

Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

February 2022

Mathematics

Supplementary Examination 2
Examiner's Version

Instructions for the standardised implementation of the supplementary examination

The following supplementary examination booklet contains five tasks that can be completed independently of one another. Each task comprises two sub-tasks: the “task” and the “guiding question”.

The preparation time is to be at least 30 minutes; the examination time is at most 25 minutes.

The use of the official formula booklet that has been approved by the relevant government authority for use in the standardised school-leaving examination in mathematics is allowed. Furthermore, the use of electronic devices (e.g. graphic display calculators or other appropriate technology) is allowed provided there is no possibility to communicate (e.g. via the internet, intranet, Bluetooth, mobile networks etc.) and there is no access to an individual's data stored on the device.

After the examination, all materials (tasks, extra sheets of paper etc.) from all candidates are to be collected in. The examination materials (tasks, extra sheets of paper, digital materials etc.) may only be made public after the time period allocated for the examination has passed.

Evaluation grid for the supplementary examination

The evaluation grid below may be used to assist in assessing the candidates' performances.

	Candidate 1		Candidate 2		Candidate 3		Candidate 4		Candidate 5	
Task 1										
Task 2										
Task 3										
Task 4										
Task 5										
Total										

Explanatory notes on assessment

Each task can be awarded zero, one or two points. There is one point available for each task for the demonstration of a core competency and one point available for each guiding question. A maximum of ten points can be achieved.

Assessment scale for the supplementary examination

Grade	Number of points achieved (core competencies + guiding questions)
Very good	7 – 10
Good	6
Satisfactory	5
Pass	4

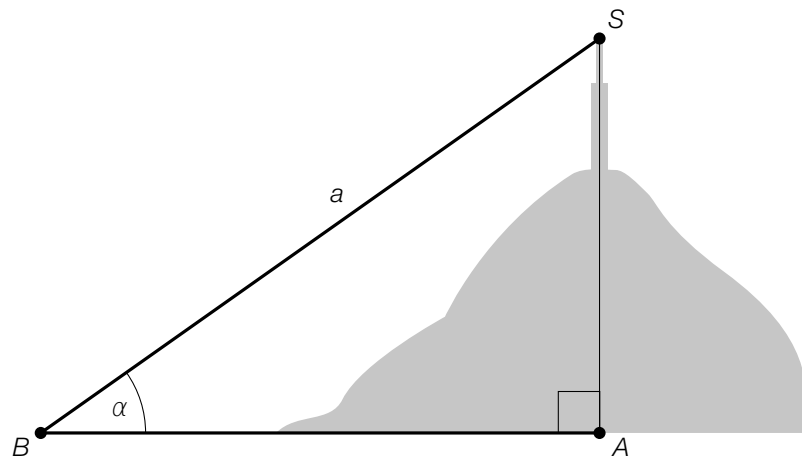
Task 1

Pyramidenkogel

The Pyramidenkogel is a mountain near Lake Wörth. At its peak stands the tallest wooden viewing tower in the world.

Task:

Bettina can see the top S of the viewing tower at an angle of α from point B on the bank of Lake Wörth (see the not-to-scale diagram below).



– Write down a formula in terms of α and a that can be used to calculate \overline{AS} .

$$\overline{AS} = \underline{\hspace{10cm}}$$

Guiding question:

There is a slide in the viewing tower. This slide is 120 m long and covers a difference in altitude of 52 m. As a simplification, it can be assumed that the gradient of the slide is constant.

– Determine the gradient as a percentage.

Solution to Task 1

Pyramidenkogel

Expected solution to the statement of the task:

$$\overline{AS} = a \cdot \sin(\alpha)$$

Answer key:

The point for the core competency is to be awarded if the formula has been written down correctly.

Expected solution to the guiding question:

$$\frac{52}{\sqrt{120^2 - 52^2}} = 0.480\dots = 48.0\dots \%$$

Answer key:

The point for the guiding question is to be awarded if the gradient has been calculated correctly. A solution for the gradient of -48% is also correct.

Task 2

Water Temperature

Water is brought to the boil in a pan.

Task:

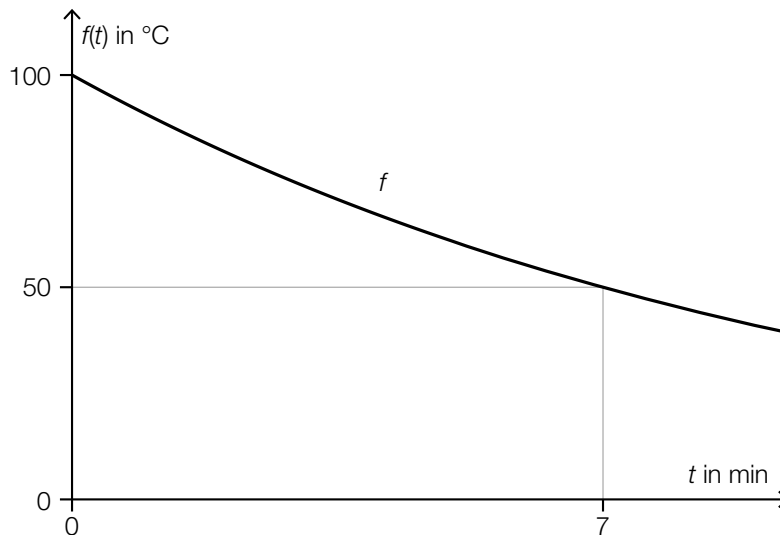
The initial temperature of the water is 22 °C.

It can be assumed that the temperature of the water increases by 8 °C per minute.

– Determine the time taken in minutes for the water to reach a temperature of 100 °C.

Guiding question:

In an experiment, the temperature of cooling water is measured. The temperature can be modelled by the exponential function f . The graph of f is shown in the diagram below.



$$f(t) = a \cdot b^t$$

t ... time in min

$f(t)$... temperature of the water at time t in °C

a, b ... parameters

– Determine the parameters a and b .

Solution to Task 2

Water Temperature

Expected solution to the statement of the task:

$$\frac{100 - 22}{8} = 9.75$$

After 9.75 min, the water has reached a temperature of 100 °C.

Answer key:

The point for the core competency is to be awarded if the time has been calculated correctly.

Expected solution to the guiding question:

$$f(0) = 100 \Rightarrow a = 100$$
$$f(7) = 50 \Rightarrow b = \sqrt[7]{\frac{50}{100}} = 0.9057\dots$$

Answer key:

The point for the guiding question is to be awarded if a and b have been calculated correctly.

Task 3

Information Plaques

In the town of Steyr, information plaques can be found on historical landmarks. The image on the right shows one such plaque.



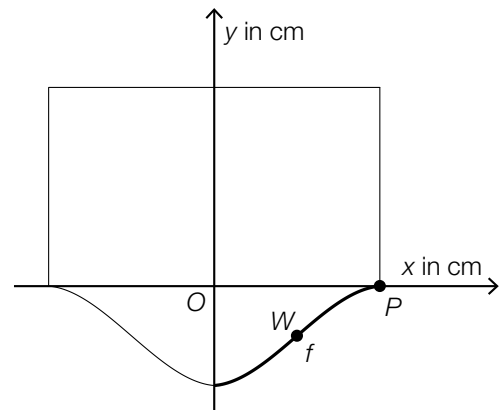
Image source: BMBWF

Task:

These plaques are to be replaced with new plaques. The diagram on the right shows the outline of one of the new plaques.

Let f be a function with:

$$f(x) = a \cdot x^3 + b \cdot x^2 + c \cdot x + d \quad \text{with } x \in [0, 25]$$

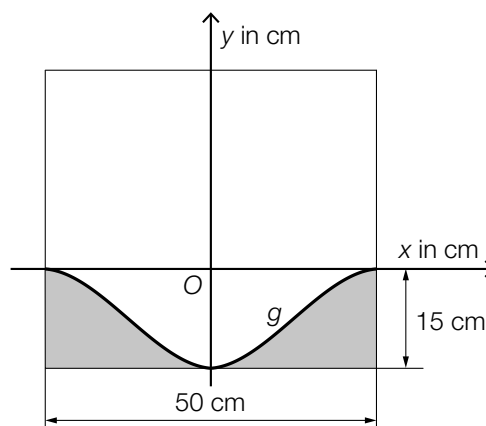


The point P has coordinates $(25, 0)$. The point of inflexion W has coordinates $(12.5, -7.5)$. The gradient of the tangent at the point of inflexion W is 0.8625 .

– Using the information about the points P and W , write down a system of equations that can be used to calculate the coefficients a , b , c and d .

Guiding question:

A new plaque is to be cut out of a rectangular metal sheet. This plaque is symmetrical about the y -axis (see diagram below).



– Write down a formula in terms of g that can be used to calculate the area A of the region shaded in grey.

$A =$ _____

Solution to Task 3

Information Plaques

Expected solution to the statement of the task:

$$f'(x) = 3 \cdot a \cdot x^2 + 2 \cdot b \cdot x + c$$

$$f''(x) = 6 \cdot a \cdot x + 2 \cdot b$$

I: $f(25) = 0$

II: $f(12.5) = -7.5$

III: $f'(12.5) = 0.8625$

IV: $f''(12.5) = 0$

or:

I: $a \cdot 25^3 + b \cdot 25^2 + c \cdot 25 + d = 0$

II: $a \cdot 12.5^3 + b \cdot 12.5^2 + c \cdot 12.5 + d = -7.5$

III: $3 \cdot a \cdot 12.5^2 + 2 \cdot b \cdot 12.5 + c = 0.8625$

IV: $6 \cdot a \cdot 12.5 + 2 \cdot b = 0$

Answer key:

The point for the core competency is to be awarded if the system of equations has been written down correctly.

Expected solution to the guiding question:

$$A = 2 \cdot \int_0^{25} (g(x) + 15) dx \quad \text{or} \quad A = 50 \cdot 15 + 2 \cdot \int_0^{25} g(x) dx \quad \text{or} \quad A = 50 \cdot 15 + \int_{-25}^{25} g(x) dx$$

Answer key:

The point for the guiding question is to be awarded if the formula has been written down correctly.

Task 4

Body Temperature

Marie has a fever and therefore takes medicine to reduce the fever.

Marie's body temperature after taking the medicine can be modelled by the function T .

t ... time after taking the medicine in h with $0 \leq t \leq 6$

$T(t)$... Marie's body temperature at time t in °C

Task:

– Interpret the result of the calculation below in the given context. Write down the corresponding unit.

$$\frac{T(5) - T(0)}{5 - 0} = -0.4$$

Guiding question:

Theo is ill and takes medicine at 9:00.

t ... time after taking the medicine in h with $t = 0$ for 9:00

$K(t)$... Theo's body temperature at time t in °C

After taking the medicine, Theo's body temperature is recorded at hourly intervals i. e. at 10:00, 11:00 etc.

A simple model is as follows:

$$K(t + 1) = K(t) - 0.5 \cdot (K(t) - 36.5)$$

$$K(0) = 39$$

– Determine the time at which Theo's temperature is first recorded as being below 37.5 °C.

Solution to Task 4

Body Temperature

Expected solution to the statement of the task:

Marie's body temperature reduces in the first 5 hours after taking the medicine by an average of 0.4 °C per hour.

Answer key:

The point for the core competency is to be awarded if the result has been interpreted correctly with the corresponding unit in the given context.

Expected solution to the guiding question:

$$K(1) = 37.75 \text{ °C}$$

$$K(2) = 37.125 \text{ °C}$$

At time $t = 2$ (11:00), Theo's body temperature is recorded as less than 37.5 °C for the first time.

Answer key:

The point for the guiding question is to be awarded if the time has been calculated correctly.

Task 5

Long Jump

Task:

Katja participates in a long jump competition.

5 of Katja's jumps are recorded. The longest jump is 4.3 m and the shortest jump is 3.7 m.

For the 3 other jumps, she achieved the same jump length w .

The mean of the 5 long jumps is 4.06 m.

– Determine the jump length w .

Guiding question:

Chiara is training for a competition. The random variable X gives Chiara's jump length in m. The following probabilities are known:

$$P(X \leq 4.5) = 0.5$$

$$P(4.5 < X \leq 4.6) = 0.3$$

Chiara jumps 2 times. The jump lengths she achieves are assumed to be independent of each other.

– Determine the probability that neither of the two jump lengths is greater than 4.6 m.

Solution to Task 5

Long Jump

Expected solution to the statement of the task:

$$\frac{4.3 + 3.7 + 3 \cdot w}{5} = 4.06 \quad \Rightarrow \quad w = 4.1 \text{ m}$$

Answer key:

The point for the core competency is to be awarded if the jump length w has been calculated correctly.

Expected solution to the guiding question:

$$0.3^2 + 2 \cdot 0.3 \cdot 0.5 + 0.5^2 = 0.64$$

The probability is 64 %.

Answer key:

The point for the guiding question is to be awarded if the probability has been calculated correctly.