Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

October 2021

Mathematics

Supplementary Examination 2
Examiner's Version

Bundesministerium
Bildung, Wissenschaft
und Forschung

Instructions for the standardised implementation of the supplementary examination

The following supplementary examination booklet contains five tasks that can be completed independently of one another. Each task comprises two sub-tasks: the “task” and the “guiding question”.

The preparation time is to be at least 30 minutes; the examination time is at most 25 minutes.

The use of the official formula booklet that has been approved by the relevant government authority for use in the standardised school-leaving examination in mathematics is allowed. Furthermore, the use of electronic devices (e. g. graphic display calculators or other appropriate technology) is allowed provided there is no possibility to communicate (e. g. via the internet, intranet, Bluetooth, mobile networks etc.) and there is no access to an individual’s data stored on the device.

After the examination, all materials (tasks, extra sheets of paper etc.) from all candidates are to be collected in. The examination materials (tasks, extra sheets of paper, digital materials etc.) may only be made public after the time period allocated for the examination has passed.

Evaluation grid for the supplementary examination

The evaluation grid below may be used to assist in assessing the candidates’ performances.

	Candidate 1		Candidate 2		Candidate 3		Candidate 4		Candidate 5	
Task 1										
Task 2										
Task 3										
Task 4										
Task 5										
Total										

Explanatory notes on assessment

Each task can be awarded zero, one or two points. There is one point available for each task for the demonstration of a core competency and one point available for each guiding question. A maximum of ten points can be achieved.

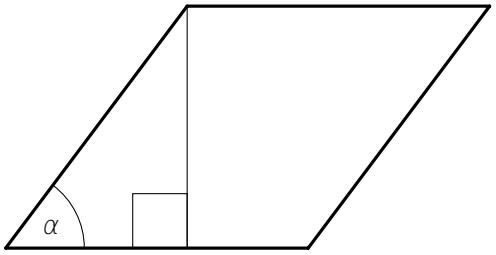
Assessment scale for the supplementary examination

Grade	Number of points achieved (core competencies + guiding questions)
Very good	7 – 10
Good	6
Satisfactory	5
Pass	4

Task 1

Rhombus

The diagram below shows a rhombus with side length a, height h and angle α ($\alpha < 90°$).



Task:

– Write down a formula in terms of a and α that can be used to calculate h.

$h =$ _____

Guiding question:

– Determine the value of α for which the area A of the rhombus is half the size of the area of a square with the same side length a.

Solution to Task 1

Rhombus

Expected solution to the statement of the task:

$\sin(\alpha) = \frac{h}{a}$
$h = \sin(\alpha) \cdot a$

Answer key:

The point for the core competency is to be awarded if the formula has been written down correctly.

Expected solution to the guiding question:

$A = a^2 \cdot \sin(\alpha)$
$a^2 \cdot \sin(\alpha) = a^2 \cdot 0.5 \quad \Rightarrow \quad \sin(\alpha) = 0.5 \quad \Rightarrow \quad \alpha = 30°$

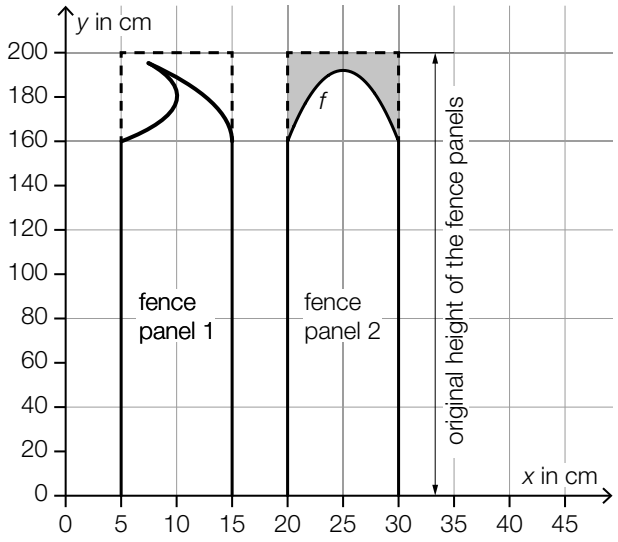
Answer key:

The point for the guiding question is to be awarded if the value of α has been determined correctly.

Task 2

Fence Panels

A carpenter cuts rectangular fence panels creatively.
The original fence panels are rectangular with a height of 200 cm and a width of 10 cm. From these, the carpenter creates the fence panel models shown in the diagram below.



Task:

Fence panel 1: The whole of the upper boundary in the region $5 \leq x \leq 15$ is to be represented by the graph of a function in terms of x.

– Justify why this is not possible.

Guiding question:

Fence panel 2: The upper boundary in the interval [20, 30] is described by the graph of the function f.

x, $f(x)$ ... coordinates in cm

The region marked in grey in the diagram above shows the wastage (i. e. the wood left over when the panel has been cut).

– Write down a formula in terms of f that can be used to calculate the area A of the region marked in grey (in cm^2).

$A =$ _____

Solution to Task 2

Fence Panels

Expected solution to the statement of the task:

This situation cannot be represented by the graph of a function because not all x-values (in the region $5 \leq x \leq 15$) correspond to exactly one y-value.

Answer key:

The point for the core competency is to be awarded if it has been justified correctly why the upper boundary cannot be represented by the graph of a function.

Expected solution to the guiding question:

$$A = 200 \cdot 10 - \int_{20}^{30} f(x)\,\mathrm{d}x$$

Answer key:

The point for the guiding question is to be awarded if the formula has been written down correctly.

Task 3

Noise

Noise can negatively affect a person's health.

Task:

The length of time a person can be exposed to a certain noise level in a day is known as the *exposure time*. This time can be modelled by the function *f*.

$f(x) = a \cdot 0.8^x$

x ... noise level in decibels (dB)
$f(x)$ ... exposure time for the noise level x in min

At a noise level of 100 dB, the exposure time is 12 min.

– Determine the parameter a.

Guiding question:

On a particular section of road, noise measurements are taken in terms of the number of vehicles per hour. On the basis of these noise measurements, the so-called *average noise level* is calculated (see table below).

number of vehicles per hour	average noise level in dB
10	52
60	58
80	61

– Show by calculation that the relationship between the number of vehicles per hour and the average noise level in dB is <u>not</u> linear.

Solution to Task 3

Noise

Expected solution to the statement of the task:

$f(100) = 12$ *or* $12 = a \cdot 0.8^{100}$

$a = 5.89... \cdot 10^{10}$

Answer key:

The point for the core competency is to be awarded if the parameter a has been determined correctly.

Expected solution to the guiding question:

$$k_1 = \frac{58 - 52}{60 - 10} = 0.12$$

$$k_2 = \frac{61 - 58}{80 - 60} = 0.15$$

$$k_3 = \frac{61 - 52}{80 - 10} = 0.128...$$

As the quotients are not the same, the relationship is not linear.
For the award of the point, the comparison of two difference quotients is sufficient.

Answer key:

The point for the guiding question is to be awarded if it has been shown by calculation that there is no linear relationship.

Task 4

Rates of Change

The quadratic function f is given by $f(x) = -x^2 + 2 \cdot x + 3$.

Task:

The average rate of change of f in the interval $[1, a]$ ($a \in \mathbb{R}$, $a > 1$) is –3.

– Determine a.

Guiding question:

– Determine the x-value x_0 for which the instantaneous rate of change of f at x_0 is equal to –3.

Solution to Task 4

Rates of Change

Expected solution to the statement of the task:

$\frac{f(a) - f(1)}{a - 1} = -3$ *or* $\frac{-a^2 + 2 \cdot a + 3 - 4}{a - 1} = -3$

Calculation using technology:

$a = 4$

Answer key:

The point for the core competency is to be awarded if a has been determined correctly.

Expected solution to the guiding question:

$f'(x) = -2 \cdot x + 2$
$f'(x_0) = -3 \quad \Rightarrow \quad x_0 = 2.5$

Answer key:

The point for the guiding question is to be awarded if x_0 has been determined correctly.

Task 5

Wheel of Fortune

A wheel of fortune is divided into multiple sectors. The probability of the spinner landing in sector *G* is *p* for each spin. The results of the individual spins are independent of each other.

Task:

Marco spins the wheel of fortune *n* times.

– Write down a formula in terms of *p* that can be used to calculate the probability shown below.

P(“the spinner lands in sector *G* at least once”) = _____

Guiding question:

Nina spins the wheel of fortune multiple times.

– State the event *E* in the given context whose probability can be calculated with the expression shown below.
$P(E) = \binom{10}{8} \cdot p^8 \cdot (1-p)^2 + \binom{10}{9} \cdot p^9 \cdot (1-p) + p^{10}$

Solution to Task 5

Wheel of Fortune

Expected solution to the statement of the task:

P(“the spinner lands in sector G at least once”) = $1 - (1 - p)^n$

Answer key:

The point for the core competency is to be awarded if the formula has been written down correctly.

Expected solution to the guiding question:

E ... “the spinner lands in sector G at least 8 times in 10 spins”

Answer key:

The point for the guiding question is to be awarded if the event has been stated correctly.