

Name:	
Class:	



Standardised Competence-Oriented
Written School-Leaving Examination

AHS

3rd May 2022

Mathematics

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Advice for Completing the Tasks

Dear candidate,

The following booklet contains Part 1 and Part 2 tasks (divided into sub-tasks). The tasks can be completed independently of one another.

Please do all of your working out solely in this booklet and on the paper provided to you. Write your name and that of your class on the cover page of the booklet in the spaces provided. Please also write your name on any separate sheets of paper used and number these pages consecutively. When responding to the instructions of each task, write the task reference (e.g. 25a1) on your sheet.

In the assessment of your work, everything that is not crossed out will be considered.

The use of the official formula booklet for this examination that has been approved by the relevant government authority is permitted. Furthermore, the use of electronic device(s) (e.g. graphic display calculators or other appropriate technology) is allowed provided there is no possibility of communicating via the internet, Bluetooth, mobile networks etc. and there is no access to your own data stored on the device.

An explanation of the task types is displayed in the examination room.

Instructions for Completing the Tasks

- Solutions must be unambiguous and clearly recognisable.
- Solutions must be given alongside their corresponding units if this has been explicitly required in the task instructions.

For tasks with open answer formats, evidence of the targeted core competency is required for the award of the point. When completing tasks with open answer formats, it is recommended that you:

- document how the solution was reached, even if electronic devices were used,
- explain any variables you have chosen yourself and give their corresponding units,
- avoid rounding prematurely,
- label diagrams or sketches.

Changing an answer for a task that requires a cross:

1. Fill in the box that contains the cross.
2. Put a cross in the box next to your new answer.

In this instance, the answer “ $5 + 5 = 9$ ” was originally chosen. The answer was later changed to be “ $2 + 2 = 4$ ”.

$1 + 1 = 3$	<input type="checkbox"/>
$2 + 2 = 4$	<input checked="" type="checkbox"/>
$3 + 3 = 5$	<input type="checkbox"/>
$4 + 4 = 4$	<input type="checkbox"/>
$5 + 5 = 9$	<input checked="" type="checkbox"/>
$6 + 6 = 10$	<input type="checkbox"/>

Selecting an item that has been filled in:

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

In this instance, the answer “ $2 + 2 = 4$ ” was filled in and then selected again.

$1 + 1 = 3$	<input type="checkbox"/>
$2 + 2 = 4$	<input checked="" type="checkbox"/>
$3 + 3 = 5$	<input type="checkbox"/>
$4 + 4 = 4$	<input checked="" type="checkbox"/>
$5 + 5 = 9$	<input type="checkbox"/>
$6 + 6 = 10$	<input type="checkbox"/>

Grading System

points awarded	grade
32–36 points	very good
27–31.5 points	good
22–26.5 points	satisfactory
17–21.5 points	pass
0–16.5 points	fail

Best-of Assessment: A best-of assessment approach will be applied to tasks 26, 27 and 28. Of these three Part 2 tasks, the task with the lowest point score will not be included in the total point score.

Good luck!

Task 1

Values of Expressions

Five expressions with $a \in \mathbb{R}$ and $a < 0$ are shown below.

Task:

Put a cross next to both expressions that always have a positive value. [2 out of 5]

$\frac{a-1}{a}$	<input type="checkbox"/>
$\frac{1-2 \cdot a}{a}$	<input type="checkbox"/>
$\frac{a}{1-a}$	<input type="checkbox"/>
$a^2 - 1$	<input type="checkbox"/>
$-a$	<input type="checkbox"/>

[0/1 p.]

Task 2

Quadratic Equation

A quadratic equation in the variable x is shown below.

$$3 \cdot x^2 + a = 2 \cdot x^2 + 6 \cdot x - 4 \quad \text{with } a \in \mathbb{R}$$

Task:

Determine all values of a for which the equation shown above has two distinct solutions in \mathbb{R} .

[0/1 p.]

Task 3

Point on a Line

Let g be a line in \mathbb{R}^3 with $g: X = \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix} + s \cdot \begin{pmatrix} -3 \\ 7 \\ 2 \end{pmatrix}$, $s \in \mathbb{R}$,

and A be a point with $A = \begin{pmatrix} 10 \\ -19 \\ a \end{pmatrix}$, $a \in \mathbb{R}$.

The point A lies on the line g .

Task:

Determine a .

$a =$ _____

[0/1 p.]

Task 4

Perpendicular Vectors

Let $\vec{v} = \begin{pmatrix} 7 \\ -3 \cdot a \end{pmatrix}$ with $a > 1$ be a vector.

Task:

Put a cross next to both vectors that are perpendicular to \vec{v} . [2 out of 5]

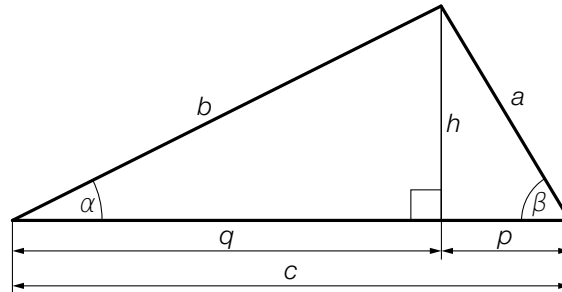
$\begin{pmatrix} -3 \cdot a \\ 7 \end{pmatrix}$	<input type="checkbox"/>
$\begin{pmatrix} 1.5 \cdot a \\ 3.5 \end{pmatrix}$	<input type="checkbox"/>
$\begin{pmatrix} -6 \cdot a^2 \\ -14 \cdot a \end{pmatrix}$	<input type="checkbox"/>
$\begin{pmatrix} 1.5 \\ 3.5 \cdot a \end{pmatrix}$	<input type="checkbox"/>
$\begin{pmatrix} 9 \cdot a^2 \\ -21 \cdot a \end{pmatrix}$	<input type="checkbox"/>

[0/1 p.]

Task 5

Calculations for a Triangle

The diagram below shows a triangle that has been divided into two right-angled triangles by the altitude h .



Task:

Match each of the four lengths to the corresponding expression that could be used to calculate that length from A to F.

a	
b	
c	
h	

A	$b \cdot \cos(\alpha)$
B	$\frac{p}{\cos(\beta)}$
C	$\frac{h}{\tan(\beta)}$
D	$q \cdot \tan(\alpha)$
E	$q + \frac{h}{\tan(\beta)}$
F	$\frac{q}{\cos(\alpha)}$

[0/1/2/1 p.]

Task 6

Intervals

Six different intervals are shown below.

For all angles α in one of these intervals, the following statements hold: $\sin(\alpha) \geq 0$ and $\sin(\alpha) \neq 1$.

Task:

Put a cross next to the correct interval. [1 out of 6]

$[270^\circ, 360^\circ)$	<input type="checkbox"/>
$[90^\circ, 180^\circ]$	<input type="checkbox"/>
$(0^\circ, 180^\circ)$	<input type="checkbox"/>
$[0^\circ, 90^\circ)$	<input type="checkbox"/>
$(90^\circ, 270^\circ]$	<input type="checkbox"/>
$[180^\circ, 270^\circ]$	<input type="checkbox"/>

[0/1 p.]

Task 7

Properties of Real Functions

Properties of a real function f are shown below.

Task:

Match each of the four properties to the corresponding statement from A to F.

For all $x \in \mathbb{R}$, $f(x) = f(-x)$.	
For a particular $m \in \mathbb{R}^+$, $f(x + m) = f(x)$ for all $x \in \mathbb{R}$.	
For all $x_1, x_2 \in \mathbb{R}$ with $x_1 < x_2$, $f(x_1) > f(x_2)$.	
For all $x \in \mathbb{R}$, $f(x) \neq 0$.	

A	f is strictly monotonically increasing.
B	The graph of f is symmetrical about the vertical axis.
C	The graph of f has an asymptote.
D	f is strictly monotonically decreasing.
E	f is periodic.
F	The graph of f has no point of intersection with the x -axis.

[0/1½/1 p.]

Task 8

Linear Function

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a linear function with $f(x) = m \cdot x + c$ and $m, c \in \mathbb{R}$.

Task:

Complete the gaps in the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

For all $x \in \mathbb{R}$, the following statement holds: $\text{\textcircled{1}}$ = $\text{\textcircled{2}}$.

①	
$f(x + 1)$	<input type="checkbox"/>
$f(x + 2)$	<input type="checkbox"/>
$f(x + 1) + f(x + 1)$	<input type="checkbox"/>

②	
$f(x) + 2 \cdot m$	<input type="checkbox"/>
$f(x) + c$	<input type="checkbox"/>
$2 \cdot f(x) + 2$	<input type="checkbox"/>

[0/1 p.]

Task 9

Indirect Proportion

Six mappings with $x \in \mathbb{R}^+$ are shown below.

Task:

Put a cross next to the mapping that describes an indirectly proportional relationship. [1 out of 6]

$x \mapsto 3 - x$	<input type="checkbox"/>
$x \mapsto -\frac{x}{3}$	<input type="checkbox"/>
$x \mapsto \frac{3}{x^2}$	<input type="checkbox"/>
$x \mapsto 3 \cdot x^{-1}$	<input type="checkbox"/>
$x \mapsto 3^{-x}$	<input type="checkbox"/>
$x \mapsto x^{-3}$	<input type="checkbox"/>

[0/1 p.]

Task 10

Odd Function

For the function $f: \mathbb{R} \rightarrow \mathbb{R}$ with $f(x) = a \cdot x^n$ ($a \in \mathbb{R}, a \neq 0$) with an odd $n \in \mathbb{N}$, the table of values shown below is given.

x	-2	0	2
$f(x)$	v	0	w

In this table, $v, w \in \mathbb{R}$.

Task:

Write down the relationship between v and w as an equation.

[0/1 p.]

Task 11

Half-Life

The half-life of a particular radioactive substance is T years.

The amount of the radioactive substance that is still present after t years is given by $m(t)$ for which $m(0) > 0$.

Task:

Put a cross next to both correct equations. [2 out of 5]

$m(T) = \frac{1}{2} \cdot m(0)$	<input type="checkbox"/>
$m(2 \cdot T) = 0$	<input type="checkbox"/>
$m(3 \cdot T) = \frac{7}{8} \cdot m(0)$	<input type="checkbox"/>
$m(4 \cdot T) = \frac{1}{4} \cdot m(T)$	<input type="checkbox"/>
$m(5 \cdot T) = \frac{1}{2} \cdot m(4 \cdot T)$	<input type="checkbox"/>

[0/1 p.]

Task 12

Sounds

The functions f , g and h each describe vibrations that create sounds in terms of the time t (in seconds).

For these functions:

$$f(t) = \sin(600 \cdot t)$$

$$g(t) = \frac{5}{4} \cdot \sin(800 \cdot t)$$

$$h(t) = \frac{6}{5} \cdot \sin(500 \cdot t)$$

The volume of a sound is higher the greater the amplitude (maximum displacement) of the corresponding vibration.

The pitch of a sound is higher the greater the frequency (number of vibrations per second) of the corresponding vibration.

Task:

Complete the gaps in the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

The vibration of the sound with the highest volume is given by the function _____^①_____;
the vibration of the sound with the lowest pitch is given by the function _____^②_____.

①	
f	<input type="checkbox"/>
g	<input type="checkbox"/>
h	<input type="checkbox"/>

②	
f	<input type="checkbox"/>
g	<input type="checkbox"/>
h	<input type="checkbox"/>

[0/1½/1 p.]

Task 13

Body Weight of a Baby

The body weights of babies in the first 6 weeks of life can be approximated by the function $G: [0, 6] \rightarrow \mathbb{R}$ with $G(t) = G_0 + 190 \cdot t$.

t ... time after the birth in weeks

$G(t)$... body weight of a baby at time t in g

G_0 ... body weight of a baby at birth in g

Nora weighed 3200 g when she was born.

Task:

Using the function G , determine the relative change in Nora's weight from her birth until 6 weeks after birth as a percentage.

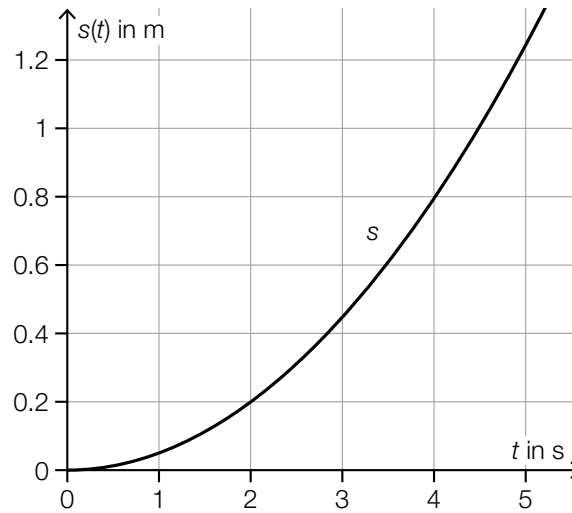
_____ %

[0/1 p.]

Task 14

Average Speed

The graph of the distance-time function s of a moving body is shown below. The time t is given in seconds and the distance $s(t)$ is given in metres.



Task:

Determine the time t_1 such that the average speed of the body in the intervals $[0, 4]$ and $[1, t_1]$ is the same.

$t_1 =$ _____ seconds

[0/1 p.]

Task 15

Differentiation Rules

Let f and g be two differentiable functions and a be a positive real number.

Task:

Put a cross next to both functions that definitely correspond to $(a^2 \cdot (f + g))'$. [2 out of 5]

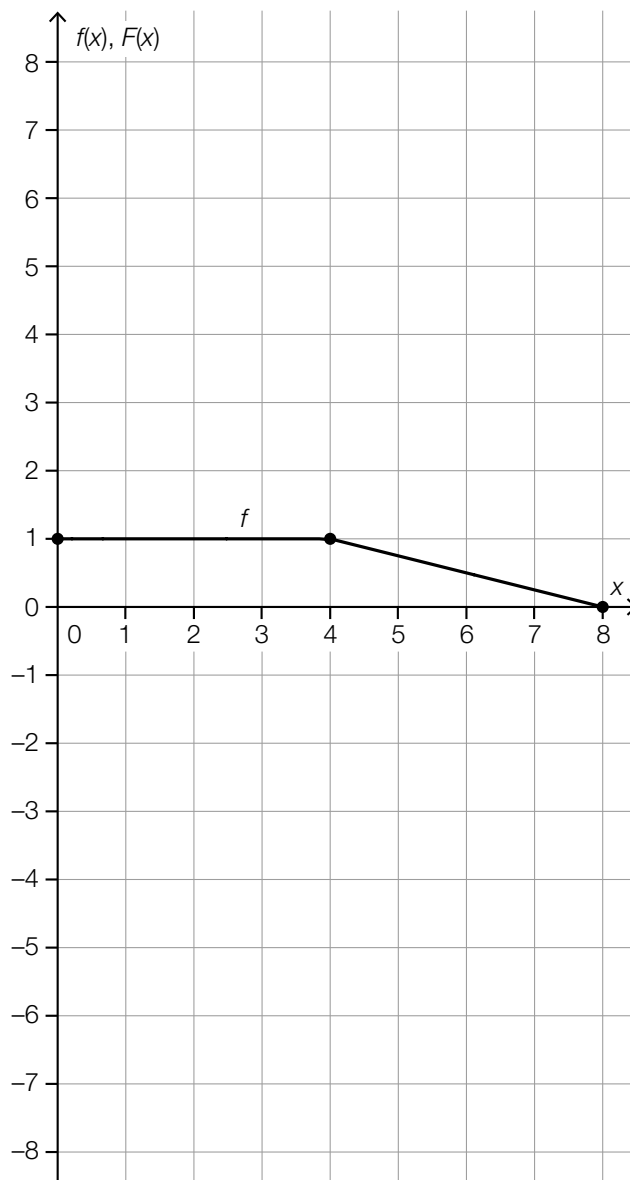
$2 \cdot a \cdot f' + 2 \cdot a \cdot g'$	<input type="checkbox"/>
$a^2 \cdot f' + a^2 \cdot g'$	<input type="checkbox"/>
$2 \cdot a \cdot (f + g)'$	<input type="checkbox"/>
$a^2 \cdot (f + g)'$	<input type="checkbox"/>
$f' + g'$	<input type="checkbox"/>

[0/1 p.]

Task 16

Antiderivative

The diagram below shows the graph of the real function $f: [0, 8] \rightarrow \mathbb{R}$, $x \mapsto f(x)$. The function F with $F(0) = 0$ is an antiderivative of f . The points shown in bold have integer coordinates.



Task:

On the diagram above, sketch the graph of F in the interval $[0, 8]$ using the values of the function $F(0)$, $F(4)$ and $F(8)$.

[0/1½/1 p.]

Task 17

Third Degree Polynomial Function

The minimum $T = (-1, 2)$ and the maximum $H = (1, 4)$ of the graph of a third degree polynomial function f are known.

Task:

Put a cross next to both true statements. [2 out of 5]

The function f is strictly monotonically decreasing in the interval $(1, 3)$.	<input type="checkbox"/>
The function f changes monotonicity in the interval $(-1, 1)$.	<input type="checkbox"/>
The function f is strictly monotonically decreasing in the interval $(-3, 1)$.	<input type="checkbox"/>
The function f is always concave down in the interval $(-1, 1)$.	<input type="checkbox"/>
The function f changes monotonicity in the interval $(0, 2)$.	<input type="checkbox"/>

[0/1 p.]

Task 18

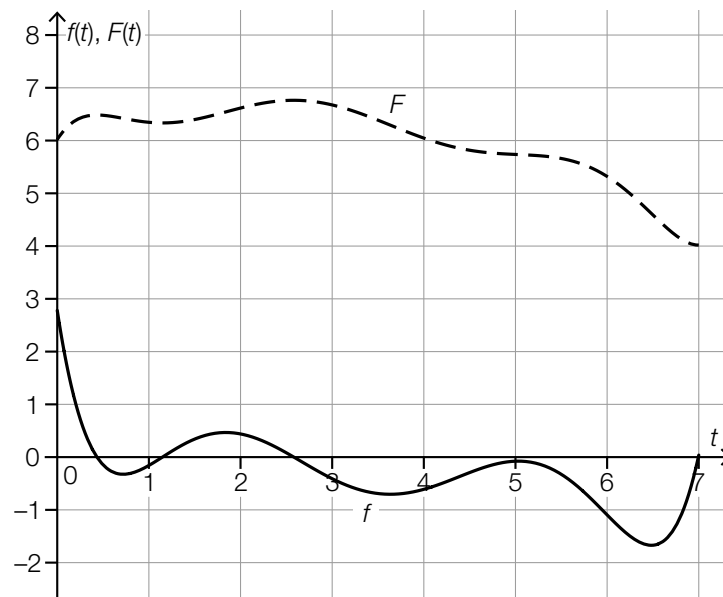
Garden Pond

The function f models the instantaneous rate of change of the water level of a particular garden pond in terms of the time t .

t ... time in days

$f(t)$... instantaneous rate of change of the water level at time t in mm/day

The function F is an antiderivative of f .



Task:

Complete the gaps in the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

The integral $\int_0^7 f(t) dt$ has the value ① , and gives the ② of the water level in the time interval $[0, 7]$.

①	
2	<input type="checkbox"/>
-2	<input type="checkbox"/>
0	<input type="checkbox"/>

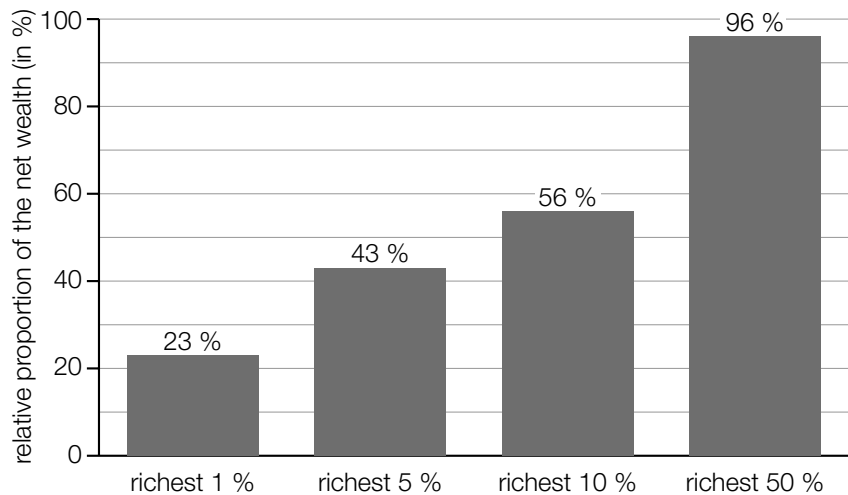
②	
average rate of change	<input type="checkbox"/>
relative change	<input type="checkbox"/>
absolute change	<input type="checkbox"/>

[0/½/1 p.]

Task 19

Wealth Distribution

The diagram below shows the relative proportion of the Austrian net wealth held by the richest members of the population in the year 2017.



Data sources: <https://awblog.at/vermoegensverteilung-oesterreich/> [04/05/2020],
<https://www.vienna.at/vermoegensverteilung-in-oesterreich-arm-und-reich-wird-meist-ererbt/6468838> [30/05/2020].

Task:

Complete the gaps in the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

In the year 2017, the ① of the population held in total ② of the Austrian net wealth.

①	
poorest 50 %	<input type="checkbox"/>
richest 6 %	<input type="checkbox"/>
poorest 95 %	<input type="checkbox"/>

②	
43 %	<input type="checkbox"/>
more than 60 %	<input type="checkbox"/>
4 %	<input type="checkbox"/>

[0/1 p.]

Task 20

Average Income

Of all the employees of a particular company, 40 % work in sales and 52 % in production. The remaining employees work in administration.

The table below shows information about the average net annual incomes in the year 2018.

	average net annual income in the year 2018 per person (in euros)
sales	26376
production	28511
administration	23427

Task:

For this company, determine the average net annual income per person in the year 2018.

[0/1 p.]

Task 21

Newborns

The table below shows the number of newborns in Austria with respect to their birth weights (weight immediately after birth) for the year 2018.

birth weight	number of newborns
less than 2 500 g	5 282
at least 2 500 g and less than 3 500 g	47 152
at least 3 500 g	32 370

Data source: https://www.statistik.at/wcm/idc/idcplg?IdcService=GET_PDF_FILE&RevisionSelectionMethod=LatestReleased&dDocName=110630 [10/04/2020].

If a newborn weighs less than 2 500 g, they are classified as “underweight”.

Task:

For the year 2018, determine the relative proportion of newborns in Austria who were classified as “underweight”.

[0/1 p.]

Task 22

Sports Competition

20 people participated in a sports competition. These people were divided into groups.

Task:

Interpret $\binom{20}{4} = 4845$ in the given context.

[0/1 p.]

Task 23

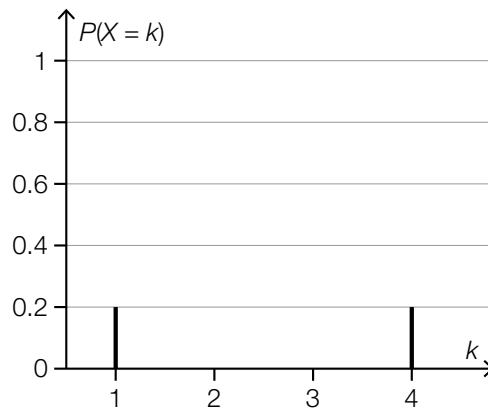
Probability Distribution of a Random Variable

Let X be a random variable that can only take the values 1, 2, 3, or 4.

It is known that $P(X = 2)$ is twice as large as $P(X = 1)$.

Task:

In the diagram below, draw the missing values $P(X = 2)$ and $P(X = 3)$ in the probability distribution of X .



[0/1 p.]

Task 24

Binomially Distributed Random Variable

A particular random experiment either results in “success” or “failure”. This random experiment is conducted 30 times. The binomially distributed random variable X gives the number of times “success” occurs. The expectation value is $E(X) = 12$.

Task:

Determine the probability $P(18 \leq X \leq 20)$.

$P(18 \leq X \leq 20) =$ _____

[0/1 p.]

Task 25 (Part 2)

Bicycle Ride

Task:

- a) Bettina goes on a 2-hour bicycle ride. Her velocity can be approximated by the function v .

$$v(t) = -0.08 \cdot t^2 + 16 \quad \text{with } 0 \leq t \leq 2$$

t ... time in h with $t = 0$ at the start of the bicycle ride

$v(t)$... velocity at time t in km/h

- 1) Determine the amount of time Bettina needs to complete the first 10 km of this bicycle ride. [0/1 p.]
- 2) Determine her acceleration at time $t = 1$. Write down the corresponding unit. [0/½/1 p.]

- b) The recommended tyre pressure for a bicycle tyre decreases as the width of the tyre increases. The recommended tyre pressure from 2 bar to 9 bar can be approximated by the function p .

$$p(x) = 19.1 \cdot e^{-0.0376 \cdot x}$$

x ... width of the tyre in mm

$p(x)$... recommended tyre pressure for a width of x in bar

- 1) Determine the largest possible interval for the width of a tyre whose recommended tyre pressure is between 2 bar and 9 bar. [0/1 p.]
- 2) Interpret the result of the calculation shown below in the given context and write down the corresponding unit.
 $p(30) - p(20) \approx -2.8$ [0/1 p.]

Task 26 (Part 2, Best-of Assessment)

Biathlon

Biathlon is a type of winter sport that combines cross-country skiing and shooting.

In a particular competition, three 2 500 m rounds must be completed.

The rules are as follows:

- After each of the first and second rounds, there is a shooting round in which five shots are taken.
- For every shot that misses the target, a 150 m penalty round must be completed, which leads to a loss of time.

Source: <https://www.sport1.de/wintersport/biathlon/2018/11/biathlon-im-ueberblick-regeln-disziplinen-wissenswertes> [15/04/2021].

Task:

a) Lisa completes the three rounds with the following average speeds (v_1 , v_2 , v_3 in m/s):

- v_1 for the first round
- v_2 for the second round
- v_3 for the third round

For each shooting event, Lisa needs a time of t^* (t^* in s).

After the first completed round, she doesn't miss any shots in the shooting round.

After the second completed round, she misses exactly 2 shots in the shooting round.

She completes the 2 penalty rounds with an average speed of v_s (v_s in m/s).

The time b (b in s) is the time Lisa needs to complete all rounds, including penalty rounds, and the shooting rounds.

1) Using v_1 , v_2 , v_3 , t^* and v_s , write down a formula that can be used to calculate b .

$$b = \underline{\hspace{15em}}$$

[0/1 p.]

- b) Hanna's velocity in the first round can be modelled by the function $v: [0, 440] \rightarrow \mathbb{R}, t \mapsto v(t)$ (t in s, $v(t)$ in m/s).

1) Interpret $\frac{1}{T} \cdot \int_0^T v(t) dt$ with $T \in (0 \text{ s}, 440 \text{ s}]$ in the given context. [0/1 p.]

There are exactly two times $t_1, t_2 \in (0 \text{ s}, 440 \text{ s})$ with $t_1 < t_2$ for which:

$$v'(t_1) = 0 \text{ and } v''(t_1) < 0$$

$$v'(t_2) = 0 \text{ and } v''(t_2) < 0$$

- 2) Complete the gaps in the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

The times t_1 and t_2 are ① of the function v , and the value of $\frac{v(t_2) - v(t_1)}{t_2 - t_1}$ corresponds to the ② in the time interval $[t_1, t_2]$. [0/1/2/1 p.]

①	
local minima	<input type="checkbox"/>
local maxima	<input type="checkbox"/>
points of inflexion	<input type="checkbox"/>

②	
average speed	<input type="checkbox"/>
distance covered	<input type="checkbox"/>
average acceleration	<input type="checkbox"/>

- c) The random variable X gives the number of shots Daria shoots successfully and is assumed to be binomially distributed. For each of the 5 shots, p is the probability of success.

1) Using p , write down a formula that can be used to calculate the probability shown below.

$$P(X \geq 4) = \underline{\hspace{10cm}}$$

[0/1 p.]

Task 27 (Part 2, Best-of Assessment)

Global Population

The table below shows estimates of the global population (in the middle of each year) for certain years.

year	global population in billions
1850	1.260
1900	1.650
1950	2.536
1960	3.030
1970	3.700
1990	5.327
2000	6.140
2010	6.957
2020	7.790

Data sources: <https://de.statista.com/statistik/daten/studie/1694/umfrage/entwicklung-der-weltbevoelkerungszahl/>,
https://www.statistik.at/web_de/statistiken/menschen_und_gesellschaft/bevoelkerung/internationale_uebersich/036446.html
[17/05/2020].

Task:

- a) In the time period from 1850 to 1950, the global population roughly doubled. Assume that for this time period, the global population increased by the same percentage each year.
- 1) Determine this percentage. [0/1 p.]
- b) From 1970, the development of the global population can be approximated by a linear function f .
- 1) Using the values for the global population in the years 1970 and 2000, write down an equation of the function f in terms of the time t (t in years with $t = 0$ for the year 1970, $f(t)$ in billions). [0/1 p.]
 - 2) Determine the percentage by which the value for 2020 calculated using the function f differs from the value for 2020 given in the table above. [0/1 p.]

- c) In another model, the development of the global population from 1970 is modelled by the function g .

$$g(t) = 3.7 \cdot e^{-0.0001 \cdot t^2 + 0.02 \cdot t}$$

t ... time from 1970 in years

$g(t)$... global population at time t in billions

According to this model, the global population will first increase before it begins to decrease.

- 1) Using the function g , determine the maximum value of the global population and the calendar year in which this value should occur according to the model.

maximum global population: around _____ billion

calendar year: _____

[0/½/1 p.]

Task 28 (Part 2, Best-of Assessment)

Vitamin C

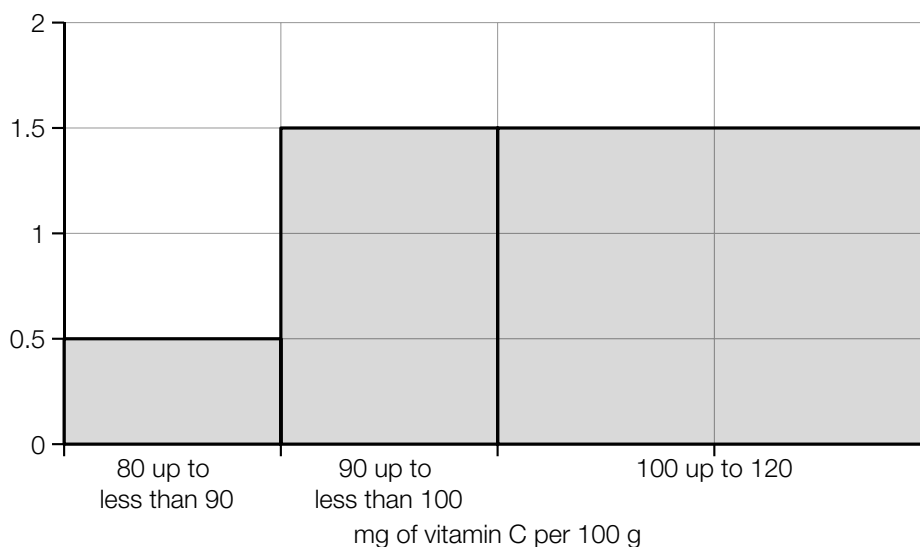
Vitamin C serves many important purposes in a human body.

Task:

- a) Broccoli contains on average 100 mg of vitamin C per 100 g.

A random sample of 50 portions of fresh broccoli is selected from a vegetable wholesaler. For each portion, the vitamin C content per 100 g is measured.

The area of a rectangle in the histogram shown below corresponds to the absolute frequency of the portions in this sample in the given region.



- 1) Determine the number of portions in the random sample that contain 100 mg to 120 mg of vitamin C per 100 g. *[0/1 p.]*

From the random sample, 3 portions are selected without replacement.

- 2) Determine the probability that at most 2 of these portions contain 100 mg to 120 mg of vitamin C per 100 g. *[0/1 p.]*

- b) A drinks manufacturer would like to fill bottles with fruit juice so that every bottle contains 100 mg of vitamin C.

The following juices are available:

- pear juice with 20 mg of vitamin C per 100 ml
- orange juice with 35 mg of vitamin C per 100 ml
- mixtures of these two juices

Emine claims that the vitamin C content of 100 mg cannot be reached for bottles that have a capacity of 250 ml.

- 1) Justify why Emine's claim is correct.

[0/1 p.]

The available fruit juices are mixed so that 350 ml of juice contains exactly 100 mg of vitamin C.

- 2) Determine the number of millilitres of pear juice and the number of millilitres of orange juice that must be combined to create this mixture.

[0/1 p.]

