrm{d}x > 0$	☐

[0/1 p.]

Task 19

Results of a Mathematics Examination

For a particular mathematics examination, in which 30 pupils participated, the maximum possible score was 48 points.
The results of this mathematics examination are represented below in a boxplot and a stem and leaf diagram.



tens digit	units digit
0	a, 6, 6, 7, 7, 8, 8
1	0, 1, 5, 5, 9
2	1, 5, 8
3	b, 3, 3, 3, 3, 4, 4, 5, 5, 7, 8, 8, 9
4	0, 0

Task:

Write down the values of a and b.

a = __________

b = __________

[0/½/1 p.]

Task 20

Changing Numbers

A particular list of data comprises 100 numbers $x_1, x_2, \ldots, x_{100}$. The mean of the list of data is 86, the minimum value is 29, and the maximum value is 103.

A second list of data also comprises 100 numbers. This list is generated by subtracting 20 from each number in the original list of data.

Task:

Write down the mean and the range of the second list of data.

mean: ______________________________________

range: ______________________________________

[0/½/1 p.]

Task 21

Two-Step Random Experiment

A random experiment can either result in a “success” with a probability of p or a “failure” with a probability of $1 - p$.

This random experiment is conducted twice. Each experiment is independent of the other. The probability that at least one of these experiments results in a “success” is 0.36.

Task:

Determine the probability p.

[0/1 p.]

Task 22

Selection Possibilities

In a particular competition, annual passes for the zoo can be won.
In this competition, 1000 people have each participated 1 time.
To determine the winners, two people are selected at random.

Task:

Write down the number of possibilities of randomly selecting these 2 people from the 1000 participants.

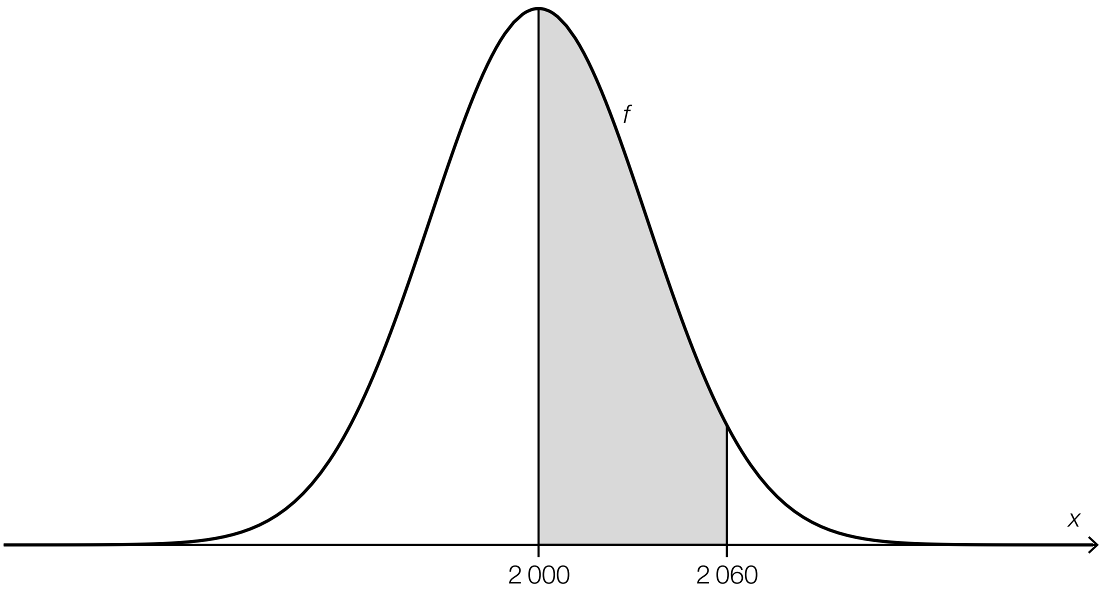
The number of selection possibilities is: ____

[0/1 p.]

Task 23

Short-Sightedness

The approximately normally distributed random variable X gives the number of short-sighted people in a sample. The function f is the density function of the random variable X and takes its maximum value when $x = 2\,000$. The graph of f is shown in the diagram below.



The area of the region shaded in grey is 0.46.

Task:

Write down the probability that there are at least 2 060 short-sighted people in this sample.

P("at least 2 060 short-sighted people") = ____

[0/1 p.]

Task 24

Binomially Distributed Random Variable

A particular random experiment with an unknown probability of success p is conducted 400 times. The binomially distributed random variable X gives the number of successes. For the expectation value the following statement holds: $\mu = 80$.

Task:

Determine the probability of success p as well as the standard deviation σ of the random variable X.

$p =$ ____

$\sigma =$ ____

[0/½/1 p.]

Task 25 (Part 2)

Prom

Task:

a) Tickets for a prom can be bought in advance or on the door. In advance, each ticket costs € 20. On the door, each ticket costs 10 % more.

Overall, 640 tickets were sold for a total price of € 13 240.

The following notation has been chosen:

x ... the number of tickets sold in advance
y ... the number of tickets sold on the door

1) Write down a system of equations that can be used to calculate x and y. *[0/½/1 p.]*

b) For entertainment, a *wheel of fortune* game is to be offered. The probability of winning is constant for each game at 25 %, independent of the other games played.

Katja plays this game 3 times.

1) Determine the probability that Katja wins exactly 2 times. *[0/1 p.]*

c) The game *hook-a-duck* is also to be offered.
From a total of 50 rubber ducks, 5 are marked underneath.

In this game, a participant chooses 2 of the 50 rubber ducks at random and without replacement. Each marked rubber duck that has been chosen results in a prize.

The random variable X describes how many of the two chosen rubber ducks are marked. The probability of one possible event in this context can be calculated with the following expression.

$$P(X = \boxed{}) = \frac{5}{50} \cdot \frac{45}{49} + \frac{45}{50} \cdot \frac{5}{49}$$

1) Write down the missing number in the box provided. *[0/1 p.]*

Martin claims: "The random variable X is binomially distributed."

2) Justify why Martin's claim is false. *[0/1 p.]*

Task 26 (Part 2, Best-of Assessment)

Changes in Temperature

The process of the cooling or heating up of a drink can be modelled by functions. For these functions, the temperature of the drink in °C can be given in terms of the time *t* in minutes.

Task:

a) Tea cooling in a teapot can be described by the function g with $g(t) = 70 \cdot e^{-0.045 \cdot t} + 18$.

At time t^* the temperature of the tea has cooled to 37 °C.

1) Determine t^*.

$t^* =$ ________________________________ min *[0/1 p.]*

2) Determine the average rate of change of g in the interval [10 min, 12 min]. Interpret the result along with the corresponding unit in the given context. *[0/½/1 p.]*

b) A particular cooled wine in a wine glass has an initial temperature of $T_0 = 5$ °C. The ambient temperature is a constant $U = 25$ °C.
The temperature of the wine is measured at regular intervals. At time t it has a value T_t.

Each minute, the temperature of the wine increases by 8 % of the difference between the ambient temperature U and the temperature T_t of the wine measured at time t. The temperature of the wine thus rises to the value T_{t+1}.

1) Complete the following difference equation for this heating process.

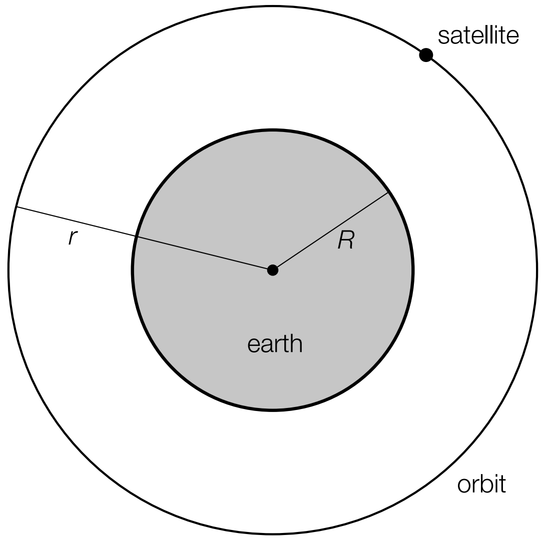
$T_{t+1} = T_t +$ ________________________________ with $T_0 = 5$ *[0/1 p.]*

2) Determine the temperature of the wine at the time $t = 3$ min. *[0/1 p.]*

Task 27 (Part 2, Best-of Assessment)

Satellites and Their Orbits

A satellite moves along an approximately circular orbit with a radius of r around the earth. The earth is assumed to be spherical with radius R.
This model is represented in the diagram below.



Task:

a) A particular satellite moves with a velocity of $v = 7\,500$ m/s along its orbit. The relationship between its velocity and the radius of its orbit is given by the equation below.

$$v = \sqrt{\frac{G \cdot M}{r}}$$

v ... velocity of the satellite in m/s
$G = 6.67 \cdot 10^{-11}$ ... universal gravitational constant in $\frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$
$M = 5.97 \cdot 10^{24}$ ... mass of the earth in kg
r ... radius of the orbit of the satellite in m

1) Determine the radius r of the orbit of this satellite.

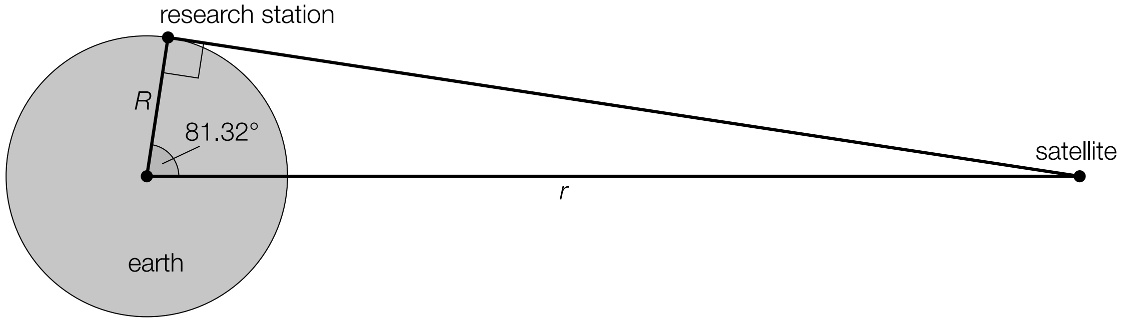
r = ____ m *[0/1 p.]*

2) Determine the time (in s) that this satellite takes to complete one orbit of the earth.

t = ____ s *[0/1 p.]*

b) The satellite dish of a research station is aligned with a particular satellite.

The following not-to-scale diagram represents this situation.



The earth's radius R is assumed to be $R = 6.37 \cdot 10^6$ m.

1) Determine the radius r of the orbit of this satellite.

r = ____ m *[0/1 p.]*

The velocity of radio signals is assumed to be $3 \cdot 10^8$ m/s.

2) Determine the time (in s) required for a radio signal to travel from the research station to this satellite. Write the solution correct to 3 decimal places. *[0/1 p.]*

Task 28 (Part 2, Best-of Assessment)

Storage Media

In recent decades, various storage media, such as memory cards, USB sticks or DVDs have been used to back up data.

Task:

a) The storage capacity of a storage medium can be given in kilobytes, megabytes or gigabytes, for example. The prefixes *kilo-*, *mega-*, *giga-* are used as follows:
1 megabyte = 1 024 kilobytes
1 gigabyte = 1 024 megabytes

A particular memory card with a capacity of 16 gigabytes is used to store photos. For the purpose of the model, it is assumed that all of the photos require the same amount of memory.
The function $N\colon \mathbb{R}^+ \to \mathbb{R}^+$ assigns the required memory F for one photo to the largest possible number $N(F)$ of photos that can be stored on this memory card (F in kilobytes).

1) Write down an equation of the function N.

$N(F) =$ ____ *[0/1 p.]*

b) Michael has 4 USB sticks labelled A, B, C and D.

- On USB stick A, he saves all of his photos.
- On the 3 other USB sticks, B, C and D, he saves exactly one third of all of his photos so that every photo is also stored on exactly 1 of these 3 USB sticks as a backup.

For each of the 4 USB sticks, the probability that it still works after 5 years is 75 % (independent of the other sticks).

As a simplification, it can be assumed that a USB stick either works fully or does not work at all.

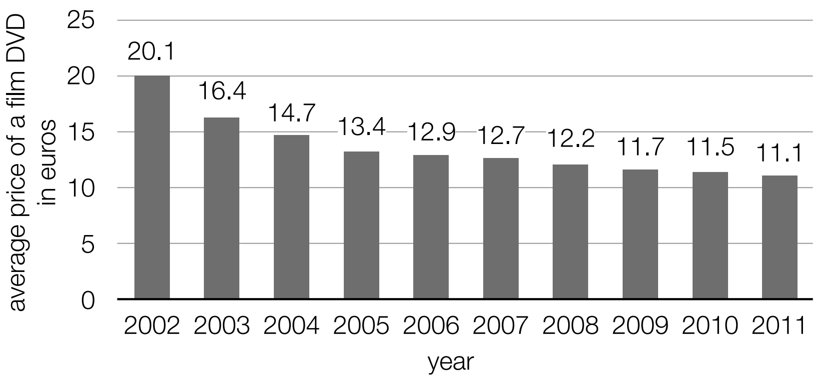
1) Determine the probability that after 5 years every one of Michael's photos is still available on at least 1 USB stick. *[0/1 p.]*

After 5 years, Michael realises that USB stick A no longer works.

2) Determine the probability that at least 2 of the 3 USB sticks B, C and D still work. *[0/1 p.]*

c) A popular storage medium for films is the DVD.

Since the beginning of the 21st century, the average price of a film DVD has decreased, as shown in the diagram below.



Data source: https://www.mkdiscpress.de/ratgeber/chronik-der-speichermedien/ [20.11.2019].

The average price of a film DVD is modelled by the function P in terms of the time t.

$P(t) = a \cdot b^t + 11$ with $a, b \in \mathbb{R}^+$

t ... time in years with $t = 0$ for the year 2002
$P(t)$ ... average price of a film DVD at time t in euros

1) Determine a and b such that P for the years 2002 and 2011 corresponds to the average price of a film DVD in the respective year according to the diagram above.

$a =$ ____________________

$b =$ ____________________ *[0/½/1 p.]*