Name:	
Class:	

Standardised Competence-Oriented
Written School-Leaving Examination

AHS

20th September 2022

Mathematics

Bundesministerium
Bildung, Wissenschaft
und Forschung

Advice for Completing the Tasks

Dear candidate,

The following booklet contains Part 1 and Part 2 tasks (divided into sub-tasks). The tasks can be completed independently of one another.

Please do all of your working out solely in this booklet and on the paper provided to you. Write your name and that of your class on the cover page of the booklet in the spaces provided. Please also write your name on any separate sheets of paper used and number these pages consecutively. When responding to the instructions of each task, write the task reference (e.g. 25a1) on your sheet.

In the assessment of your work, everything that is not crossed out will be considered.

The use of the official formula booklet for this examination that has been approved by the relevant government authority is permitted. Furthermore, the use of electronic device(s) (e.g. graphic display calculators or other appropriate technology) is allowed provided there is no possibility of communicating via the internet, Bluetooth, mobile networks etc. and there is no access to your own data stored on the device.

An explanation of the task types is displayed in the examination room.

Instructions for Completing the Tasks

– Solutions must be unambiguous and clearly recognisable.
– Solutions must be given alongside their corresponding units if this has been explicitly required in the task instructions.

For tasks with open answer formats, evidence of the targeted core competency is required for the award of the point. When completing tasks with open answer formats, it is recommended that you:

– document how the solution was reached, even if electronic devices were used,
– explain any variables you have chosen yourself and give their corresponding units,
– avoid rounding prematurely,
– label diagrams or sketches.

Changing an answer for a task that requires a cross:

1. Fill in the box that contains the cross.
2. Put a cross in the box next to your new answer.

In this instance, the answer “5 + 5 = 9” was originally chosen. The answer was later changed to be “2 + 2 = 4”.

1 + 1 = 3	☐
2 + 2 = 4	☒
3 + 3 = 5	☐
4 + 4 = 4	☐
5 + 5 = 9	■
6 + 6 = 10	☐

Selecting an item that has been filled in:

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

In this instance, the answer “2 + 2 = 4” was filled in and then selected again.

1 + 1 = 3	☐
2 + 2 = 4	■
3 + 3 = 5	☐
4 + 4 = 4	■
5 + 5 = 9	☐
6 + 6 = 10	☐

Grading System

points awarded	grade
32–36 points	very good
27–31.5 points	good
22–26.5 points	satisfactory
17–21.5 points	pass
0–16.5 points	fail

Best-of Assessment: A best-of assessment approach will be applied to tasks 26, 27 and 28. Of these three Part 2 tasks, the task with the lowest point score will not be included in the total point score.

Good luck!

Task 1

Sets of Numbers

Statements about sets of numbers are shown below.

Task:

Put a cross next to each of the two correct statements. *[2 out of 5]*

The set of integers is a subset of the set of natural numbers.	☐
The set of rational numbers contains all integers.	☐
The set of rational numbers contains all real numbers.	☐
The set of complex numbers is a subset of the set of real numbers.	☐
All irrational numbers are contained within the set of real numbers.	☐

[0/1 p.]

Task 2

Museum Visits

The entrance fees for a particular museum are determined as follows:
The entrance fee for an adult is x euros. For students, this entrance fee is reduced by p %.
Children and youths pay nothing to enter.

On a particular weekend, E people pay the entrance fee for adults and S people pay the entrance fee for students. In addition, K children and J youths visit the museum on this weekend.
The total income of the museum from entrance fees on this weekend is given by G.

Task:

Write down a formula that can be used to calculate G.

G = ____

[0/1 p.]

Task 3

Changing Schools

At a particular academic secondary school (AHS), k pupils decide to attend the upper secondary school at this school at the end of the 8th class. All of the remaining m pupils decide to change to a college for higher vocational education (BHS).
The following statements hold:

- A third of the pupils of this 8th class change to a BHS.
- The number of pupils who will attend the upper secondary school at this school is 47 greater than the number of pupils who will change to a BHS.

Task:

Put a cross next to each of the two correct equations. *[2 out of 5]*

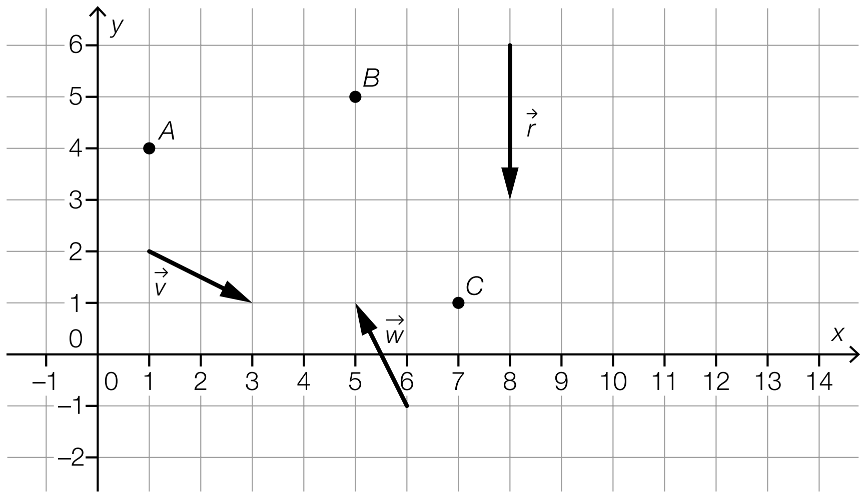
$k + m = 3 \cdot m$	☐
$k = 2 \cdot m - 47$	☐
$m = k - 47$	☐
$k = 3 \cdot m$	☐
$3 \cdot k - m = 47$	☐

[0/1 p.]

Task 4

Points and Vectors

Three points A, B and C as well as three vectors $\vec{r}$, $\vec{v}$ and $\vec{w}$ are shown in the coordinate system below.
The points have integer coordinates, and the vectors have integer components.



Task:

Put a cross next to each of the two correct statements. *[2 out of 5]*

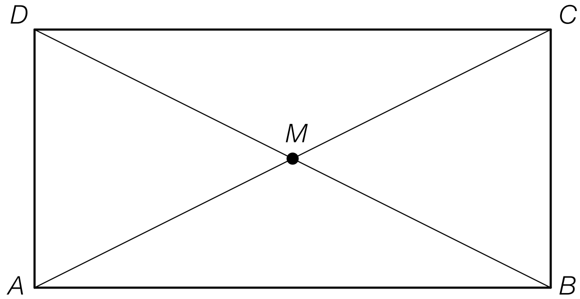
$A = B + t \cdot \vec{r}$ for a $t \in \mathbb{R}$	☐
$B = C + t \cdot \vec{v}$ for a $t \in \mathbb{R}$	☐
$C = B + t \cdot \vec{w}$ for a $t \in \mathbb{R}$	☐
$B = A + t \cdot \vec{w}$ for a $t \in \mathbb{R}$	☐
$C = A + t \cdot \vec{v}$ for a $t \in \mathbb{R}$	☐

[0/1 p.]

Task 5

Vectors in a Rectangle

A rectangle with vertices A, B, C and D is shown below. The point of intersection of the two diagonals is given by M.



Task:

Put a cross next to each of the two correct statements. *[2 out of 5]*

$\overrightarrow{AD} = \frac{1}{2} \cdot \overrightarrow{AC} + \frac{1}{2} \cdot \overrightarrow{BD}$	☐
$\overrightarrow{MA} = \frac{1}{2} \cdot \overrightarrow{CM}$	☐
$\frac{3}{5} \cdot \overrightarrow{CD} = -\frac{2}{5} \cdot \overrightarrow{AB}$	☐
$\overrightarrow{DC} = \overrightarrow{BD} - \overrightarrow{AD}$	☐
$\frac{1}{2} \cdot \overrightarrow{AD} = -\frac{1}{2} \cdot \overrightarrow{CB}$	☐

[0/1 p.]

Task 6

Perpendicular Lines

The vector equation of the line g is given by:

$$g\colon X = \begin{pmatrix} -2 \\ 0 \\ 7 \end{pmatrix} + s \cdot \begin{pmatrix} 4 \\ -4 \\ 2 \end{pmatrix} \text{ with } s \in \mathbb{R}$$

For a line n, the following statements hold:

- n is perpendicular to g.
- n intersects with g at the point $P = (2, -4, 9)$.

Task:

Write down a vector equation of one such line n.

$n\colon X =$ ____

[0/1 p.]

Task 7

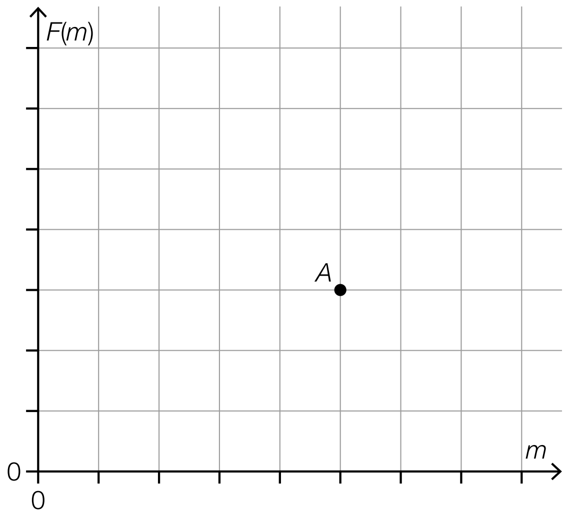
Centripetal Force

When a body moves along a circular path of radius r at constant velocity v, the absolute value of the centripetal force F is a function of the mass m of this body.

The following equation holds: $F(m) = \frac{m \cdot v^2}{r}$

Task:

On the diagram below, sketch the graph of F so that it goes through the point A.



[0/1 p.]

Task 8

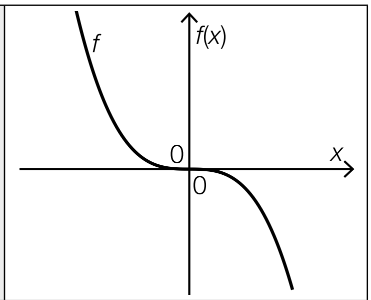
Graphs of Functions

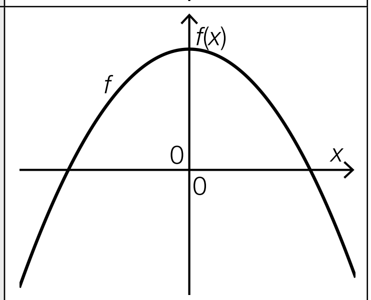
Four types of functions as well as characteristic sections of six graphs of functions are shown below.

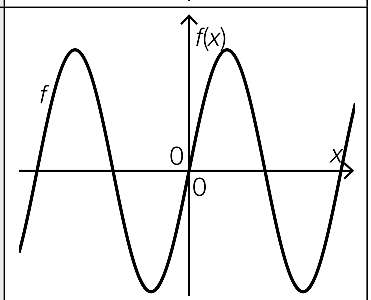
Task:

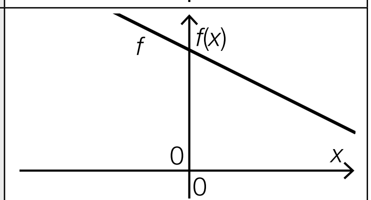
Match each of the four types of functions to its corresponding graph from A to F.

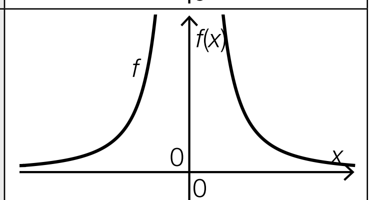
exponential function	
linear function	
2nd degree polynomial function	
sine function	

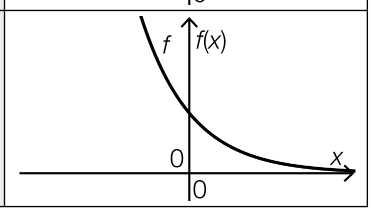
A


B


C


D


E


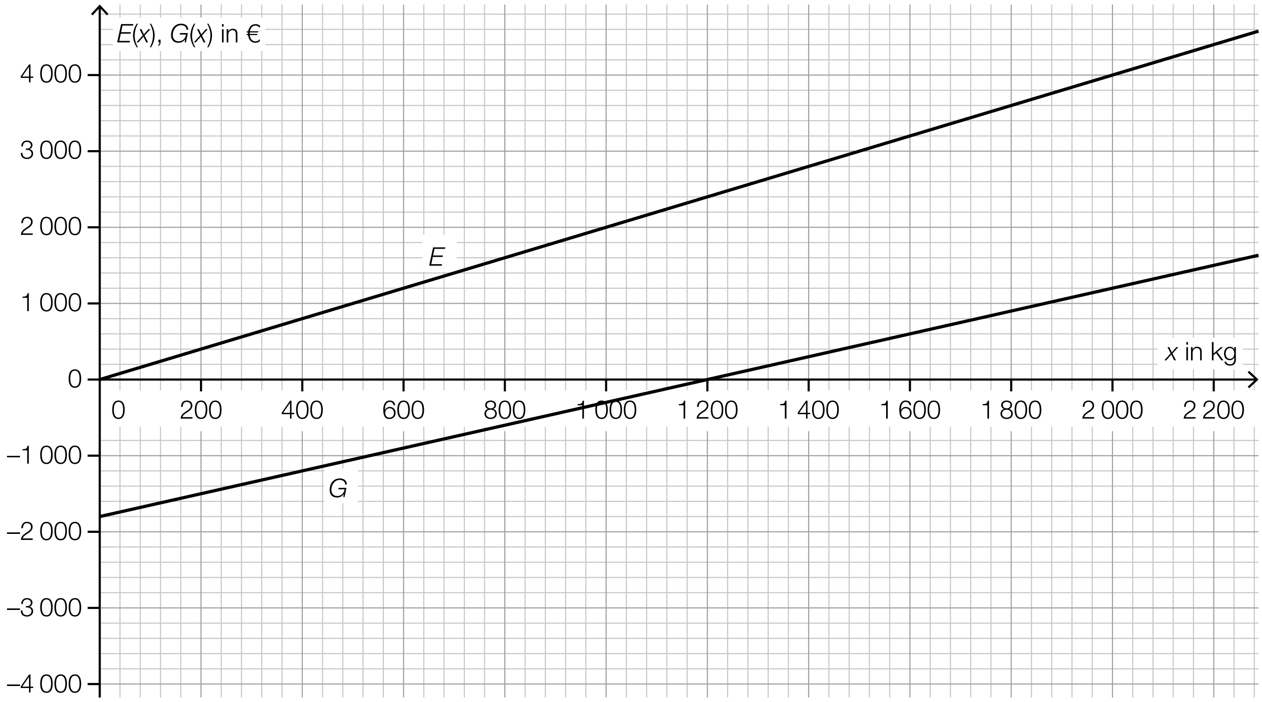
F


[0 / ½ / 1 p.]

Task 9

Revenue and Profit

The diagram below shows the graph of the linear revenue function $E: x \mapsto E(x)$ and the graph of the linear profit function $G: x \mapsto G(x)$ (x in kg, $E(x)$ and $G(x)$ in €).



Task:

Write down the sales price and the fixed costs.

sales price: ____________________ €/kg

fixed costs: € ____________________

[0/½/1 p.]

Task 10

Filling Machines

If four equally fast filling machines are used simultaneously, they require 24 minutes to fill 6000 bottles of mineral water.

The function f assigns a number n of such simultaneously working filling machines to the time $f(n)$ required to fill 6000 bottles ($n \in \mathbb{N}\setminus\{0\}$ and $f(n)$ in minutes).

Task:

Write down an equation of the function f.

$f(n) =$ ____

[0/1 p.]

Task 11

Flu Infections

On the evening of the 10th February 2019, 2000 people were infected with flu in a particular country; on the evening of the 21st February 2019, 4000 people were infected. It can be assumed that the number of people infected with flu rose by the same percentage each day in this country in February 2019.

Task:

Determine this percentage.

[0/1 p.]

Task 12

Properties of a Sine Function

Let $f\colon \mathbb{R} \to \mathbb{R}$ be a function with $f(x) = a \cdot \sin(b \cdot x)$ with $a, b \in \mathbb{R}^+$.

Task:

Put a cross next to each of the two statements that are true about the function f. *[2 out of 5]*

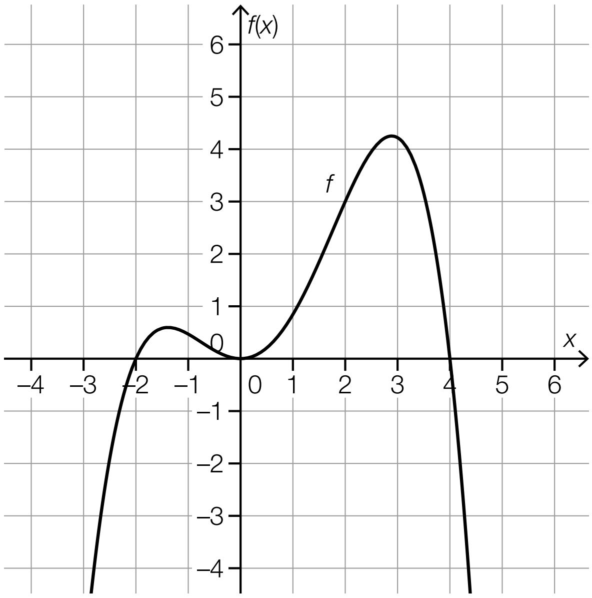
The greater the value of b, the greater the length of the (shortest) period is.	☐
The smaller the value of a, the greater the length of the (shortest) period is.	☐
The smaller the value of a, the smaller the number of zeros in the interval $[0, 2 \cdot \pi]$ is.	☐
The greater the value of a, the greater the difference between the greatest and lowest value of the function is.	☐
The greater the value of b, the smaller the distance between two consecutive zeros is.	☐

[0/1 p.]

Task 13

Relative Change of a Polynomial Function

The graph of the polynomial function *f* is shown below.



Task:

Determine the relative change of *f* in the interval [2, 4].

[0/1 p.]

Task 14

Decline of a Population

The number $f(t)$ of individuals in a population during an observation period of 100 weeks is modelled by a function f. The time t is given in weeks.

Task:

Put a cross next to the statement that correctly describes the relationship $\frac{f(100) - f(0)}{100} = -35$ in the given context. *[1 out of 6]*

The number of individuals reduced by 35 per week during the observation period.	☐
At the beginning of the observation period, there were 35 % more individuals than at the end of this time period.	☐
The number of individuals reduced by an average of 35 per week during the observation period.	☐
The number of individuals reduced to 35 % of the original population during the observation period.	☐
The number of individuals reduced by 35 % per week during the observation period.	☐
The number of individuals reduced by a total of 35 during the observation period.	☐

[0/1 p.]

Task 15

First Derivative

Let $f\colon \mathbb{R} \to \mathbb{R}, x \mapsto f(x)$ be a differentiable function.
The following statement holds: $f'(0) = 2$

Let $g\colon \mathbb{R} \to \mathbb{R}$ be a function with $g(x) = a \cdot f(k \cdot x)$ for two numbers $a, k \in \mathbb{R}$.

Task:

Write down a formula that can be used to calculate $g'(0)$ in terms of a and k.

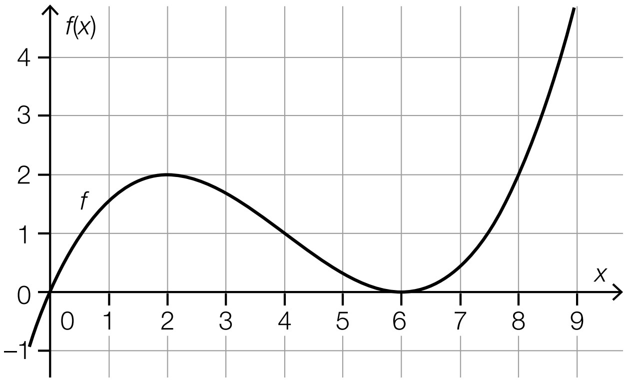
$g'(0) =$ ____

[0/1 p.]

Task 16

Derivative and Antiderivative

The diagram below shows the graph of the 3rd degree polynomial function f. All local maxima and minima and the point of inflexion of f have integer coordinates.



Task:

Complete the gaps in the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

The graph of the 1st derivative of f ______①______ and the graphs of all antiderivatives of f ______②______.

①	
crosses the x-axis when $x = 4$	☐
is strictly monotonically decreasing in the interval $(-\infty, 4)$	☐
is concave down in the interval $(-\infty, 4)$	☐

②	
have a point of inflexion with a horizontal tangent when $x = 6$	☐
cross the x-axis when $x = 6$	☐
are strictly monotonically decreasing in the interval $(2, 6)$	☐

[0/½/1 p.]

Task 17

Derivative of a Third-Degree Polynomial Function

A 3rd degree polynomial function f has a local maximum at $x_1 = -2$ and a local minimum at $x_2 = 2$.
The function has the 1st derivative f'.

Task:

Put a cross next to each of the two correct statements. *[2 out of 5]*

f' is positive over the whole interval $(-2, 2)$.	☐
f' has the same value at x_1 as at x_2.	☐
f' is negative over the whole interval $(-3, -2)$.	☐
f' has a positive value at $x = 4$.	☐
f' has the value 0 at $x = 0$.	☐

[0/1 p.]

Task 18

Fungal Spores

Fungi reproduce using spores.

In an experiment, the spores of a particular fungus cover an area of 5 μm² at time $t = 0$.

The function f models the speed at which the covered area increases in terms of the time t.
t… time in h
$f(t)$… speed at which the covered area increases at time t in μm²/h

Task:

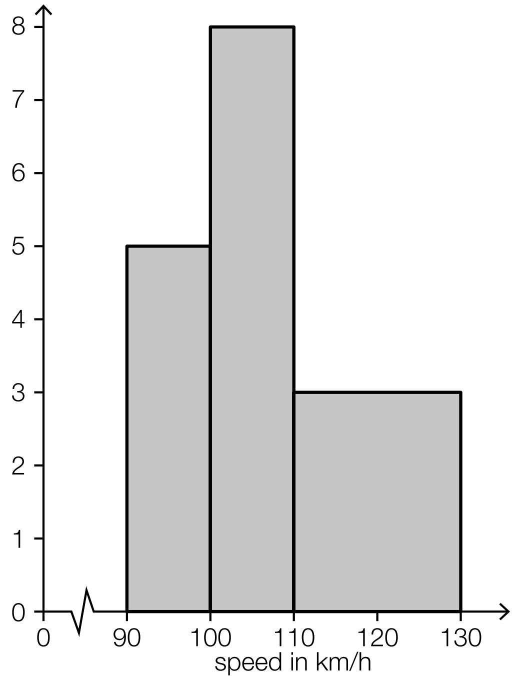
Interpret $5 + \int_0^3 f(t)\,\mathrm{d}t$ in the given context.

[0 / 1 p.]

Task 19

Speed Check

On a section of motorway the speeds of vehicles are measured, and then the histogram shown below is created. The area of a rectangle corresponds to the absolute frequency of the speeds in each class.



Task:

Determine the number of vehicles that were included in the creation of the histogram.

[0/1 p.]

Task 20

Points on a Test

In the lower secondary school, Sophie took 16 mathematics tests. In each of these mathematics tests, 48 points were available. The mean of the total number of points achieved by Sophie was 38.5 points.

In the first two mathematics tests in the upper secondary school, Sophie scored 41 points and 47 points respectively out of a maximum of 48 available points.

Task:

Determine the mean $\bar{x}$ of the number of points achieved by Sophie across all 18 mathematics tests.

[0/1 p.]

Task 21

Median and Mean

For a particular group of 11 people, the following statements hold: the mean of their gross salaries is € 5,690; the median of their gross salaries is € 3,200.

Task:

Put a cross next to the two statements that are always true. *[2 out of 5]*

At least 1 person in this group has a gross salary of exactly € 3,200.	☐
At least 1 person in this group has a gross salary of exactly € 5,690.	☐
At least 6 people in this group have a gross salary of at most € 3,200.	☐
At most 1 person in this group has a gross salary of more than € 5,690.	☐
At least 5 people in this group have a gross salary of more than € 5,690.	☐

[0/1 p.]

Task 22

Christmas Presents

According to a survey, 87 % of the Austrian population buy Christmas presents. Out of this section of the population, 3 % are "last-minute shoppers", who only start buying presents a few days before Christmas.

Data source: https://ooe.orf.at/stories/3020487/ [07.11.2019].

Task:

Using the data from this survey, determine the proportion p of "last-minute shoppers" of the Austrian population as a percentage.

p = ____________________ %

[0/1 p.]

Task 23

Binomial Coefficients

Let a and b be two natural numbers with $0 \leq a < b \leq 9$.

For two binomial coefficients, the following statement holds:
$\binom{9}{a} = \binom{9}{b}$

Task:

Write down an expression for a in terms of b.

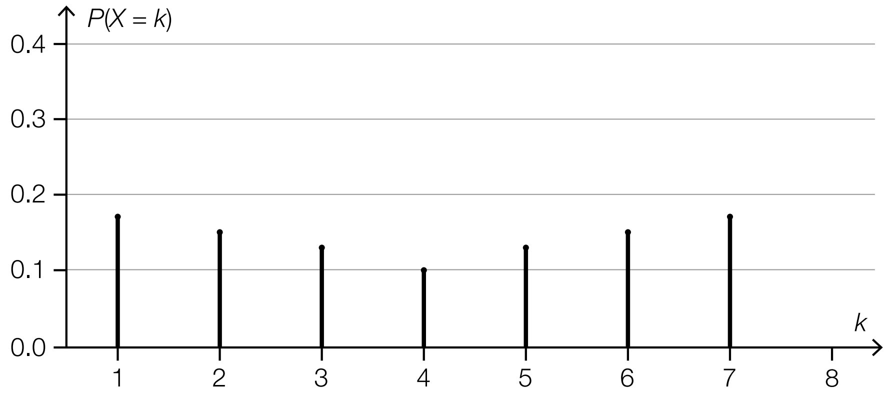
a = ____

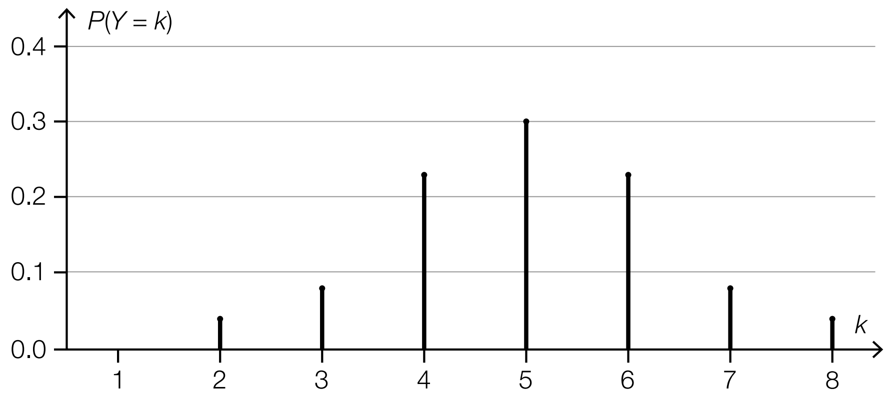
[0/1 p.]

Task 24

Expected Values and Standard Deviations

Let X and Y be two random variables, which can each take exactly 7 integer values with a positive probability. The probability distributions for X and Y are shown below.





Task:

Complete the gaps in the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

For the expected values $E(X)$ and $E(Y)$, the statement ______①______ holds;
for the standard deviations $\sigma(X)$ and $\sigma(Y)$, the statement ______②______ holds.

①	
$E(X) < E(Y)$	☐
$E(X) = E(Y)$	☐
$E(X) > E(Y)$	☐

②	
$\sigma(X) < \sigma(Y)$	☐
$\sigma(X) = \sigma(Y)$	☐
$\sigma(X) > \sigma(Y)$	☐

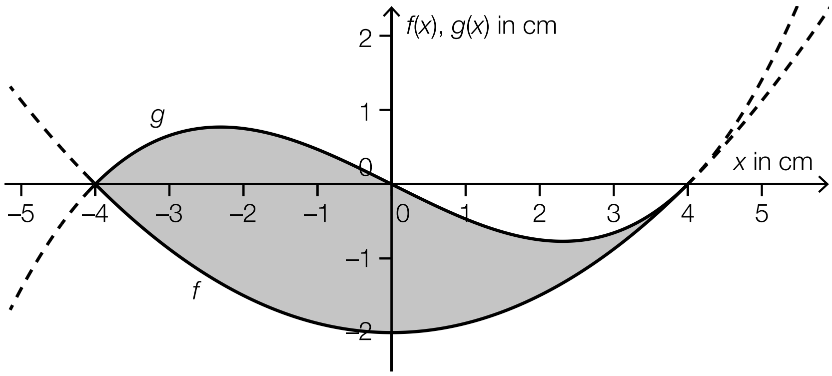
[0/½/1 p.]

Task 25 (Part 2)

Company Logos

Task:

a) The diagram below shows a company logo shaded in grey.



The lower boundary line is defined by a section of the graph of the function *f*:
$f(x) = \frac{1}{8} \cdot x^2 - 2$

The upper boundary line is defined by a section of the graph of the function *g*:
$g(x) = a \cdot (x^3 - 16 \cdot x)$ with $a \in \mathbb{R}$

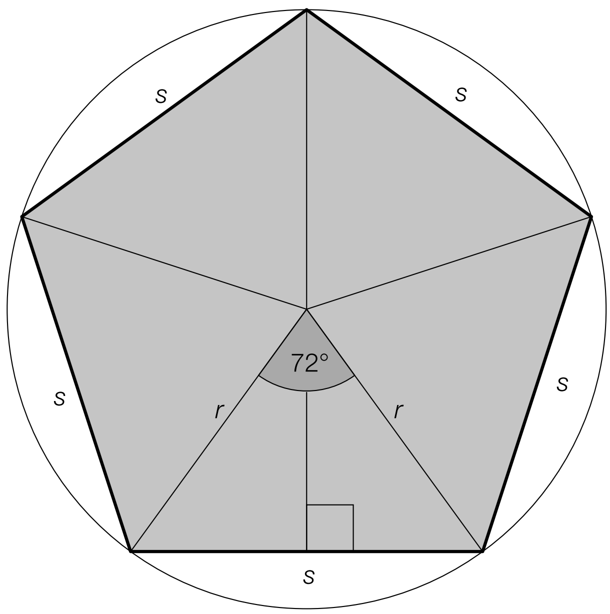
When $x = 4$, *f* and *g* have the same gradient.

1) Determine the value of the parameter *a*. *[0/1 p.]*

The point (0, 0) is a point of inflexion of the graph of *g*.

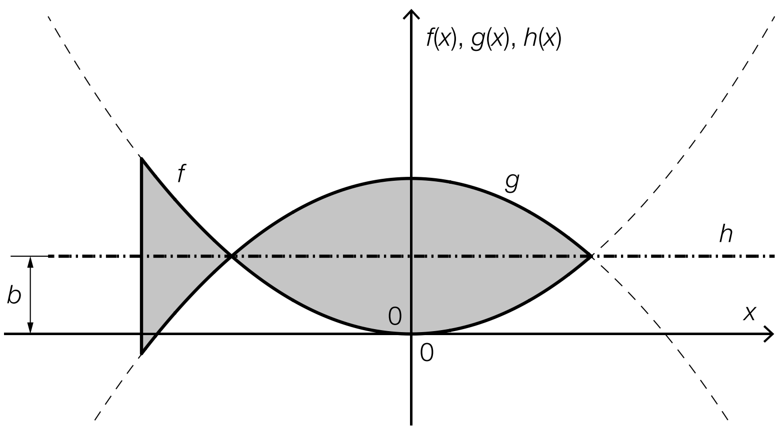
2) Justify why the graph of the function *g* cannot have any further points of inflexion. *[0/1 p.]*

b) The logo of a car manufacturer has the shape of a regular pentagon (see the not-to-scale diagram shown below).



1) For $r = 3$ cm, determine the perimeter u of this regular pentagon. *[0/1 p.]*

c) The logo for a fish restaurant is shown in grey in the coordinate system below.



The logo is symmetrical about the graph of the constant function h with $h(x) = b$ with $b \in \mathbb{R}^+$. The boundary lines of the logo are sections of the graphs of the functions f and g (see diagram above).

For the function f:

$f(x) = a \cdot x^2$ with $a \in \mathbb{R}^+$

1) Write down an equation of the function g in terms of a and b. *[0/1 p.]*

Task 26 (Part 2, Best-of Assessment)

Pellet Heating

In households in Austria, various methods of heating are used such as, for example, oil heating (which uses heating oil as fuel) or pellet heating (which uses pellets – small, compacted wood chippings – as fuel).

The table below shows the average annual heating prices for heating oil and pellets for the years 2006 and 2019 in cents per kilowatt-hour (cents/kWh).

	2006	2019
heating oil	6.80	7.95
pellets	4.40	4.84

Data source: https://www.propellets.at/haeufige-fragen-und-antworten-zu-pellets [13.10.2021].

Task:

a) 1) Determine the average rate of change of the average annual price for pellets (in cents/kWh per year) for the time period from 2006 to 2019. *[0/1 p.]*

b) Family Buchner lives in a house and heats with heating oil. The family is thinking about switching to heating with pellets.
In order to estimate the total cost of heating with heating oil or with pellets from the year 2019 onwards, the following assumptions are made for family Buchner:

- Family Buchner uses around 15000 kWh of energy per year to heat their house.
- The average annual price for heating with heating oil (0.0795 €/kWh) and for heating with pellets (0.0484 €/kWh) remain the same from the year 2019 onwards.
- The transition from oil heating to pellet heating incurs a one-time cost of € 10,000.

t ... time since the start of 2019 in years
$K_{oil}(t)$ ... estimated total cost of heating with heating oil up to time t in €
$K_{pellets}(t)$ ... estimated total cost of heating with pellets up to time t in €

1) Based on these assumptions, write down an equation of a function for each of K_{oil} and $K_{pellets}$.

$K_{oil}(t)$ = ______________________________

$K_{pellets}(t)$ = ______________________________ *[0/½/1 p.]*

2) Determine the time t_1 at which the estimated total costs for family Buchner for heating with pellets are equal to the estimated total costs for heating with heating oil. *[0/1 p.]*

c)* The number of pellet heating systems in Austria can be modelled by the equation shown below for the time period from 1997 to 2019.

$$A(t) = \frac{147\,130}{1 + 31 \cdot e^{-0.28 \cdot t}}$$

t ... time since the beginning of the year 1997 in years
$A(t)$ ... number of pellet heating systems in Austria at time t

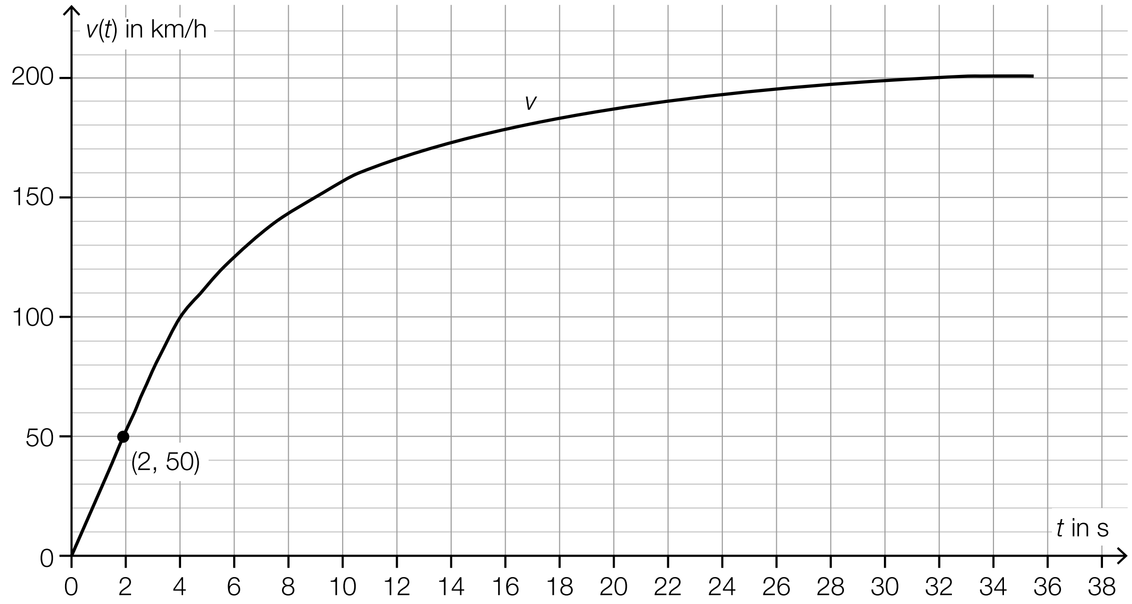
1) Determine the year during the time period from 1997 to 2019 when the instantaneous rate of change of the number of pellet heating systems in Austria was greatest according to this model. *[0/1 p.]*

Task 27 (Part 2, Best-of Assessment)

Acceleration Test

For an acceleration test, a vehicle accelerates from rest (initial velocity = 0 km/h).

The diagram below shows the graph of the velocity-time function v for an acceleration test for a sports car. The sports car moves with velocity $v(t)$ in km/h t seconds after it has started accelerating.



Task:

a) It is assumed that the velocity v_1 of the sports car in the time period [0, 2] is directly proportional to the time t (t in s, $v_1(t)$ in km/h).

1) Write down an equation for the function v_1.

$v_1(t) =$ ____ *[0/1 p.]*

b) Using another model, the velocity of the sports car in the time period [0, 20] can be described by the function v_2 in terms of the time t.

$v_2(t) = -0.001 \cdot t^4 + 0.078 \cdot t^3 - 2.23 \cdot t^2 + 32 \cdot t$

t ... time in s
$v_2(t)$ ... velocity at time t in km/h

1) Using v_2, determine the time $t_2 \in [0, 20]$ at which the velocity of the sports car is 130 km/h. *[0/1 p.]*

c) The velocity-acceleration function a assigns each velocity $v \in [80, 160]$ of the sports car to the approximate corresponding acceleration $a(v)$.

$a(v) = 0.0003 \cdot v^2 + b \cdot v + c$ with $b, c \in \mathbb{R}$

v ... velocity in km/h
$a(v)$ ... acceleration at velocity v in m/s²

The table below shows two values for the acceleration.

v in km/h	80	160
$a(v)$ in m/s²	6.7	1.4

1) Determine b and c. *[0/1 p.]*

2) Using the function a and the diagram in the introductory text, determine the time t_3 at which the acceleration is 3.7 m/s². *[0/1 p.]*

Task 28 (Part 2, Best-of Assessment)

Dice Game

In a dice game, five six-sided dice are rolled simultaneously. The numbers 1, 2, 3, 4, 5, and 6 occur on each dice with equal probability. The five dice are rolled independently of each other. The results of the rolls are independent of each other.

Three possible events are described below.

Five-of-a-kind	Any number occurs five times e.g. 4, 4, 4, 4, 4
Full House	Any number occurs exactly three times. Any other number occurs exactly twice e.g. 1, 1, 1, 4, 4
Straight	The numbers 1, 2, 3, 4, 5 or 2, 3, 4, 5, 6 each occur exactly once.

Task:

a) 1) Determine the probability of *Five-of-a-kind* if the five dice are rolled once. *[0/1 p.]*

The numbers 2, 2, 2, 4 and 5 are rolled. When the player rolls again, only the two dice with the numbers 4 and 5 are rolled again; the other three dice are left on the table.
The probability of *Five-of-a-kind* occurring on this second roll is p_1.
The probability of a *Full House* occurring on this second roll is p_2.

2) Determine the two probabilities p_1 and p_2.

p_1 = ____________________

p_2 = ____________________ *[0/½/1 p.]*

b) The expression below gives the probability of an event E occurring when the five dice are rolled once:

$$P(E) = 6 \cdot \left[\binom{5}{4} \cdot \left(\frac{1}{6}\right)^4 \cdot \frac{5}{6}\right]$$

1) Describe a possible event E in the given context. *[0/1 p.]*

c) The probability of rolling a *Straight* with the five dice is around 3.09 %.
The probability of rolling a *Full House* with the five dice is around 3.86 %.

Franz rolls all five dice once. Anna gives Franz 40 euros if he gets a *Straight* or a *Full House*.
In all other cases, Anna receives x euros from Franz.

1) Determine the value of x such that the expected amounts that Anna and Franz pay out to each other are approximately equal. *[0/1 p.]*