Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

October 2022

Mathematics

Supplementary Examination 1
Examiner's Version

Instructions for the standardized implementation of the supplementary examination

The following supplementary examination booklet contains four tasks that can be completed independently of one another as well as the corresponding solutions.

Each task comprises three competencies to be demonstrated.

The preparation time is to be at least 30 minutes; the examination time is at most 25 minutes.

The use of the official formula booklet that has been approved by the relevant government authority for use in the standardized school-leaving examination in mathematics is allowed. Furthermore, the use of electronic devices (e.g. graphic display calculators or other appropriate technology) is allowed provided there is no possibility to communicate (e.g. via the internet, intranet, Bluetooth, mobile networks etc.) and there is no access to an individual's data stored on the device.

After the examination, all materials (tasks, extra sheets of paper etc.) from the candidates are to be collected in. The examination materials (tasks, extra sheets of paper, data that has been produced digitally etc.) may only be made public after the time period allocated for the examination has passed.

Evaluation grid for the supplementary examination

The evaluation grid below may be used to assist in assessing the candidates' performances.

	Candidate 1			Candidate 2			Candidate 3			Candidate 4			Candidate 5		
Task 1															
Task 2															
Task 3															
Task 4															
Total															

Explanatory notes on assessment

Each task can be awarded zero, one, two or three points. A maximum of twelve points can be achieved.

Assessment scale for the supplementary examination

Total number of competencies demonstrated	Assessment of the oral supplementary examination
12	Very good
10–11	Good
8–9	Satisfactory
6–7	Pass
0–5	Fail

Task 1

Plant Pots

Cylindrical plant pots can be arranged in so-called *square packing* or *hexagonal packing* (see the model bird's-eye view diagrams shown below).
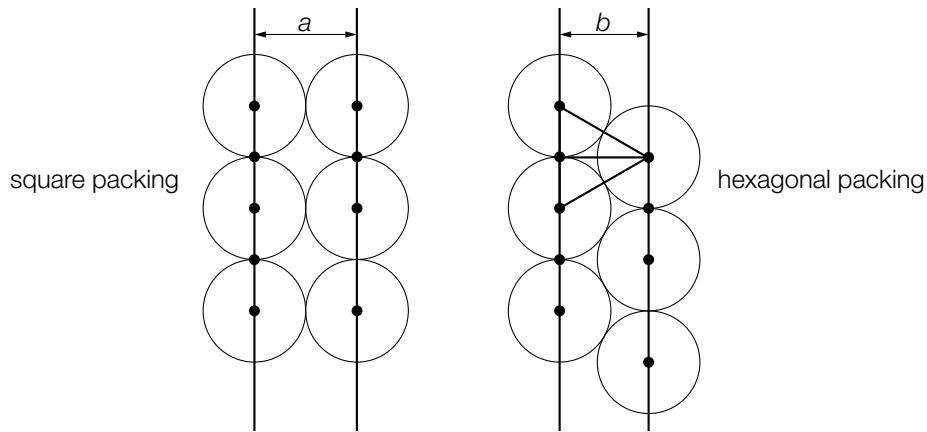
a) The distance *b* in hexagonal packing is smaller than the distance *a* in square packing.

   1) Determine the difference $a - b$ for the case when the diameter of the plant pots is 40 cm.

b) Two cylindrical plant pots with a circular base are compared to each other.

   Plant pot *A* has radius *r* and height *h*.
   The volume of the plant pot is V_A.
   Plant pot *B* has the same height *h* but a radius that is 10 % larger than plant pot *A*.

   1) Show that the volume V_B of plant pot B is 21 % greater than V_A.

c) In a plant pot with a height of 20 cm, there is a plant with height *h* (in cm). The rays of sunlight that fall on the plant make an angle of α with the horizontal (see diagram on the right).
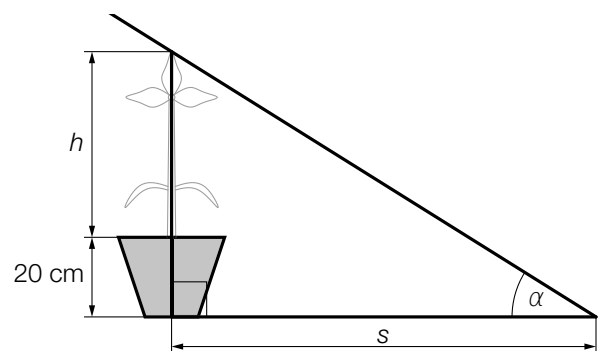
   1) Write down a formula in terms of *h* and α that can be used to determine the length *s* (in cm) of the shadow.

   $s =$ _____

Solution to Task 1

Plant Pots

a1) $a = 40$

b is the height of an equilateral triangle with side length 40.

$b = \dfrac{a}{2} \cdot \sqrt{3}\ = 34.64...$

or:

$b = \sqrt{40^2 - 20^2} = 34.64...$

$a - b = 40 - 34.64...$
$a - b = 5.35...$ cm

b1) $V_A = r^2 \cdot \pi \cdot h$
$V_B = (1.1 \cdot r)^2 \cdot \pi \cdot h = 1.21 \cdot V_A$

c1) $s = \dfrac{h + 20}{\tan(\alpha)}$

Task 2

Desk Lamps

Various models of desk lamp are available. The light source is suspended in different ways depending on design of the model. The suspension methods are modelled by a thick black line in the diagrams below.
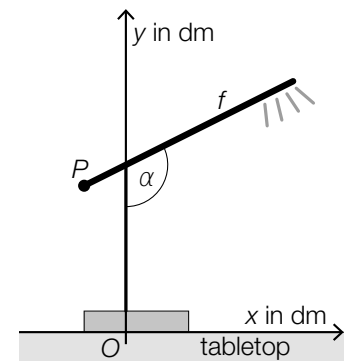
a) The suspension of the light source in model *A* is shown in the diagram on the right.

   1) Justify why this method of suspension cannot be described by the graph of a single function (*y* in terms of *x*).
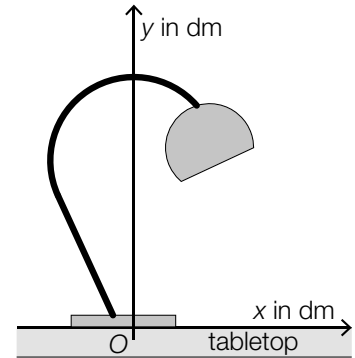
b) The suspension of the light source in model *B* can be described by the graph of the linear function *f* (see diagram on the right).
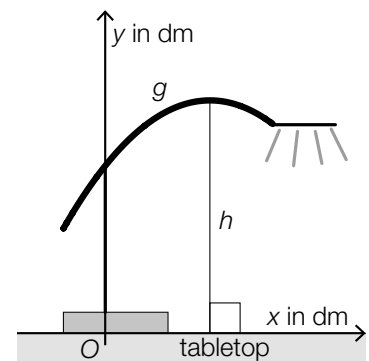
   1) Using $P = (-1, 3.5)$ and $\alpha = 116.56°$, write down an equation of the function *f*.

c) The suspension of the light source in model *C* can be described by the graph of the quadratic function *g* (see diagram on the right).

   The following statement holds: $g(x) = -0.25 \cdot x^2 + 1.25 \cdot x + 4$

   1) Determine the maximum height *h* of the lamp above the tabletop.

Solution to Task 2

Desk Lamps

a1) A function assigns each *x*-value to exactly one *y*-value. As there is a region for which 2 points of the desk lamp are directly above each other, this method of suspension cannot be described by the graph of a single function.

b1) $f(x) = k \cdot x + d$

$k = \tan(116{,}56° - 90°) = 0.499...$
$-1 \cdot 0.499... + d = 3.5$
$d = 3.99...$

$f(x) = 0.5 \cdot x + 4$ *(coefficients rounded)*

c1) $g'(x) = 0$ *or* $-0.5 \cdot x + 1.25 = 0$

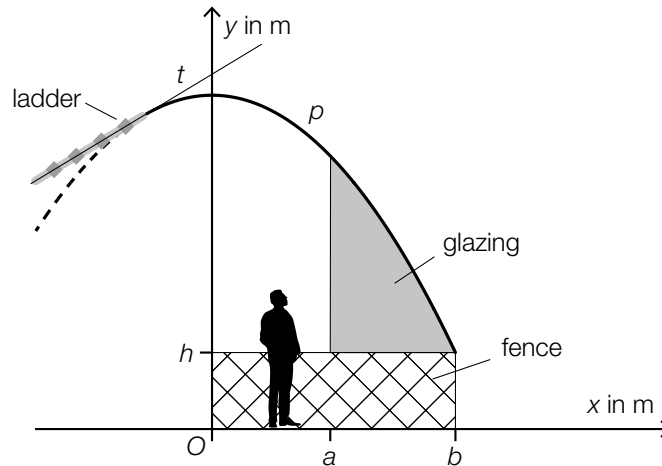Calculation using technology:

$x = 2.5$
$g(2.5) = 5.56...$

The maximum height *h* of the desk lamp above the tabletop is around 5.6 dm.

Task 3

Viewing Platform

The diagram below shows a covered viewing platform shown from the side.

a) The roof is modelled by the graph of the quadratic function p.

$p(x) = -0.302 \cdot x^2 + 4.8$

$x, p(x) \ldots$ coordinates in m

For cleaning purposes, a ladder is mounted on the roof. The ladder runs along the tangent t to the graph p when $x = -1$.

1) Determine the angle of elevation of the tangent t.

b) The platform is to be glazed at the side. The glazing will cover the space between the top of the fence and the roof (see diagram above).

1) Write down a formula that can be used to calculate the size A of the area shaded grey.

$A =$ _____

c) For safety reasons, the roof requires a supporting beam of length $\ell = p(a) - h$.

1) Draw the length ℓ in the diagram above.

Solution to Task 3

Viewing Platform

a1) $p'(x) = -0.604 \cdot x$
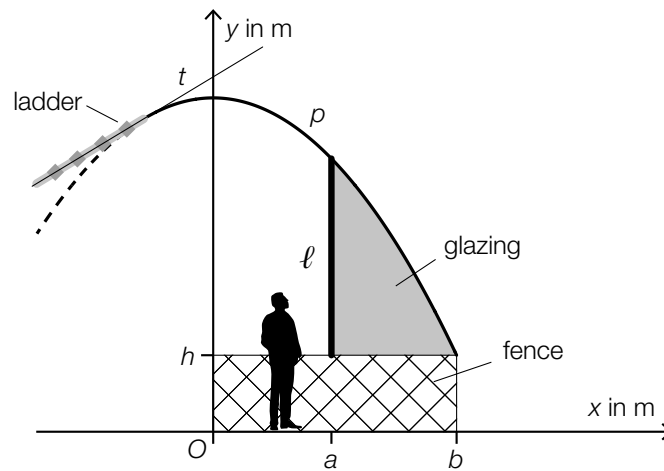
$p'(-1) = 0.604$
$\alpha = \arctan(0.604) = 31.13...°$

The angle of elevation of the tangent t is around 31.1°.

b1) $A = \displaystyle\int_a^b (p(x) - h)\,dx \quad or \quad A = \int_a^b p(x)\,dx - (b - a) \cdot h$

c1)

Task 4

Cigarettes

Many of the substances contained in cigarette smoke are hazardous to health.

a) The amount in mg of substances contained in cigarette smoke in the cigarettes of
    100 smokers is investigated. These have been sorted into 3 categories (see table below).

class	amount in mg of substances per cigarette	class midpoint	absolute frequency
1	[0, 10[	5	55
2	[10, 30[	20	40
3	[30, 50[	40	5

An estimate of the mean of the amount of substances is to be calculated. The respective
class midpoints are determined for this purpose.

1) Determine the mean of the amount of substances.

2) Explain why the median of the amount of substances is in class 1.

b) The probability that a randomly chosen smoker smokes more than one cigarette per day is p.

The probability that exactly 5 out of 100 smokers smoke more than one cigarette per day is to
be calculated.

1) Write down a formula that can be used to determine this probability.

Solution to Task 4

Cigarettes

a1) $\dfrac{5 \cdot 55 + 20 \cdot 40 + 40 \cdot 5}{100} = 12.75$

The mean of the amount of substances is 12.75 mg.

a2) The median of an ordered list always lies in the middle of all values. Of the given 100 values, 55, so more than half, are in class 1. Therefore, the median must also lie in this class.

b1) X ... The number of smokers that smoke more than one cigarette per day

Binomial distribution with $n = 100$ and p

$$P(X = 5) = \binom{100}{5} \cdot p^5 \cdot (1 - p)^{95}$$