

Standardised Competence-Oriented Written
School-Leaving Examination/
School-Leaving and Diploma Examination

Formula Booklet

Mathematics (AHS)
Applied Mathematics (BHS)
Higher Education Entrance
Examination

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1 Sets

\in	is an element of...
\notin	is not an element of...
\cap	intersection
\cup	union
\subset	proper subset
\subseteq	subset
\setminus	difference ("without")
$\{\}$	empty set

Sets of numbers

$\mathbb{N} = \{0, 1, 2, \dots\}$	natural numbers
\mathbb{Z}	integers
\mathbb{Q}	rational numbers
\mathbb{R}	real numbers
\mathbb{C}	complex numbers
\mathbb{R}^+ or \mathbb{R}^-	positive or negative real numbers
\mathbb{R}_0^+ or \mathbb{R}_0^-	positive or negative real numbers including zero

2 Prefixes

tera-	T	10^{12}	deci-	d	10^{-1}
giga-	G	10^9	centi-	c	10^{-2}
mega-	M	10^6	milli-	m	10^{-3}
kilo-	k	10^3	micro-	μ	10^{-6}
hecto-	h	10^2	nano-	n	10^{-9}
deca-	da	10^1	pico-	p	10^{-12}

3 Powers

Powers with integer exponents

$$a \in \mathbb{R}; n \in \mathbb{N} \setminus \{0\} \qquad a \in \mathbb{R} \setminus \{0\}; n \in \mathbb{N} \setminus \{0\}$$

$$a^n = \underbrace{a \cdot a \cdot \dots \cdot a}_{n \text{ factors}} \qquad a^1 = a \qquad a^{-n} = \frac{1}{a^n} = \left(\frac{1}{a}\right)^n \qquad a^{-1} = \frac{1}{a} \qquad a^0 = 1$$

Powers with rational exponents (roots)

$$a, b \in \mathbb{R}_0^+; n, k \in \mathbb{N} \setminus \{0\} \quad \text{where } n \geq 2$$

$$a = \sqrt[n]{b} \iff a^n = b \qquad a^{\frac{1}{n}} = \sqrt[n]{a} \qquad a^{\frac{k}{n}} = \sqrt[n]{a^k} \qquad a^{-\frac{k}{n}} = \frac{1}{\sqrt[n]{a^k}} \quad \text{where } a > 0$$

Calculation rules

$$a, b \in \mathbb{R} \setminus \{0\}; r, s \in \mathbb{Z}$$

$$\text{or } a, b \in \mathbb{R}^+; r, s \in \mathbb{Q}$$

$$a, b \in \mathbb{R}_0^+; m, n, k \in \mathbb{N} \setminus \{0\} \quad \text{where } m, n \geq 2$$

$$a^r \cdot a^s = a^{r+s}$$

$$\frac{a^r}{a^s} = a^{r-s}$$

$$(a^r)^s = a^{r \cdot s}$$

$$(a \cdot b)^r = a^r \cdot b^r$$

$$\left(\frac{a}{b}\right)^r = \frac{a^r}{b^r}$$

$$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

$$\sqrt[n]{a^k} = (\sqrt[n]{a})^k$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \quad (b \neq 0)$$

$$\sqrt[n]{\sqrt[m]{a}} = \sqrt[n \cdot m]{a}$$

Binomial formulae

$$a, b \in \mathbb{R}; n \in \mathbb{N}$$

$$(a + b)^2 = a^2 + 2 \cdot a \cdot b + b^2$$

$$(a - b)^2 = a^2 - 2 \cdot a \cdot b + b^2$$

$$(a + b) \cdot (a - b) = a^2 - b^2$$

$$(a + b)^n = \sum_{k=0}^n \binom{n}{k} \cdot a^{n-k} \cdot b^k$$

$$(a - b)^n = \sum_{k=0}^n (-1)^k \cdot \binom{n}{k} \cdot a^{n-k} \cdot b^k$$

4 Logarithms

$$a, b, c \in \mathbb{R}^+ \quad \text{where } a \neq 1; x, r \in \mathbb{R}$$

$$x = \log_a(b) \Leftrightarrow a^x = b$$

$$\log_a(b \cdot c) = \log_a(b) + \log_a(c)$$

$$\log_a\left(\frac{b}{c}\right) = \log_a(b) - \log_a(c)$$

$$\log_a(b^r) = r \cdot \log_a(b)$$

$$\log_a(a^x) = x$$

$$\log_a(a) = 1$$

$$\log_a(1) = 0$$

$$\log_a\left(\frac{1}{a}\right) = -1$$

$$a^{\log_a(b)} = b$$

natural logarithm (logarithm with base e): $\ln(b) = \log_e(b)$

common logarithm (logarithm with base 10): $\lg(b) = \log_{10}(b)$

5 Quadratic Equations

$$p, q \in \mathbb{R}$$

$$a, b, c \in \mathbb{R} \quad \text{where } a \neq 0$$

$$x^2 + p \cdot x + q = 0$$

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

$$a \cdot x^2 + b \cdot x + c = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}$$

Vieta's Theorem

x_1 and x_2 are the solutions to the equation $x^2 + p \cdot x + q = 0$ if and only if:

$$x_1 + x_2 = -p$$

$$x_1 \cdot x_2 = q$$

Linear factorisation

$$x^2 + p \cdot x + q = (x - x_1) \cdot (x - x_2)$$

6 Two-Dimensional Shapes

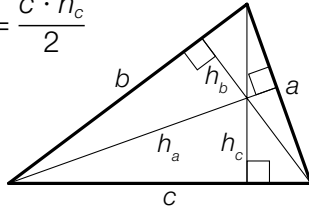
A ... area
u ... perimeter

Triangle

General triangle

$$A = \frac{a \cdot h_a}{2} = \frac{b \cdot h_b}{2} = \frac{c \cdot h_c}{2}$$

$$u = a + b + c$$

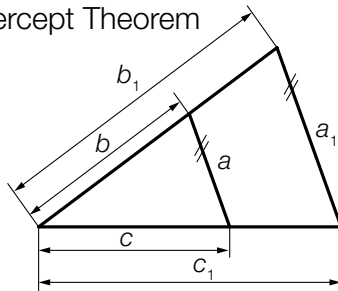


Heron's Formula

$$A = \sqrt{s \cdot (s - a) \cdot (s - b) \cdot (s - c)} \text{ where } s = \frac{a + b + c}{2}$$

Similarity and the Intercept Theorem

$$\frac{a}{a_1} = \frac{b}{b_1} = \frac{c}{c_1}$$



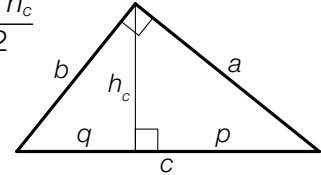
Right-angled triangle with hypotenuse c and sides a, b

$$A = \frac{a \cdot b}{2} = \frac{c \cdot h_c}{2}$$

$$h_c^2 = p \cdot q$$

$$a^2 = c \cdot p$$

$$b^2 = c \cdot q$$



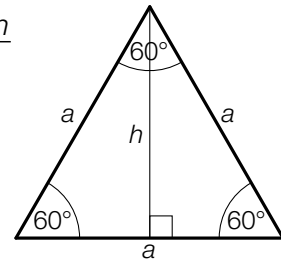
Pythagorean theorem

$$a^2 + b^2 = c^2$$

Equilateral triangle

$$A = \frac{a^2}{4} \cdot \sqrt{3} = \frac{a \cdot h}{2}$$

$$h = \frac{a}{2} \cdot \sqrt{3}$$

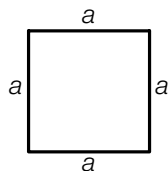


Quadrilateral

Square

$$A = a^2$$

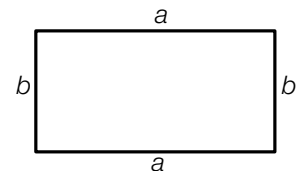
$$u = 4 \cdot a$$



Rectangle

$$A = a \cdot b$$

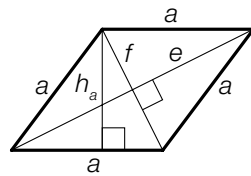
$$u = 2 \cdot a + 2 \cdot b$$



Rhombus

$$A = a \cdot h_a = \frac{e \cdot f}{2}$$

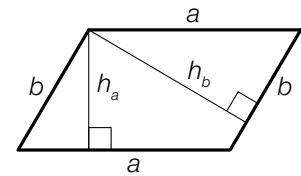
$$u = 4 \cdot a$$



Parallelogram

$$A = a \cdot h_a = b \cdot h_b$$

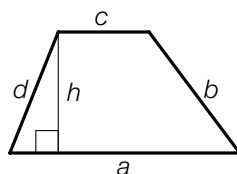
$$u = 2 \cdot a + 2 \cdot b$$



Trapezium

$$A = \frac{(a + c) \cdot h}{2}$$

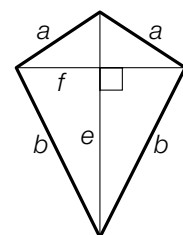
$$u = a + b + c + d$$



Kite

$$A = \frac{e \cdot f}{2}$$

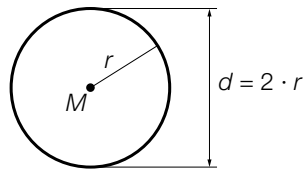
$$u = 2 \cdot a + 2 \cdot b$$



Circle

$$A = \pi \cdot r^2 = \frac{\pi \cdot d^2}{4}$$

$$u = 2 \cdot \pi \cdot r = \pi \cdot d$$

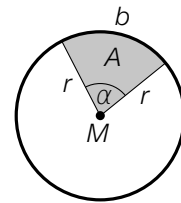


Arc length and sector of a circle

α in degrees ($^\circ$)

$$b = \pi \cdot r \cdot \frac{\alpha}{180^\circ}$$

$$A = \pi \cdot r^2 \cdot \frac{\alpha}{360^\circ} = \frac{b \cdot r}{2}$$



7 Solids

V ... volume

O ... surface area

G ... area of the base

M ... lateral surface area

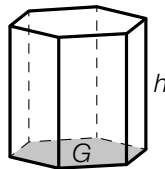
u_G ... perimeter of the base

Prism

$$V = G \cdot h$$

$$M = u_G \cdot h$$

$$O = 2 \cdot G + M$$

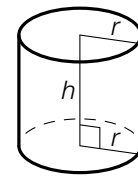


Cylinder

$$V = \pi \cdot r^2 \cdot h$$

$$M = 2 \cdot \pi \cdot r \cdot h$$

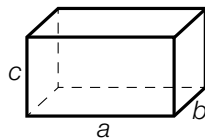
$$O = 2 \cdot \pi \cdot r^2 + 2 \cdot \pi \cdot r \cdot h$$



Cuboid

$$V = a \cdot b \cdot c$$

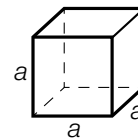
$$O = 2 \cdot (a \cdot b + a \cdot c + b \cdot c)$$



Cube

$$V = a^3$$

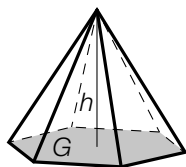
$$O = 6 \cdot a^2$$



Pyramid

$$V = \frac{G \cdot h}{3}$$

$$O = G + M$$



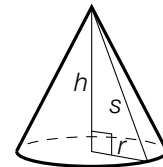
Cone

$$V = \frac{1}{3} \cdot \pi \cdot r^2 \cdot h$$

$$M = \pi \cdot r \cdot s$$

$$O = \pi \cdot r^2 + \pi \cdot r \cdot s$$

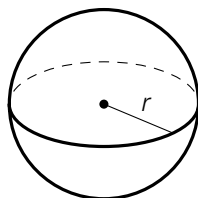
$$s = \sqrt{h^2 + r^2}$$



Sphere

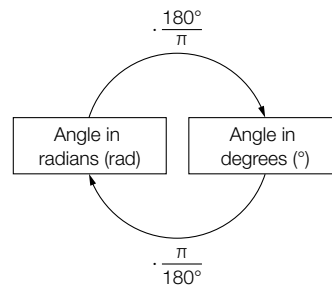
$$V = \frac{4}{3} \cdot \pi \cdot r^3$$

$$O = 4 \cdot \pi \cdot r^2$$



8 Trigonometry

Converting between degrees and radians

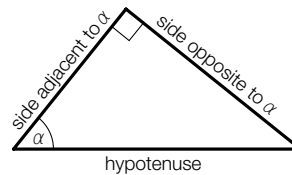


Right-angled triangle trigonometry

Sine: $\sin(\alpha) = \frac{\text{side opposite to } \alpha}{\text{hypotenuse}}$

Cosine: $\cos(\alpha) = \frac{\text{side adjacent to } \alpha}{\text{hypotenuse}}$

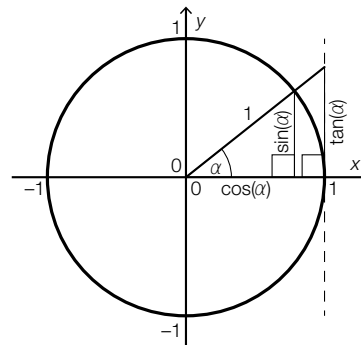
Tangent: $\tan(\alpha) = \frac{\text{side opposite to } \alpha}{\text{side adjacent to } \alpha}$



Unit circle trigonometry

$$\sin^2(\alpha) + \cos^2(\alpha) = 1$$

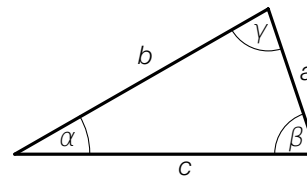
$$\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)} \quad \text{for } \cos(\alpha) \neq 0$$



Trigonometry in general triangles

Sine Rule: $\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}$

Cosine Rule: $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos(\alpha)$
 $b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos(\beta)$
 $c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos(\gamma)$



Trigonometric formula for the area of a triangle

$$A = \frac{1}{2} \cdot b \cdot c \cdot \sin(\alpha) = \frac{1}{2} \cdot a \cdot c \cdot \sin(\beta) = \frac{1}{2} \cdot a \cdot b \cdot \sin(\gamma)$$

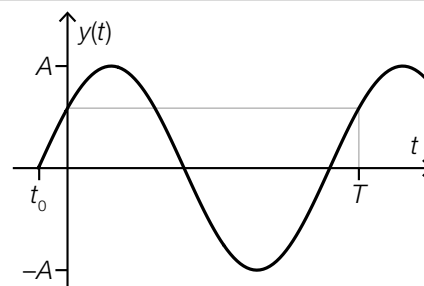
General sine function (in terms of time t)

A ... amplitude	T ... oscillation period (period length)
ω ... angular frequency	f ... frequency
φ ... zero phase angle	

$$y(t) = A \cdot \sin(\omega \cdot t + \varphi)$$

$$T = \frac{2 \cdot \pi}{\omega} = \frac{1}{f}$$

$$t_0 = -\frac{\varphi}{\omega}$$



9 Complex Numbers

j or i ... imaginary unit with $j^2 = -1$ or $i^2 = -1$

a ... real part, $a \in \mathbb{R}$

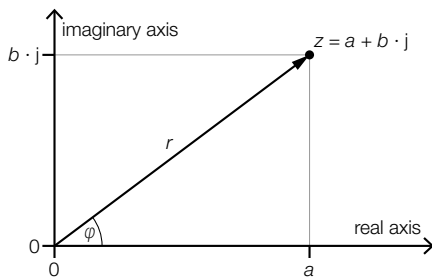
b ... imaginary part, $b \in \mathbb{R}$

r ... modulus, $r \in \mathbb{R}_0^+$

φ ... argument, $\varphi \in \mathbb{R}$

Cartesian form

$$z = a + b \cdot j$$



Polar forms

$$z = r \cdot [\cos(\varphi) + j \cdot \sin(\varphi)] = r \cdot e^{j \cdot \varphi} = (r; \varphi) = r \angle \varphi$$

Conversions

$$a = r \cdot \cos(\varphi)$$

$$b = r \cdot \sin(\varphi)$$

$$r = \sqrt{a^2 + b^2}$$

$$\tan(\varphi) = \frac{b}{a}$$

10 Vectors

P, Q ... points

Vectors in \mathbb{R}^2

Arrow from P to Q :

$$P = (p_1, p_2), Q = (q_1, q_2)$$

$$\vec{PQ} = \begin{pmatrix} q_1 - p_1 \\ q_2 - p_2 \end{pmatrix}$$

Calculation rules in \mathbb{R}^2

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \vec{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}, \vec{a} \pm \vec{b} = \begin{pmatrix} a_1 \pm b_1 \\ a_2 \pm b_2 \end{pmatrix}$$

$$k \cdot \vec{a} = k \cdot \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} k \cdot a_1 \\ k \cdot a_2 \end{pmatrix} \quad \text{where } k \in \mathbb{R}$$

Scalar product in \mathbb{R}^2

$$\vec{a} \cdot \vec{b} = a_1 \cdot b_1 + a_2 \cdot b_2$$

Absolute value (length) of a vector in \mathbb{R}^2

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2}$$

\vec{n} , a vector perpendicular to $\vec{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ in \mathbb{R}^2

$$\vec{n} = k \cdot \begin{pmatrix} -a_2 \\ a_1 \end{pmatrix} \quad \text{for } |\vec{a}| \neq 0 \text{ and } k \in \mathbb{R} \setminus \{0\}$$

Vectors in \mathbb{R}^n

Arrow from P to Q :

$$P = (p_1, p_2, \dots, p_n), Q = (q_1, q_2, \dots, q_n)$$

$$\vec{PQ} = \begin{pmatrix} q_1 - p_1 \\ q_2 - p_2 \\ \vdots \\ q_n - p_n \end{pmatrix}$$

Calculation rules in \mathbb{R}^n

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}, \vec{b} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}, \vec{a} \pm \vec{b} = \begin{pmatrix} a_1 \pm b_1 \\ a_2 \pm b_2 \\ \vdots \\ a_n \pm b_n \end{pmatrix}$$

$$k \cdot \vec{a} = k \cdot \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} = \begin{pmatrix} k \cdot a_1 \\ k \cdot a_2 \\ \vdots \\ k \cdot a_n \end{pmatrix} \quad \text{where } k \in \mathbb{R}$$

Scalar product in \mathbb{R}^n

$$\vec{a} \cdot \vec{b} = a_1 \cdot b_1 + a_2 \cdot b_2 + \dots + a_n \cdot b_n$$

Absolute value (length) of a vector in \mathbb{R}^n

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + \dots + a_n^2}$$

Criterion for two vectors to be perpendicular in \mathbb{R}^2 and \mathbb{R}^3

$$\vec{a} \cdot \vec{b} = 0 \Leftrightarrow \vec{a} \perp \vec{b} \text{ for } |\vec{a}| \neq 0 \text{ and } |\vec{b}| \neq 0$$

Criterion for two vectors to be parallel in \mathbb{R}^2 and \mathbb{R}^3

$$\vec{a} \parallel \vec{b} \Leftrightarrow \vec{a} = k \cdot \vec{b} \text{ for } |\vec{a}| \neq 0, |\vec{b}| \neq 0 \text{ and } k \in \mathbb{R} \setminus \{0\}$$

Angle φ between \vec{a} and \vec{b} in \mathbb{R}^2 and \mathbb{R}^3

$$\cos(\varphi) = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|} \text{ for } |\vec{a}| \neq 0 \text{ and } |\vec{b}| \neq 0$$

Unit vector \vec{a}_0 in the direction of \vec{a}

$$\vec{a}_0 = \frac{1}{|\vec{a}|} \cdot \vec{a} \text{ for } |\vec{a}| \neq 0$$

Vector product in \mathbb{R}^3

$$\vec{a} \times \vec{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2 \cdot b_3 - a_3 \cdot b_2 \\ a_3 \cdot b_1 - a_1 \cdot b_3 \\ a_1 \cdot b_2 - a_2 \cdot b_1 \end{pmatrix}$$

11 Straight Lines

g ... line	\vec{g} ... a direction vector for the line g
	\vec{n} ... a vector perpendicular to the line g
	X, P ... points on the line g
	m ... gradient of the line g
	α ... angle of slope of the line g
	$a, b, c, d, m \in \mathbb{R}$

Vector equation of a line g in \mathbb{R}^2 and \mathbb{R}^3

$$g: X = P + t \cdot \vec{g} \text{ where } t \in \mathbb{R}$$

Equation of a line g in \mathbb{R}^2

the explicit equation of a line:

$$g: y = m \cdot x + c \quad \text{where } m = \tan(\alpha)$$

a general equation of a line:

$$g: a \cdot x + b \cdot y = d$$

a normal vector representation:

$$g: \vec{n} \cdot X = \vec{n} \cdot P \quad \left. \vphantom{g: \vec{n} \cdot X = \vec{n} \cdot P} \right\} \text{ where } \vec{n} \parallel \begin{pmatrix} a \\ b \end{pmatrix} \text{ for } \begin{pmatrix} a \\ b \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

12 Matrices

$$a_{ij}, b_{ij} \in \mathbb{R}; i, j, m, n, p \in \mathbb{N} \setminus \{0\}; k \in \mathbb{R}$$

Addition/subtraction of matrices

$$\begin{pmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \dots & a_{mn} \end{pmatrix} \pm \begin{pmatrix} b_{11} & \dots & b_{1n} \\ \vdots & \ddots & \vdots \\ b_{m1} & \dots & b_{mn} \end{pmatrix} = \begin{pmatrix} a_{11} \pm b_{11} & \dots & a_{1n} \pm b_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} \pm b_{m1} & \dots & a_{mn} \pm b_{mn} \end{pmatrix}$$

Multiplication of a matrix by a number k

$$k \cdot \begin{pmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \dots & a_{mn} \end{pmatrix} = \begin{pmatrix} k \cdot a_{11} & \dots & k \cdot a_{1n} \\ \vdots & \ddots & \vdots \\ k \cdot a_{m1} & \dots & k \cdot a_{mn} \end{pmatrix}$$

Matrix multiplication

A ... $m \times p$ -matrix

B ... $p \times n$ -matrix

$C = A \cdot B$... $m \times n$ -matrix

$$\begin{pmatrix} a_{11} & \dots & a_{1p} \\ \vdots & \ddots & \vdots \\ a_{i1} & \dots & a_{ip} \\ \vdots & \ddots & \vdots \\ a_{m1} & \dots & a_{mp} \end{pmatrix} \cdot \begin{pmatrix} b_{11} & \dots & b_{1j} & \dots & b_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{p1} & \dots & b_{pj} & \dots & b_{pn} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & \dots & b_{nj} & \dots & b_{nn} \end{pmatrix} = \begin{pmatrix} c_{11} & \dots & c_{1j} & \dots & c_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & \dots & c_{ij} & \dots & c_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{m1} & \dots & c_{mj} & \dots & c_{mn} \end{pmatrix} \quad \text{where} \quad c_{ij} = a_{i1} \cdot b_{1j} + a_{i2} \cdot b_{2j} + \dots + a_{ip} \cdot b_{pj}$$

Identity matrix I

$$I = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & 1 \end{pmatrix}$$

Transposed matrix A^T

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

$$A^T = \begin{pmatrix} a_{11} & a_{21} & \dots & a_{m1} \\ a_{12} & a_{22} & \dots & a_{m2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & \dots & a_{mn} \end{pmatrix}$$

Inverse matrix A^{-1} of a square matrix A

$$A \cdot A^{-1} = A^{-1} \cdot A = I$$

Systems of linear equations in matrix notation (n equations with n unknowns)

$$a_{11} \cdot x_1 + a_{12} \cdot x_2 + \dots + a_{1n} \cdot x_n = b_1$$

$$a_{21} \cdot x_1 + a_{22} \cdot x_2 + \dots + a_{2n} \cdot x_n = b_2$$

...

$$a_{n1} \cdot x_1 + a_{n2} \cdot x_2 + \dots + a_{nn} \cdot x_n = b_n$$

$$\underbrace{\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}}_A \cdot \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}}_{\vec{x}} = \underbrace{\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}}_{\vec{b}}$$

If the inverse matrix A^{-1} exists, then $\vec{x} = A^{-1} \cdot \vec{b}$ holds

Manufacturing processes

A ... square material consumption matrix

I ... identity matrix

\vec{x} ... production vector

\vec{n} ... demand vector

$$\vec{x} = A \cdot \vec{x} + \vec{n}$$

$$\vec{x} = (I - A)^{-1} \cdot \vec{n}$$

$$\vec{n} = (I - A) \cdot \vec{x}$$

13 Sequences and Series

Arithmetic sequence

$$(a_n) = (a_1, a_2, a_3, \dots)$$

$$d = a_{n+1} - a_n$$

Recursive rule

$$a_{n+1} = a_n + d \text{ with } a_1 \text{ given}$$

Explicit rule

$$a_n = a_1 + (n - 1) \cdot d$$

Finite arithmetic series

Sum s_n of the first n terms

$$s_n = \sum_{i=1}^n a_i = a_1 + a_2 + \dots + a_{n-1} + a_n$$

$$s_n = \frac{n}{2} \cdot (a_1 + a_n) = \frac{n}{2} \cdot [2 \cdot a_1 + (n - 1) \cdot d]$$

Geometric sequence

$$(b_n) = (b_1, b_2, b_3, \dots)$$

$$q = \frac{b_{n+1}}{b_n}$$

Recursive rule

$$b_{n+1} = b_n \cdot q \text{ with } b_1 \text{ given}$$

Explicit rule

$$b_n = b_1 \cdot q^{n-1}$$

Finite geometric series

Sum s_n of the first n terms

$$s_n = \sum_{i=1}^n b_i = b_1 + b_2 + \dots + b_{n-1} + b_n$$

$$s_n = b_1 \cdot \frac{q^n - 1}{q - 1} \text{ for } q \neq 1$$

Infinite geometric series

$\sum_{n=1}^{\infty} b_n$ is convergent if and only if
 $|q| < 1$

$$s = \lim_{n \rightarrow \infty} s_n = \frac{b_1}{1 - q} \text{ for } |q| < 1$$

14 Rates of Change

For a real function f defined over an interval $[a, b]$:

Absolute change of f in $[a, b]$

$$f(b) - f(a)$$

Relative (percentage) change of f in $[a, b]$

$$\frac{f(b) - f(a)}{f(a)} \text{ for } f(a) \neq 0$$

Difference quotient (average rate of change) of f in $[a, b]$ or in $[x, x + \Delta x]$

$$\frac{f(b) - f(a)}{b - a} \text{ or } \frac{f(x + \Delta x) - f(x)}{\Delta x} \text{ for } b \neq a \text{ or } \Delta x \neq 0$$

Differential quotient (instantaneous rate of change) of f at the point x

$$f'(x) = \lim_{x_1 \rightarrow x} \frac{f(x_1) - f(x)}{x_1 - x} \text{ or } f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

15 Growth and Decay Processes

t ... time

$N(t)$... amount at time t

$N_0 = N(0)$... amount at time $t = 0$

Linear

$k \in \mathbb{R}^+$

linear growth $N(t) = N_0 + k \cdot t$

linear decay $N(t) = N_0 - k \cdot t$

Exponential

$a, \lambda \in \mathbb{R}^+$ where $a \neq 1$ and $N_0 > 0$

a ... growth factor

exponential growth $N(t) = N_0 \cdot a^t$ for $a > 1$ $N(t) = N_0 \cdot e^{\lambda \cdot t}$

exponential decay $N(t) = N_0 \cdot a^t$ for $0 < a < 1$ $N(t) = N_0 \cdot e^{-\lambda \cdot t}$

Limited

$S, a, \lambda \in \mathbb{R}^+$ where $0 < a < 1$

S ... saturation value, carrying capacity

limited growth (saturation function) $N(t) = S - b \cdot a^t$ where $b = S - N_0$ $N(t) = S - b \cdot e^{-\lambda \cdot t}$ where $b = S - N_0$

limited decay $N(t) = S + b \cdot a^t$ where $b = |S - N_0|$ $N(t) = S + b \cdot e^{-\lambda \cdot t}$ where $b = |S - N_0|$

Logistic

$S, a, \lambda \in \mathbb{R}^+$ where $0 < a < 1$ and $N_0 > 0$

S ... saturation value, carrying capacity

logistic growth $N(t) = \frac{S}{1 + c \cdot a^t}$ where $c = \frac{S - N_0}{N_0}$ $N(t) = \frac{S}{1 + c \cdot e^{-\lambda \cdot t}}$ where $c = \frac{S - N_0}{N_0}$

16 Differentiation and Integration

$f, g, h \dots$ functions that are differentiable over \mathbb{R} or over a defined interval

$f', g', h' \dots$ derivative functions

$F, G, H \dots$ antiderivatives

$C, k, q \in \mathbb{R}; a \in \mathbb{R}^+ \setminus \{1\}$

Indefinite integral

$$\int f(x) dx = F(x) + C \quad \text{where} \quad F' = f$$

Definite integral

$$\int_a^b f(x) dx = F(x) \Big|_a^b = F(b) - F(a)$$

Function	Derivative	Antiderivative
$f(x) = k$	$f'(x) = 0$	$F(x) = k \cdot x$
$f(x) = x^q$	$f'(x) = q \cdot x^{q-1}$	$F(x) = \frac{x^{q+1}}{q+1}$ for $q \neq -1$ $F(x) = \ln(x)$ for $q = -1$
$f(x) = e^x$	$f'(x) = e^x$	$F(x) = e^x$
$f(x) = a^x$	$f'(x) = \ln(a) \cdot a^x$	$F(x) = \frac{a^x}{\ln(a)}$
$f(x) = \ln(x)$	$f'(x) = \frac{1}{x}$	$F(x) = x \cdot \ln(x) - x$
$f(x) = \log_a(x)$	$f'(x) = \frac{1}{x \cdot \ln(a)}$	$F(x) = \frac{1}{\ln(a)} \cdot (x \cdot \ln(x) - x)$
$f(x) = \sin(x)$	$f'(x) = \cos(x)$	$F(x) = -\cos(x)$
$f(x) = \cos(x)$	$f'(x) = -\sin(x)$	$F(x) = \sin(x)$
$f(x) = \tan(x)$	$f'(x) = 1 + \tan^2(x) = \frac{1}{\cos^2(x)}$	$F(x) = -\ln(\cos(x))$
$g(x) = k \cdot f(x)$	$g'(x) = k \cdot f'(x)$	$G(x) = k \cdot F(x)$
$h(x) = f(x) \pm g(x)$	$h'(x) = f'(x) \pm g'(x)$	$H(x) = F(x) \pm G(x)$
$g(x) = f(k \cdot x)$	$g'(x) = k \cdot f'(k \cdot x)$	$G(x) = \frac{1}{k} \cdot F(k \cdot x)$

Differentiation rules

multiplication by a constant $(k \cdot f)' = k \cdot f'$

sum rule $(f \pm g)' = f' \pm g'$

product rule $(f \cdot g)' = f' \cdot g + f \cdot g'$

quotient rule $\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$ for $g(x) \neq 0$

chain rule $h(x) = f(g(x)) \Rightarrow h'(x) = f'(g(x)) \cdot g'(x)$

Method for integration – linear substitution

$$\int f(a \cdot x + b) dx = \frac{F(a \cdot x + b)}{a} + C$$

Volume V of solids of revolution

Rotation of the graph of a function f with $y = f(x)$ about an axis

Rotation about the x-axis ($a \leq x \leq b$)

$$V_x = \pi \cdot \int_a^b y^2 dx$$

Rotation about the y-axis ($c \leq y \leq d$)

$$V_y = \pi \cdot \int_c^d x^2 dy$$

Arc length s of the graph of a function f in the interval $[a, b]$

$$s = \int_a^b \sqrt{1 + (f'(x))^2} dx$$

Mean m of a function f in the interval $[a, b]$

$$m = \frac{1}{b-a} \cdot \int_a^b f(x) dx$$

17 1st Order Differential Equations

Separable differential equations

$$y' = f(x) \cdot g(y) \text{ or } \frac{dy}{dx} = f(x) \cdot g(y) \text{ where } y = y(x)$$

1st order linear differential equation with constant coefficients

y ... general solution of a nonhomogeneous differential equation

y_h ... general solution of the homogeneous differential equation $y' + a \cdot y = 0$

y_p ... particular solution of the nonhomogeneous differential equation

s ... interference function

$$y' + a \cdot y = s(x) \text{ where } a \in \mathbb{R}, y = y(x)$$

$$y = y_h + y_p$$

18 Statistics

x_1, x_2, \dots, x_n ... a list of n real numbers
for which k different values x_1, x_2, \dots, x_k occur.

H_i ... absolute frequency of x_i with $H_1 + H_2 + \dots + H_k = n$

Relative frequency h_i of x_i

$$h_i = \frac{H_i}{n}$$

Measures of central tendency

Arithmetic mean \bar{x}

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{1}{n} \cdot \sum_{i=1}^n x_i$$

$$\bar{x} = \frac{x_1 \cdot H_1 + x_2 \cdot H_2 + \dots + x_k \cdot H_k}{n} = \frac{1}{n} \cdot \sum_{i=1}^k x_i \cdot H_i$$

Geometric mean \bar{x}_{geo}

$$\bar{x}_{\text{geo}} = \sqrt[n]{x_1 \cdot x_2 \cdot \dots \cdot x_n} \quad \text{for } x_i > 0$$

Median \tilde{x} for metric data

$x_{(1)} \leq x_{(2)} \leq \dots \leq x_{(n)}$... ordered list of n values

$$\tilde{x} = \begin{cases} x_{(\frac{n+1}{2})} & \dots \text{ when } n \text{ is odd} \\ \frac{1}{2} \cdot (x_{(\frac{n}{2})} + x_{(\frac{n}{2}+1)}) & \dots \text{ when } n \text{ is even} \end{cases}$$

Quartiles

q_1 : At least 25 % of the values are less than or equal to q_1 , and at least 75 % of the values are greater than or equal to q_1 .

$q_2 = \tilde{x}$: At least 50 % of the values are less than or equal to q_2 , and at least 50 % of the values are greater than or equal to q_2 .

q_3 : At least 75 % of the values are less than or equal to q_3 , and at least 25 % of the values are greater than or equal to q_3 .

Measures of spread

Range: $x_{\text{max}} - x_{\text{min}}$

Interquartile range: $q_3 - q_1$

s^2 ... (empirical) variance of a sample

s ... (empirical) standard deviation of a sample

$$s^2 = \frac{1}{n} \cdot \sum_{i=1}^n (x_i - \bar{x})^2$$

$$s^2 = \frac{1}{n} \cdot \sum_{i=1}^k (x_i - \bar{x})^2 \cdot H_i$$

$$s = \sqrt{\frac{1}{n} \cdot \sum_{i=1}^n (x_i - \bar{x})^2}$$

$$s = \sqrt{\frac{1}{n} \cdot \sum_{i=1}^k (x_i - \bar{x})^2 \cdot H_i}$$

If the variance of a population should be estimated using a sample of size n .

$$s_{n-1}^2 = \frac{1}{n-1} \cdot \sum_{i=1}^n (x_i - \bar{x})^2$$

$$s_{n-1}^2 = \frac{1}{n-1} \cdot \sum_{i=1}^k (x_i - \bar{x})^2 \cdot H_i$$

$$s_{n-1} = \sqrt{\frac{1}{n-1} \cdot \sum_{i=1}^n (x_i - \bar{x})^2}$$

$$s_{n-1} = \sqrt{\frac{1}{n-1} \cdot \sum_{i=1}^k (x_i - \bar{x})^2 \cdot H_i}$$

19 Probability

$n \in \mathbb{N} \setminus \{0\}; k \in \mathbb{N}$ where $k \leq n$

$A, B \dots$ events

\bar{A} or $\neg A \dots$ complementary event of A

$A \cap B$ or $A \wedge B \dots$ A and B (the event A and the event B both occur)

$A \cup B$ or $A \vee B \dots$ A or B (at least one of the two events A or B occurs)

$P(A) \dots$ probability of event A occurring

$P(A|B) \dots$ probability of event A occurring given that event B has occurred (conditional probability)

Factorial

$$n! = n \cdot (n-1) \cdot \dots \cdot 1$$

$$0! = 1$$

$$1! = 1$$

Binomial coefficient

$$\binom{n}{k} = \frac{n!}{k! \cdot (n-k)!}$$

Probability for a Laplace experiment

$$P(A) = \frac{\text{number of successful outcomes for } A}{\text{number of possible outcomes}}$$

Elementary rules

$$P(\bar{A}) = 1 - P(A)$$

$$\text{or } P(\neg A) = 1 - P(A)$$

$$P(A \cap B) = P(A) \cdot P(B|A) = P(B) \cdot P(A|B)$$

$$\text{or } P(A \wedge B) = P(A) \cdot P(B|A) = P(B) \cdot P(A|B)$$

If A and B are (stochastically) independent of one another:

$$P(A \cap B) = P(A) \cdot P(B)$$

$$\text{or } P(A \wedge B) = P(A) \cdot P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\text{or } P(A \vee B) = P(A) + P(B) - P(A \wedge B)$$

If A and B are mutually exclusive:

$$P(A \cup B) = P(A) + P(B)$$

$$\text{or } P(A \vee B) = P(A) + P(B)$$

Conditional probability of A given the condition B

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\text{or } P(A|B) = \frac{P(A \wedge B)}{P(B)}$$

Bayes' Theorem

$$P(A|B) = \frac{P(A) \cdot P(B|A)}{P(B)} = \frac{P(A) \cdot P(B|A)}{P(A) \cdot P(B|A) + P(\bar{A}) \cdot P(B|\bar{A})}$$

or

$$P(A|B) = \frac{P(A) \cdot P(B|A)}{P(B)} = \frac{P(A) \cdot P(B|A)}{P(A) \cdot P(B|A) + P(\neg A) \cdot P(B|\neg A)}$$

Expectation value μ of a discrete random variable X with values x_1, x_2, \dots, x_n

$$\mu = E(X) = x_1 \cdot P(X = x_1) + x_2 \cdot P(X = x_2) + \dots + x_n \cdot P(X = x_n) = \sum_{i=1}^n x_i \cdot P(X = x_i)$$

Variance σ^2 of a discrete random variable X with values x_1, x_2, \dots, x_n

$$\sigma^2 = V(X) = \sum_{i=1}^n (x_i - \mu)^2 \cdot P(X = x_i)$$

Standard deviation σ

$$\sigma = \sqrt{V(X)}$$

Binomial distribution

$$n \in \mathbb{N} \setminus \{0\}; k \in \mathbb{N}; p \in \mathbb{R} \text{ where } k \leq n \text{ and } 0 \leq p \leq 1$$

The random variable X is binomially distributed with parameters n and p

$$P(X = k) = \binom{n}{k} \cdot p^k \cdot (1 - p)^{n-k}$$

Expectation value: $E(X) = \mu = n \cdot p$

Variance: $V(X) = \sigma^2 = n \cdot p \cdot (1 - p)$

Normal distribution

$\mu, \sigma \in \mathbb{R}$ where $\sigma > 0$

f ... probability density function

F ... cumulative distribution function

φ ... probability density function of the standard normal distribution

Φ ... cumulative density function of the standard normal distribution

Normal distribution $N(\mu; \sigma^2)$: The random variable X is normally distributed with expectation value (μ), standard deviation (σ) and variance (σ^2)

$$P(X \leq x_1) = F(x_1) = \int_{-\infty}^{x_1} f(x) dx = \int_{-\infty}^{x_1} \frac{1}{\sigma \cdot \sqrt{2 \cdot \pi}} \cdot e^{-\frac{1}{2} \cdot \left(\frac{x-\mu}{\sigma}\right)^2} dx$$

Probabilities for the empirical rule

$$P(\mu - \sigma \leq X \leq \mu + \sigma) \approx 0.683$$

$$P(\mu - 2 \cdot \sigma \leq X \leq \mu + 2 \cdot \sigma) \approx 0.954$$

$$P(\mu - 3 \cdot \sigma \leq X \leq \mu + 3 \cdot \sigma) \approx 0.997$$

Standard normal distribution $N(0, 1)$

$$z = \frac{x - \mu}{\sigma}$$

$$\phi(z) = P(Z \leq z) = \int_{-\infty}^z \varphi(x) dx = \frac{1}{\sqrt{2 \cdot \pi}} \cdot \int_{-\infty}^z e^{-\frac{x^2}{2}} dx$$

$$\phi(-z) = 1 - \phi(z)$$

$$P(-z \leq Z \leq z) = 2 \cdot \phi(z) - 1$$

$P(-z \leq Z \leq z)$	= 90 %	= 95 %	= 99 %
z	≈ 1.645	≈ 1.960	≈ 2.576

Prediction Intervals and Confidence Intervals

$\mu, \sigma, \alpha \in \mathbb{R}$ where $\sigma > 0$ and $0 < \alpha < 1$

\bar{x} ... sample mean

s_{n-1} ... sample standard deviation

n ... sample size

$z_{1-\frac{\alpha}{2}}$... $(1-\frac{\alpha}{2})$ -quantile of the standard normal distribution

$t_{f, 1-\frac{\alpha}{2}}$... $(1-\frac{\alpha}{2})$ -quantile of the t -distribution with f degrees of freedom

Two-sided $(1-\alpha)$ -prediction interval for a single value of a normally distributed random variable

$$\left[\mu - z_{1-\frac{\alpha}{2}} \cdot \sigma, \mu + z_{1-\frac{\alpha}{2}} \cdot \sigma \right]$$

Two-sided $(1-\alpha)$ -prediction interval for the sample mean of normally distributed values

$$\left[\mu - z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}, \mu + z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}} \right]$$

Two-sided $(1-\alpha)$ -confidence interval for the expectation value of a normally distributed random variable

known σ : $\left[\bar{x} - z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}, \bar{x} + z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}} \right]$

unknown σ : $\left[\bar{x} - t_{f, 1-\frac{\alpha}{2}} \cdot \frac{s_{n-1}}{\sqrt{n}}, \bar{x} + t_{f, 1-\frac{\alpha}{2}} \cdot \frac{s_{n-1}}{\sqrt{n}} \right]$ where $f = n - 1$

20 Linear Regression

$(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$... pairs of values

\bar{x}, \bar{y} ... mean of x_i and y_i

linear regression function f with $f(x) = m \cdot x + c$

$$m = \frac{\sum_{i=1}^n (x_i - \bar{x}) \cdot (y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2}$$

$$c = \bar{y} - m \cdot \bar{x}$$

Pearson's correlation coefficient

$$r = \frac{\sum_{i=1}^n (x_i - \bar{x}) \cdot (y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 \cdot \sum_{i=1}^n (y_i - \bar{y})^2}}$$

21 Financial Mathematics

Compound interest calculation

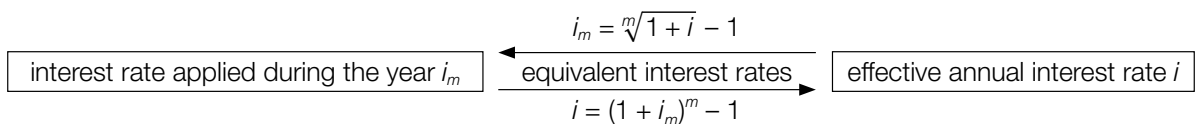
K_0 ... initial investment
 K_n ... final capital after n years
 i ... annual percentage rate of interest

simple interest: $K_n = K_0 \cdot (1 + i \cdot n)$
 compound interest: $K_n = K_0 \cdot (1 + i)^n$

Interest calculated during the year

m ... number of compounding periods per year The following abbreviations are used for compounding periods:
 p.a. ... per year
 p.s. ... per semester
 p.q. ... per quarter
 p.m. ... per month

$$K_n = K_0 \cdot (1 + i_m)^{n \cdot m}$$



Annuities

R ... amount paid per time period
 n ... number of payments
 i ... interest rate
 $q = 1 + i$... accumulation factor

Requirement: annuity period = interest period

	ordinary annuity	annuity due
final value E	$E_{\text{ordinary}} = R \cdot \frac{q^n - 1}{q - 1}$	$E_{\text{due}} = R \cdot \frac{q^n - 1}{q - 1} \cdot q$
present value B	$B_{\text{ordinary}} = R \cdot \frac{q^n - 1}{q - 1} \cdot \frac{1}{q^n}$	$B_{\text{due}} = R \cdot \frac{q^n - 1}{q - 1} \cdot \frac{1}{q^{n-1}}$

Amortisation table

period	interest amount	repayment amount	annuity	residual debt
0				K_0
1	$K_0 \cdot i$	T_1	$A_1 = K_0 \cdot i + T_1$	$K_1 = K_0 - T_1$
...

22 Investments

E_t ... revenue in year t

A_t ... expenses in year t

A_0 ... acquisition costs

R_t ... returns in year t

i ... imputed interest rate (annual interest rate)

n ... operating duration in years

i_w ... reinvestment interest rate (annual interest rate)

E ... final value of the reinvested returns

$$R_t = E_t - A_t$$

Net present value C_0

$$C_0 = -A_0 + \left[\frac{R_1}{(1+i)} + \frac{R_2}{(1+i)^2} + \dots + \frac{R_n}{(1+i)^n} \right]$$

Internal rate of return i_{internal}

$$-A_0 + \left[\frac{R_1}{(1+i_{\text{internal}})} + \frac{R_2}{(1+i_{\text{internal}})^2} + \dots + \frac{R_n}{(1+i_{\text{internal}})^n} \right] = 0$$

Modified internal rate of return i_{mod}

$$A_0 \cdot (1+i_{\text{mod}})^n = E \quad \text{where} \quad E = R_1 \cdot (1+i_w)^{n-1} + R_2 \cdot (1+i_w)^{n-2} + \dots + R_{n-1} \cdot (1+i_w) + R_n$$

23 Cost-of-Production and Theory of Value

x ... amount produced, offered, required or sold ($x \geq 0$)

cost function K	$K(x)$
fixed costs F	$K(0)$
variable cost function K_v	$K_v(x) = K(x) - F$
marginal cost function K'	$K'(x)$
unit cost function (average cost function) \bar{K}	$\bar{K}(x) = \frac{K(x)}{x}$
variable unit cost function (variable average cost function) \bar{K}_v	$\bar{K}_v(x) = \frac{K_v(x)}{x}$
minimum efficient scale x_{opt}	$\bar{K}'(x_{\text{opt}}) = 0$ (minimum of \bar{K})
long-term break-even price (cost-covering price)	$\bar{K}(x_{\text{opt}})$
operating minimum x_{min}	$\bar{K}_v'(x_{\text{min}}) = 0$ (minimum of \bar{K}_v)
short-term break-even price	$\bar{K}_v(x_{\text{min}})$
point of inflexion of the cost function	$K''(x) = 0$
progressive costs	$K''(x) > 0$
degressive costs	$K''(x) < 0$
price p	
price function of demand (price-demand function) p_N	$p_N(x)$
price function of supply p_A	$p_A(x)$
market equilibrium	$p_A(x) = p_N(x)$
ceiling price	$p_N(0)$
saturation amount	$p_N(x) = 0$
revenue function E	$E(x) = p \cdot x$ or $E(x) = p_N(x) \cdot x$
marginal revenue function E'	$E'(x)$
profit function G	$G(x) = E(x) - K(x)$
marginal profit function G'	$G'(x)$
break-even point x_u	$G(x_u) = G(x_o) = 0$ where $x_u \leq x_o$
upper profit limit x_o	
profit range	$[x_u, x_o]$
Cournot's point C	$C = (x_C, p_N(x_C))$ where $G'(x_C) = 0$

24 Technical and Scientific Basics

ρ ... density	t ... time
m ... mass	s ... distance
V ... volume	v ... velocity
F ... force	a ... acceleration
W ... work done	v_0 ... initial velocity
P ... power	

density	$\rho = \frac{m}{V}$
---------	----------------------

force	$F = m \cdot a$
-------	-----------------

work done	$W = F \cdot s$
-----------	-----------------

power	$P = \frac{W}{t}$
-------	-------------------

Motion

velocity for uniform linear motion	$v = \frac{s}{t}$
------------------------------------	-------------------

velocity for uniformly accelerated linear motion	$v = a \cdot t + v_0$
--	-----------------------

velocity in terms of the time t	$v(t) = s'(t)$
-----------------------------------	----------------

acceleration in terms of the time t	$a(t) = v'(t) = s''(t)$
---------------------------------------	-------------------------

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