Name:	
Class:	

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

February 2023

Mathematics

Supplementary Examination 1
Candidate's Version

Bundesministerium
Bildung, Wissenschaft
und Forschung

Instructions for the supplementary examination

Dear candidate,

The following supplementary examination booklet contains four tasks that can be completed independently of one another.

Each task comprises three competencies to be demonstrated.

The preparation time is to be at least 30 minutes; the examination time is at most 25 minutes.

The use of the official formula booklet that has been approved by the relevant government authority for use in the standardized school-leaving examination in mathematics is allowed. Furthermore, the use of electronic devices (e.g. graphic display calculators or other appropriate technology) is allowed provided there is no possibility to communicate (e.g. via the internet, intranet, Bluetooth, mobile networks etc.) and there is no access to an individual’s data stored on the device.

Assessment

Each task can be awarded zero, one, two or three points. A maximum of twelve points can be achieved.

Assessment scale for the supplementary examination

Total number of competencies demonstrated	Assessment of the oral supplementary examination
12	Very good
10–11	Good
8–9	Satisfactory
6–7	Pass
0–5	Fail

Good luck!

Task 1

The Number π

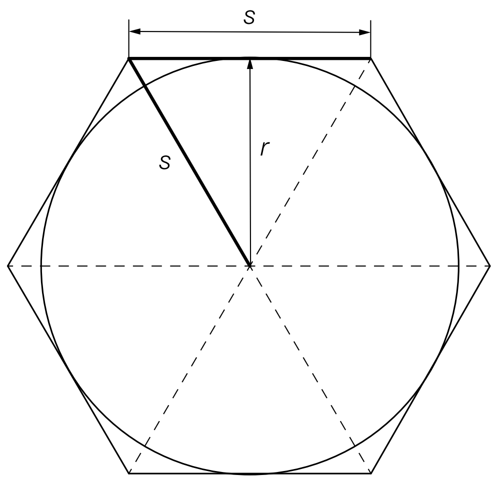
Over the course of history, various methods have been used to determine the value of the number $\pi = 3.141...$ as accurately as possible.

a) In the oldest known book of calculations (*Rhind Mathematical Papyrus*), the following approximation π_N is given:

$$\pi_N = \left(\frac{16}{9}\right)^2$$

1) Determine in how far the approximation π_N differs from π as a percentage.

b) In another method, the circumference of a circle with radius r is approximated by the perimeter of a circumscribed hexagon (see diagram on the right).

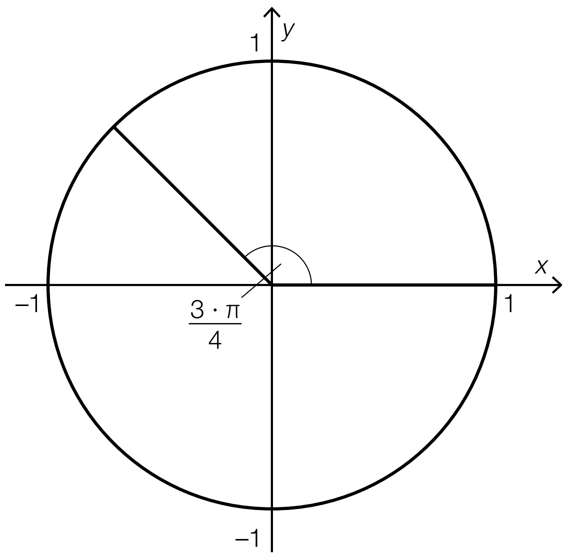


1) Write down a formula in terms of r that can be used to determine the perimeter u of the circumscribed hexagon.

$u =$ ____

c) 1) On the unit circle shown below, sketch the angle α with $\alpha \neq \frac{3 \cdot \pi}{4}$ for which the following statement holds:

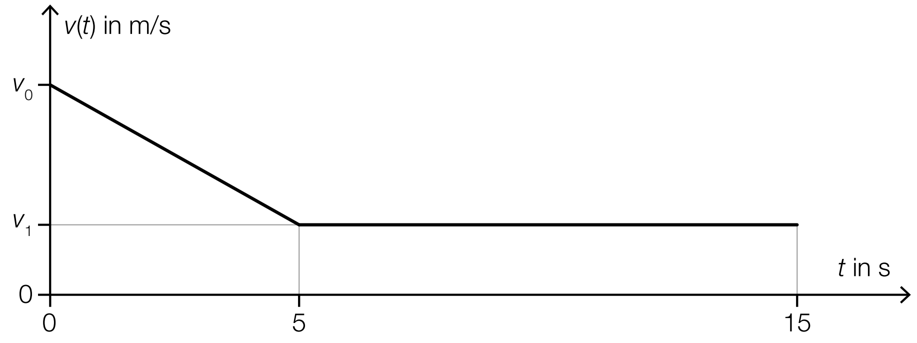
$$\sin\left(\frac{3 \cdot \pi}{4}\right) = \sin(\alpha)$$



Task 2

Car Journey

a) The diagram below shows the velocity in terms of time for a particular car journey for a time period of 15 s.



The distance covered in the first 5 s is the same as the distance covered in the following 10 s.

1) Write down an equation in terms of v_0 and v_1 that describes this situation correctly.

b) For another car journey, the velocity can be approximated by the function v_A.

$v_A(t) = 70 \cdot t^3 - 260 \cdot t^2 + 230 \cdot t + 80 \quad \text{with} \quad 0 \leq t \leq 1.5$

t ... time in h
$v_A(t)$ ... velocity at time t in km/h

1) Determine the maximum velocity for this car journey.
2) Interpret the result of the calculation shown below in the given context.

   $v_A'(0) = 230$

Task 3

Bacteria

A sample of bacteria is investigated in a laboratory.

The number of bacteria is 1 200 when the observations begin.
After 6 days, the number of bacteria is 1 800.

a) The development of the number of bacteria over time can be modelled by the linear function f.

t ... time in days with $t = 0$ corresponding to the beginning of the observations
$f(t)$ ... number of bacteria at time t

1) Write down an equation of the linear function f.

b) In a different model, the development of the number of these bacteria over time is described by the exponential function g.

t ... time in days with $t = 0$ corresponding to the beginning of the observations
$g(t)$ ... number of bacteria at time t

1) Complete the boxes below with the missing symbol (<, > or =).

$g'(t)$ ☐ $g'(t+1)$

$g''(t)$ ☐ 0

2) Determine the time at which the number of bacteria reaches 6 000 according to this model.

Task 4

Dice

a) Two fair six-sided dice with faces showing the values 1, 2, 3, 4, 5 and 6 are rolled simultaneously. The sum of the values shown is determined.

The random variable X describes the sum of the values shown on the pair of dice.

1) Justify why the following statement holds: $P(X = 11) = 2 \cdot P(X = 12)$.

b) Alex participates in a competition. In this competition, a fair six-sided dice with faces showing the values 1, 2, 2, 3, 3 and 3 is rolled once.

Before Alex rolls the dice, the game master collects an amount of e euros.

If the dice shows the value 1, the game master will pay Alex x euros.
If the dice shows the value 2, the game master will pay Alex 2 euros.
If the dice shows the value 3, the game master will not pay Alex any money.

From experience, the game master knows that he can expect a profit of 0.50 euros per roll of the dice.

1) Write down an equation in terms of e that can be used to calculate x.

c) In the production of a particular six-sided dice with faces showing the values 1, 2, 3, 4, 5 and 6, inaccuracies have occurred. This has resulted in one particular face being rolled with a different probability from the others.

The dice is rolled 500 times. The results are shown in the table below.

value shown on the face	1	2	3	4	5	6
absolute frequency of the value shown on the face	75	98	65	110	80	72

1) Based on these results, determine an estimate for the probability shown below.

P(“the value shown on the face is greater than 3 when the dice has been rolled once”) = ________________