Name:	
Class:	

Standardised Competence-Oriented
Written School-Leaving Examination

AHS

11th January 2023

Mathematics

Bundesministerium
Bildung, Wissenschaft
und Forschung

Advice for Completing the Tasks

Dear candidate,

The following booklet contains Part 1 and Part 2 tasks (divided into sub-tasks). The tasks can be completed independently of one another.

Please do all of your working out solely in this booklet and on the paper provided to you. Write your name and that of your class on the cover page of the booklet in the spaces provided. Please also write your name on any separate sheets of paper used and number these pages consecutively. When responding to the instructions of each task, write the task reference (e.g. 25a1) on your sheet.

In the assessment of your work, everything that is not crossed out will be considered.

The use of the official formula booklet for this examination that has been approved by the relevant government authority is permitted. Furthermore, the use of electronic device(s) (e.g. graphic display calculators or other appropriate technology) is allowed provided there is no possibility of communicating via the internet, Bluetooth, mobile networks etc. and there is no access to your own data stored on the device.

An explanation of the task types is displayed in the examination room.

Instructions for Completing the Tasks

- Solutions must be unambiguous and clearly recognisable.
- Solutions must be given alongside their corresponding units if this has been explicitly required in the task instructions.

For tasks with open answer formats, evidence of the targeted core competency is required for the award of the point. When completing tasks with open answer formats, it is recommended that you:

- document how the solution was reached, even if electronic devices were used,
- explain any variables you have chosen yourself and give their corresponding units,
- avoid rounding prematurely,
- label diagrams or sketches.

Changing an answer for a task that requires a cross:

1. Fill in the box that contains the cross.
2. Put a cross in the box next to your new answer.

In this instance, the answer “5 + 5 = 9” was originally chosen. The answer was later changed to be “2 + 2 = 4”.

1 + 1 = 3	☐
2 + 2 = 4	☒
3 + 3 = 5	☐
4 + 4 = 4	☐
5 + 5 = 9	■
6 + 6 = 10	☐

Selecting an item that has been filled in:

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

In this instance, the answer “2 + 2 = 4” was filled in and then selected again.

1 + 1 = 3	☐
2 + 2 = 4	■
3 + 3 = 5	☐
4 + 4 = 4	■
5 + 5 = 9	☐
6 + 6 = 10	☐

Grading System

points awarded	grade
32–36 points	very good
27–31.5 points	good
22–26.5 points	satisfactory
17–21.5 points	pass
0–16.5 points	fail

Best-of Assessment: A best-of assessment approach will be applied to tasks 26, 27 and 28. Of these three Part 2 tasks, the task with the lowest point score will not be included in the total point score.

Good luck!

Task 1

Sum and Product of Two Numbers

Let a and b be two numbers with $a, b \in \mathbb{R}$ such that $a + b = a \cdot b$

Task:

Justify in general why it is not possible for both a and b to be negative under this condition.

[0/1 p.]

Task 2

Pure Water

Pure water consists solely of water molecules. It can be assumed that one water molecule has a mass of $3 \cdot 10^{-23}$ g.

Task:

Determine the number of water molecules in 3 kg of pure water.

[0/1 p.]

Task 3

Rental Properties

Alexander rents out four apartments.
The table below shows the gross rents and the running costs for a particular year.

	gross rent (in €)	running costs (in €)
apartment 1	4 800	1 200
apartment 2	5 500	1 400
apartment 3	6 000	1 800
apartment 4	7 000	1 900

The columns of the table can be written as vectors. The vector B gives the gross rents, and the vector K gives the running costs.

The gross rents are the sum of the net rents and the running costs. The profit (after tax) is 60 % of the net rents.

Task:

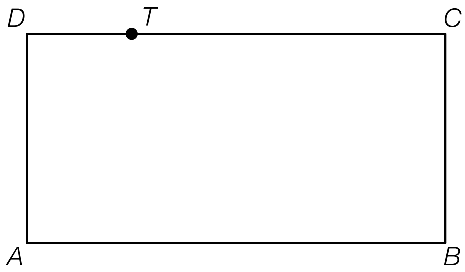
Determine the vector G, whose components give Alexander's profits from renting the four apartments.

[0/1 p.]

Task 4

Point on the Side of a Rectangle

A rectangle with vertices A, B, C and D is shown below. The point T divides the line segment CD in a ratio of 3 : 1 (see diagram below).



For the point T, the following statement holds:
$T = A + r \cdot \overrightarrow{AB} + s \cdot \overrightarrow{DA}$ with $r, s \in \mathbb{R}$

Task:

Determine r and s.

$r =$ ____

$s =$ ____

[0/½/1 p.]

Task 5

Two Lines in Three-Dimensional Space

Let g and h be two lines in $\mathbb{R}^3$.

- $g: X = A + t \cdot \vec{a}$ with $t \in \mathbb{R}$
- $h: X = B + s \cdot \vec{b}$ with $s \in \mathbb{R}$

Task:

Complete the gaps in the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

If _____①_____ holds, then the lines g and h must _____②_____.

①	
$A \notin h$ and $\vec{a} = \vec{b}$	☐
$B \in g$ and $\vec{a} \cdot \vec{b} = 0$	☐
$\vec{a} = r \cdot \vec{b}$ with $r \in \mathbb{R}\backslash\{0\}$ and $B \notin g$	☐

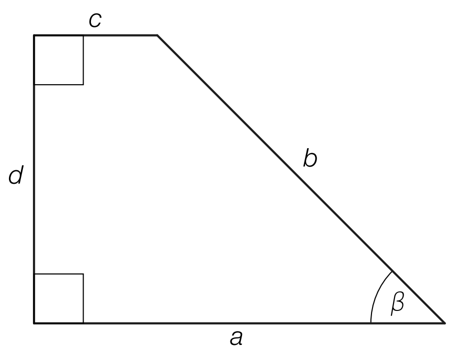
②	
intersect	☐
be identical	☐
be skew	☐

[0/1 p.]

Task 6

Quadrilateral

The diagram below shows a quadrilateral.



Task:

Using the required side lengths, write down a formula that can be used to calculate $\tan(\beta)$.

$\tan(\beta) =$ ____

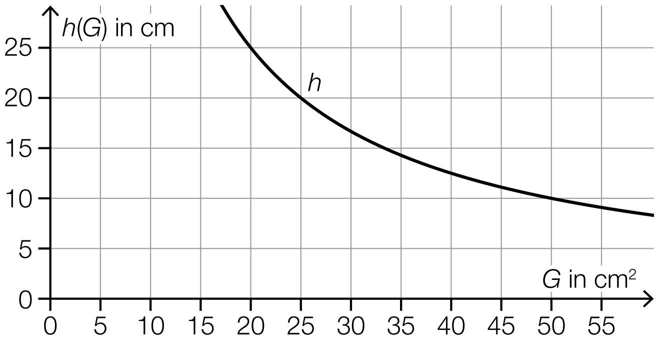
[0/1 p.]

Task 7

Containers

Cylindrical containers are produced that all have the same volume V_0.

The function h describes the height of one such container in terms of the area G of its base (G in cm², $h(G)$ in cm). The graph of the function h is shown in the diagram below.



Task:

Determine V_0.

[0/1 p.]

Task 8

Properties of Functions

Real functions including the parameters $a \in \mathbb{R}^+$ and $b \in (0, 1)$ are shown below.

Task:

Match each of the four equations of functions shown below to the corresponding property from A to F.

$f(x) = a \cdot x + b$	
$f(x) = a \cdot x^2 + b$	
$f(x) = a \cdot b^x$	
$f(x) = a \cdot \sin(b \cdot x)$	

A	$f(x) = f(-x)$ for all $x \in \mathbb{R}$ holds.
B	$f(x) = -f(-x)$ for all $x \in \mathbb{R}$ holds.
C	f is strictly monotonically decreasing in $\mathbb{R}$.
D	f has exactly two zeros.
E	f is concave down for all $x \in \mathbb{R}$.
F	f has exactly one zero.

[0/½/1 p.]

Task 9

Falling Ball

A ball falls from a viewing platform. The function h models the height of the falling ball above the ground in terms of the time t.

The following statement holds: $h\colon \mathbb{R}_0^+ \to \mathbb{R}$, $h(t) = 30 - 4.9 \cdot t^2$ (t in s, $h(t)$ in m).

Task:

Determine the point in time at which the ball is 4 m above the ground.

[0/1 p.]

Task 10

Costs of a Business

The function K with $K(x) = 100 \cdot x^3 - 1\,800 \cdot x^2 + 11\,200 \cdot x + 20\,000$ gives the total cost in euros to a company when it produces x (in tonnes) of a particular product.

Task:

Determine the production amount (in tonnes) for which the total cost is € 48,000 higher than the fixed costs.

[0/1 p.]

Task 11

Height of a Tree

The height of a particular tree can be modelled by an exponential function for the first 15 years after planting.

This tree has a height of 2.2 m 10 years after planting and a height of 2.7 m 15 years after planting.

Task:

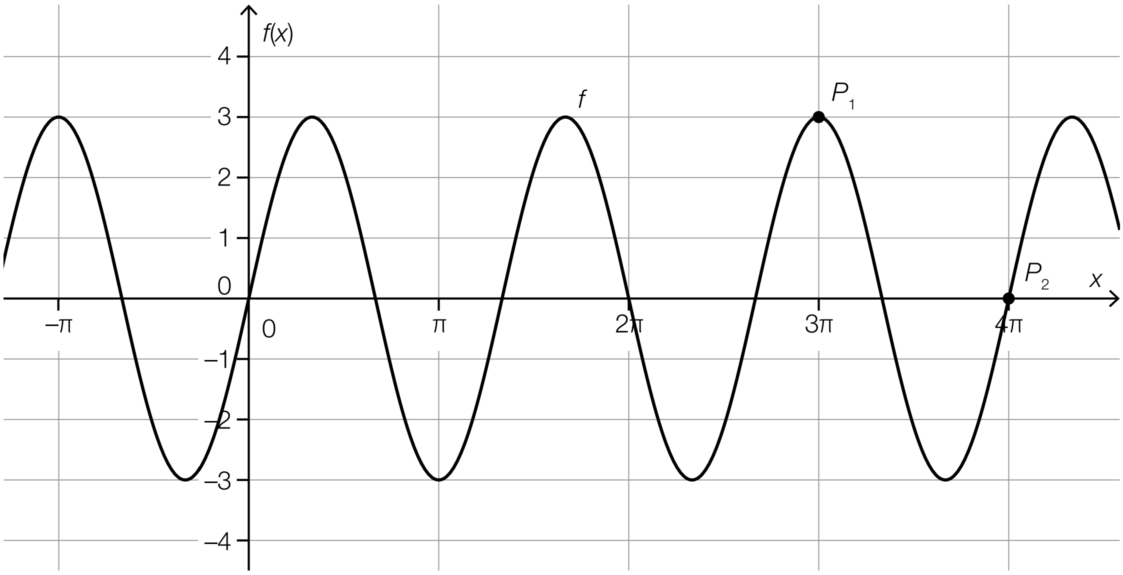
Determine the height of this tree at the time of planting.

[0/1 p.]

Task 12

Graph of a Sine Function

The diagram below shows the graph of the sine function f with $f(x) = a \cdot \sin(b \cdot x)$ with $a, b \in \mathbb{R}^+$.
The graph of f goes through the points $P_1 = (3\pi, 3)$ and $P_2 = (4\pi, 0)$.



Task:

Write down the values of a and b.

a = ____________________

b = ____________________

[0/½/1 p.]

Task 13

Population Growth

In a particular country, the population has increased rapidly since 1960.
$B(t)$ gives the population of this country in year t.

Task:

Interpret $\frac{B(2017) - B(1960)}{B(1960)} = 3.23$ in the given context.

[0/1 p.]

Task 14

Fuel Consumption

The function V describes the amount of fuel in the tank of a car in terms of the distance x covered. After a journey of x kilometres, there are $V(x)$ litres of fuel in the tank.

The car has completed a journey of 180 km without refuelling.

Task:

Using the function V, write down an expression that can be used to calculate the average fuel consumption (in litres per kilometre) for this journey.

[0/1 p.]

Task 15

Differentiation Rules

Let $g: \mathbb{R} \to \mathbb{R}$ and $h: \mathbb{R} \to \mathbb{R}$ be differentiable functions and $k \in \mathbb{R}$.

Task:

Put a cross next to each of the two statements that are always true. *[2 out of 5]*

For the real function f with $f(x) = g(x) - h(x)$ the statement $f'(x) = g'(x) - h'(x)$ holds.	☐
For the real function f with $f(x) = h(k \cdot x)$ the statement $f'(x) = h'(k \cdot x)$ holds.	☐
For the real function f with $f(x) = k \cdot g(x)$ the statement $f'(x) = k \cdot g'(x)$ holds.	☐
For the real function f with $f(x) = g(x) + k$ the statement $f'(x) = g'(x) + k \cdot x$ holds.	☐
For the real function f with $f(x) = g(x) + h(x)$ the statement $f'(x) = g'(x) \cdot h'(x)$ holds.	☐

[0/1 p.]

Task 16

Overtaking

The acceleration of a particular vehicle whilst it is overtaking is described by the function a.

The following statement holds:
$a(t) = -t^3 + 3 \cdot t^2$ with $0 \leq t \leq 3$

t ... time since the vehicle starts to overtake in s
$a(t)$ ... acceleration of the vehicle at time t in m/s^2

The function v assigns each time t to the velocity of the vehicle $v(t)$ (in m/s).

When the vehicle starts to overtake, the vehicle's velocity is $v(0) = 20$ m/s.

Task:

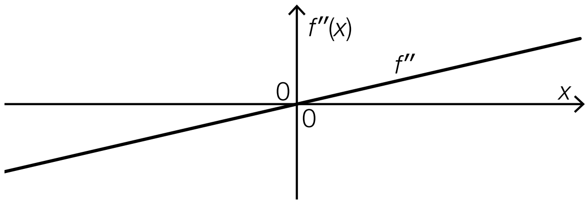
Write down an equation of the function v.

[0/1 p.]

Task 17

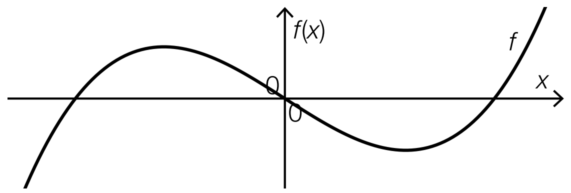
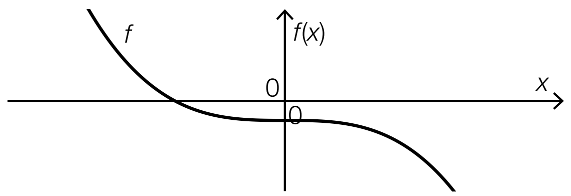
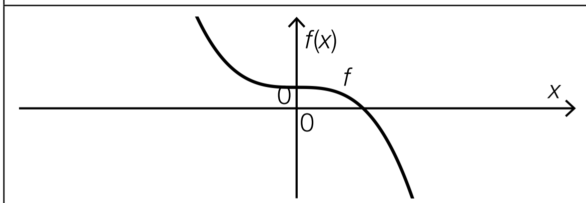
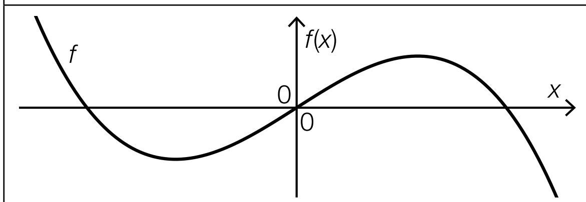
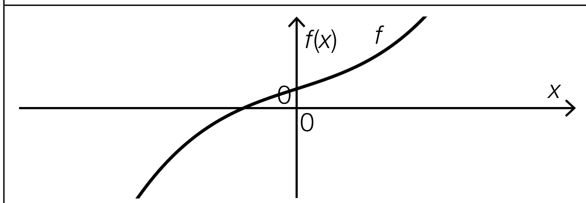
Second Derivative

The diagram below shows the graph of the 2nd derivative f'' of a 3rd degree polynomial function f. The graph of f'' is a straight line that goes through the origin.



Task:

Put a cross next to each of the two diagrams that could represent the graph of a polynomial function f as described above. *[2 out of 5]*

	☐
	☐
	☐
	☐
	☐

[0/1 p.]

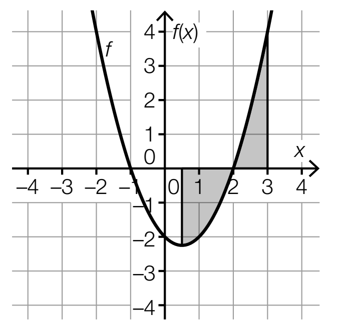
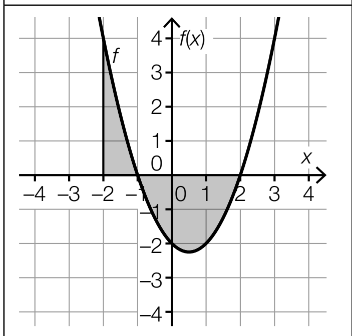
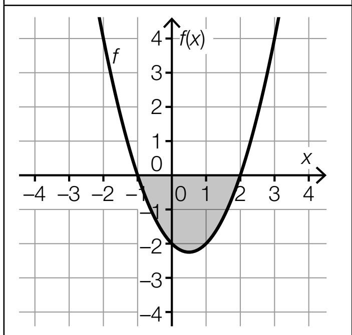
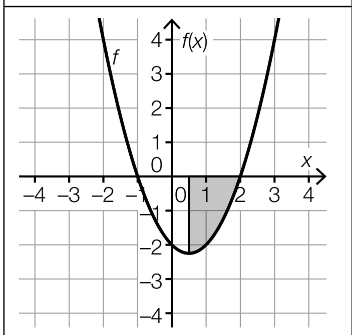
Task 18

Definite Integral

Each of the four diagrams below shows the graph of the quadratic function *f*. The graph of *f* crosses the *x*-axis at $x = -1$ and $x = 2$. The local minimum of *f* is at $x = 0.5$.

Task:

Match each of the areas shaded in grey in the four diagrams to the corresponding expression that can be used to calculate the size of the area from A to F.

A	$-\int_{0.5}^{2} f(x)\,\mathrm{d}x$
B	$-\int_{0.5}^{2} f(x)\,\mathrm{d}x + \int_{2}^{3} f(x)\,\mathrm{d}x$
C	$\int_{-2}^{-1} f(x)\,\mathrm{d}x + \int_{-1}^{2} f(x)\,\mathrm{d}x$
D	$\int_{-2}^{-1} f(x)\,\mathrm{d}x - \int_{-1}^{2} f(x)\,\mathrm{d}x$
E	$\int_{-2}^{0.5} f(x)\,\mathrm{d}x$
F	$-2 \cdot \int_{0.5}^{2} f(x)\,\mathrm{d}x$

[0/½/1 p.]

Task 19

Course Participation

In the time period from 2015 to 2020, a particular course was offered every year in an educational institution. The table below shows the number of course participants for each year in this time period. The number of course participants in 2016 is given by x.

year	number of course participants
2015	12
2016	x
2017	11
2018	12
2019	12
2020	15

The mean number of course participants in the time period from 2015 to 2020 is 12.

Task:

Determine x.

[0/1 p.]

Task 20

Success and Failure

A particular random experiment comprises n independent trials ($n \in \mathbb{N}\setminus\{0\}$). Each trial results in "success" with a probability of p; otherwise the trial results in "failure".

Task:

Describe a possible event E for this random experiment that occurs with probability $1 - (1 - p)^n$.

[0/1 p.]

Task 21

Coin Toss

A random experiment involves tossing a coin many times. After each toss, the coin shows either "heads" or "tails". The probability that the coin shows "heads" is equal to the probability that it shows "tails" after each toss. The results of the tosses are independent of each other. The coin is tossed as many times as it takes until the second "heads" or the second "tails" appears.

The random variable X describes the number of required tosses.

Task:

Determine the probability $P(X = 3)$.

[0/1 p.]

Task 22

Binomial Coefficient

$\binom{10}{2}$ is a binomial coefficient.

Task:

Put a cross next to each of the two amounts that correspond to the binomial coefficient $\binom{10}{2}$.
[2 out of 5]

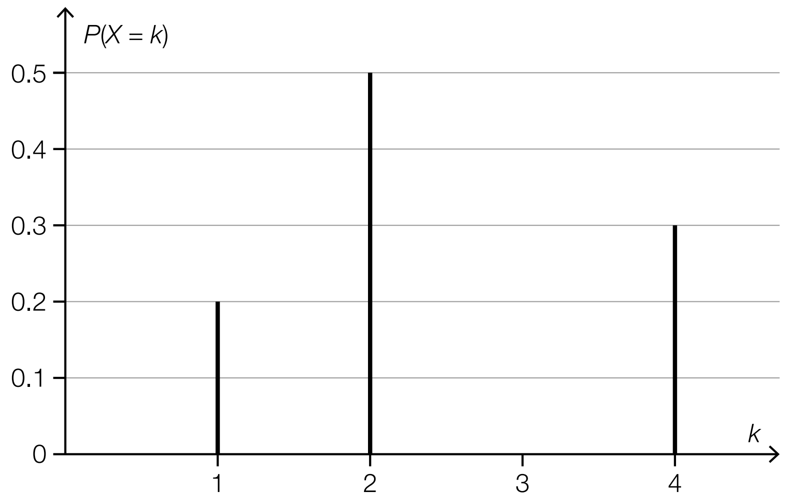
the amount of two-element subsets of a set with ten elements	☐
the amount of numbers that can be created with two digits	☐
the amount of possible ways of selecting two people from a group of ten people	☐
the amount of possible results when a coin is tossed ten times	☐
the amount of possible results when two ten-sided dice each with the numbers from 1 to 10 on their faces are rolled	☐

[0/1 p.]

Task 23

Probability Distribution

The diagram below shows the probability distribution for the random variable X.



The random variable X only takes the values 1, 2 and 4 with a positive probability.

Task:

Determine the expected value $E(X)$.

[0/1 p.]

Task 24

Wheel of Fortune

A wheel of fortune with 24 equally sized sectors is spun. Two of the sectors are green; all the others are red.

For each spin:

- The pointer lands in each sector with equal probability.
- If the pointer lands in a green sector, a prize is won.
- If the pointer lands in a red sector, no prize is won.

The wheel of fortune is spun n times. The results of the spins are independent of each other.

Task:

Write down the expected value for the number of prizes won in terms of n.

[0/1 p.]

Task 25 (Part 2)

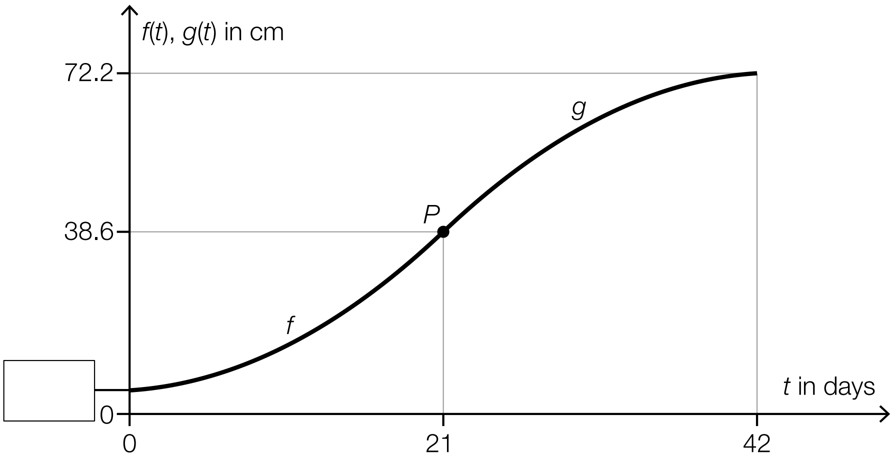
Sunflowers

Task:

a) The height of a particular sunflower in terms of the time t can be approximated by the two quadratic functions f and g. The graphs of these two functions have the same gradient at their point P of intersection (see diagram below).

$f(t) = \frac{1}{15} \cdot t^2 + 0.2 \cdot t + 5$ with $0 \leq t \leq 21$
$g(t) = a \cdot t^2 + b \cdot t + c$ with $21 \leq t \leq 42$

$t \in [0, 42]$ ... time since the start of the observations in days
$f(t)$ ... height of the sunflower at time t in cm
$g(t)$ ... height of the sunflower at time t in cm



1) Complete the missing value on the axis of the diagram above in the box provided. *[0/1 p.]*

2) Write down a system of equations that can be used to calculate the coefficients a, b and c of the function g. *[0/½/1 p.]*

3) Interpret the expression below in the given context along with the corresponding unit.
For $t_1 = 2$ days, $t_2 = 42$ days:

$\frac{g(t_2) - f(t_1)}{t_2 - t_1}$ *[0/1 p.]*

b) The height of a different sunflower in terms of the time t can be approximated over a particular time period by the function h.

$h(t) = 6.2 \cdot a^t$

t ... time since the start of the observations in days
$h(t)$ ... height of the sunflower at time t in cm

At time $t = 17$, the height of this sunflower is 38.6 cm.

1) Determine a. *[0/1 p.]*

Task 26 (Part 2, Best-of Assessment)

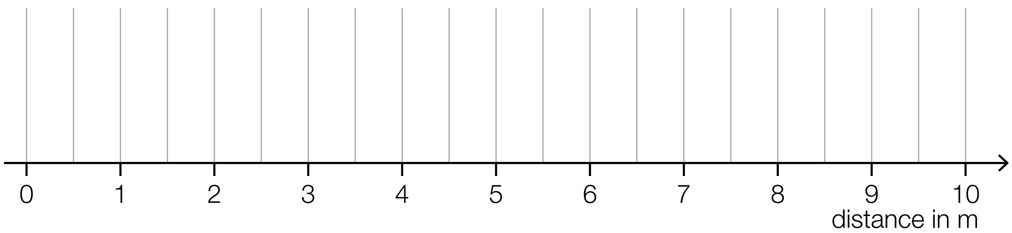
Swimming Course

Task:

a) During a children's swimming course, a swimming teacher records the distances that each child completes during their first unassisted swim. She determines the following values:

minimum: 1.5 m
median: 3 m
3rd quartile: 4 m
range: 5.5 m
interquartile range (difference between the 3rd and 1st quartiles): 2 m

1) Draw the boxplot that corresponds to these values in the space below. *[0/1 p.]*



During a different children's swimming course, the distances swum by 17 children were recorded.
The median of these distances swum is 12 m.

Someone claims that 10 children swam a distance less than 12 m.

2) Justify why this claim is <u>not</u> correct. *[0/1 p.]*

b) The behaviour of children in a particular swimming group when they first try to jump into the water from the side of the pool can be divided into 3 categories:

	absolute frequency	relative frequency
children that jump in straightaway	20	
children that jump in with hesitation		0.4
children that refuse to jump in	10	

1) Complete the 3 missing values in the table above. *[0/1 p.]*

c) In a box, there are 12 red, 10 yellow and 8 blue swim disks. A swimming teacher takes 3 swim disks out of this box one after the other at random and without replacement. (At each stage, each of the remaining swim disks in the box is selected with equal probability.)

The probability of the swimming teacher selecting swim disks in 3 different colours is to be determined.

1) Put a cross next to the expression that gives the probability to be determined. *[1 out of 6]*
[0/1 p.]

$\frac{12}{30} \cdot \frac{10}{30} \cdot \frac{8}{30}$	☐
$\frac{12}{30} \cdot \frac{10}{30} \cdot \frac{8}{30} \cdot 3$	☐
$\frac{12}{30} \cdot \frac{10}{29} \cdot \frac{8}{28}$	☐
$\frac{12}{30} \cdot \frac{10}{29} \cdot \frac{8}{28} \cdot 3$	☐
$\frac{12}{30} \cdot \frac{10}{29} \cdot \frac{8}{28} \cdot 6$	☐
$\left(\frac{12}{30} \cdot \frac{10}{29} \cdot \frac{8}{28}\right)^3$	☐

Task 27 (Part 2, Best-of Assessment)

Special Fourth Degree Polynomial Function

Let f be a polynomial function with $f(x) = a \cdot x^4 + b \cdot x^2 + c$ with $a, b, c \in \mathbb{R}\backslash\{0\}$.

Task:

a) 1) Write down an equation in terms of a and b that can be used to calculate the point of inflexion of f. *[0/1 p.]*

b) 1) Show by calculation using the 1st and 2nd derivatives of f that a maximum or minimum P of the graph of f lies on the vertical axis. *[0/1 p.]*

Exactly one of the coefficients a, b and c determines whether the point P is a maximum.

2) Complete the gaps in the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement. *[0/1 p.]*

For the point P to be a maximum, the coefficient _____①_____ must be _____②_____.

①	
a	☐
b	☐
c	☐

②	
less than 0	☐
equal to 1	☐
greater than 0	☐

c) Let g be a polynomial function with $g(x) = d \cdot (x + e)^2 \cdot (x - e)^2$ with $d \neq 0$ and $e \in \mathbb{R}$.
The graph of g goes through the point $N = (2, 0)$.

1) Under these conditions, determine all possible values of e. *[0/1 p.]*

Task 28 (Part 2, Best-of Assessment)

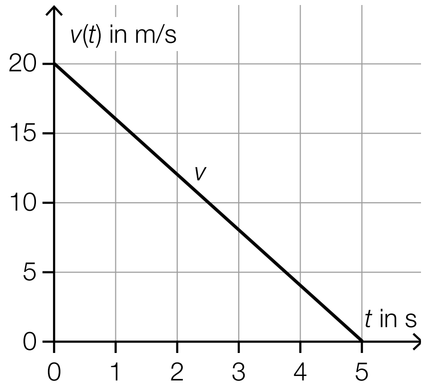
Braking

Braking causes a negative acceleration, which reduces the velocity of a moving vehicle.

Task:

a) A particular vehicle is brought to a complete stop through braking. The distance covered by the vehicle while braking is known as the *braking distance*.

The diagram below shows the velocity-time diagram for a vehicle that comes to a complete stop through braking in 5 s.



The following conditions hold for the velocity-time function v:

$v(t) = -4 \cdot t + 20 \quad \text{with} \quad t \in [0, 5]$

t ... time in s
$v(t)$ ... velocity at time t in m/s

1) Interpret the coefficients –4 and 20 in the equation of the function shown above in the given context. *[0/½/1 p.]*

The length of the braking distance of this vehicle until it comes to a complete stop is given by s_B. If the initial velocity is halved and the negative acceleration remains the same, then the length of the braking distance reduces to $k \cdot s_B$ with $k \in \mathbb{R}$.

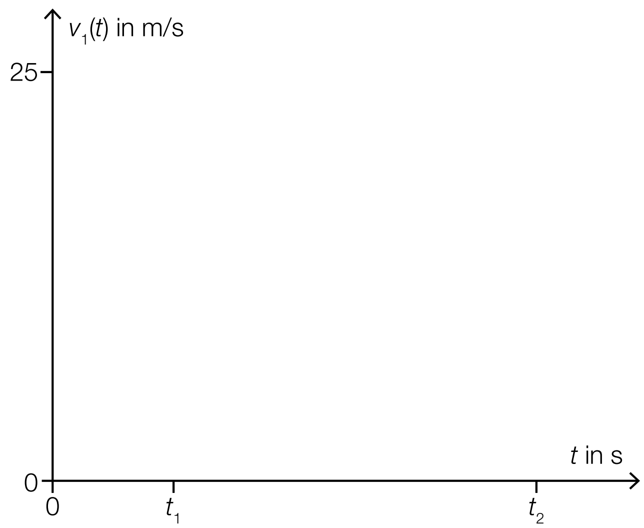
2) Determine k. *[0/1 p.]*

b) A vehicle travels with a constant velocity of 25 m/s. At time $t = 0$, the driver sees an obstacle on the road.

The following conditions hold:

- The driver requires a certain amount of time to react. During this time, the vehicle continues to travel at a constant velocity of 25 m/s.
- The driver begins to brake at time t_1 with a constant deceleration due to braking (negative acceleration).
- At time t_2, the vehicle comes to a complete stop.

1) On the velocity-time diagram below, draw the corresponding velocity-time graph for the situation described above (t in s, $v_1(t)$ in m/s). *[0/1 p.]*



The distance covered by the vehicle in the time interval $[0, t_2]$ is known as the *stopping distance* s_A (s_A in m).

2) Write down a formula that can be used to calculate s_A in terms of t_1 and t_2.

s_A = ______________________ *[0/1 p.]*