Name:	
Class:	

Standardised Competence-Oriented
Written School-Leaving Examination

AHS

3rd May 2023

Mathematics

Bundesministerium
Bildung, Wissenschaft
und Forschung

Advice for Completing the Tasks

Dear candidate,

The following booklet contains Part 1 and Part 2 tasks (divided into sub-tasks). The tasks can be completed independently of one another. You have a total of *270 minutes* available in which to work through this booklet.

Please do all of your working out solely in this booklet and on the paper provided to you. Write your name and that of your class on the cover page of the booklet in the spaces provided. Please also write your name on any separate sheets of paper used and number these pages consecutively. When responding to the instructions of each task, write the task reference (e. g. 25a1) on your sheet.

In the assessment of your work, everything that is not crossed out will be considered.

The use of the official formula booklet for this examination that has been approved by the relevant government authority is permitted. Furthermore, the use of electronic device(s) (e. g. graphic display calculators or other appropriate technology) is allowed provided there is no possibility of communicating via the internet, Bluetooth, mobile networks etc. and there is no access to your own data stored on the device.

An explanation of the task types is displayed in the examination room.

Instructions for Completing the Tasks

– Solutions must be unambiguous and clearly recognisable.
– Solutions must be given alongside their corresponding units if this has been explicitly required in the task instructions.

For tasks with open answer formats, evidence of the targeted core competency is required for the award of the point. When completing tasks with open answer formats, it is recommended that you:

– document how the solution was reached, even if electronic devices were used,
– explain any variables you have chosen yourself and give their corresponding units,
– avoid rounding prematurely,
– label diagrams or sketches.

Changing an answer for a task that requires a cross:

1. Fill in the box that contains the cross.
2. Put a cross in the box next to your new answer.

In this instance, the answer “5 + 5 = 9” was originally chosen. The answer was later changed to be “2 + 2 = 4”.

1 + 1 = 3	☐
2 + 2 = 4	☒
3 + 3 = 5	☐
4 + 4 = 4	☐
5 + 5 = 9	■
6 + 6 = 10	☐

Selecting an item that has been filled in:

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

In this instance, the answer “2 + 2 = 4” was filled in and then selected again.

1 + 1 = 3	☐
2 + 2 = 4	◉
3 + 3 = 5	☐
4 + 4 = 4	■
5 + 5 = 9	☐
6 + 6 = 10	☐

Grading System

points awarded	grade
32 – 36 points	very good
27 – 31.5 points	good
22 – 26.5 points	satisfactory
17 – 21.5 points	pass
0 – 16.5 points	fail

Best-of Assessment: A best-of assessment approach will be applied to tasks 26, 27 and 28. Of these three Part 2 tasks, the task with the lowest point score will not be included in the total point score.

Good luck!

Task 1

Numbers and Sets of Numbers

Five statements about numbers and sets of numbers are stated below.

Task:

Put a cross next to each of the two true statements. *[2 out of 5]*

$\sqrt{\frac{9}{2}}$ is a rational number.	☐
$-\sqrt{100}$ is an integer.	☐
$\sqrt{15}$ is a terminating, non-recurring decimal number.	☐
Every rational number is also a real number.	☐
$\sqrt{-4}$ is a real number.	☐

[0/1 p.]

Task 2

Flight Tickets

A fifth of the tickets for a particular flight are sold to private passengers; the rest are sold to travel companies.

Each ticket sold to a travel company is 5 % cheaper than a ticket sold to a private passenger.

The variable x gives the price per ticket for a private passenger.

Task:

Write down an expression that can be used to calculate the average price per ticket in terms of x.

average price per ticket: _____

[0/1 p.]

Task 3

Smoothie

The vitamin C content of blackcurrants is on average 177 mg per 100 g; the vitamin C content of kiwis is on average 46 mg per 100 g.

These two types of fruit are to be mixed in a smoothie so that a total of 75 g of smoothie contains 100 mg of vitamin C.

Task:

Determine the amount of blackcurrants (in g) and the amount of kiwis (in g) that need to be mixed for this smoothie.

[0 / 1 p.]

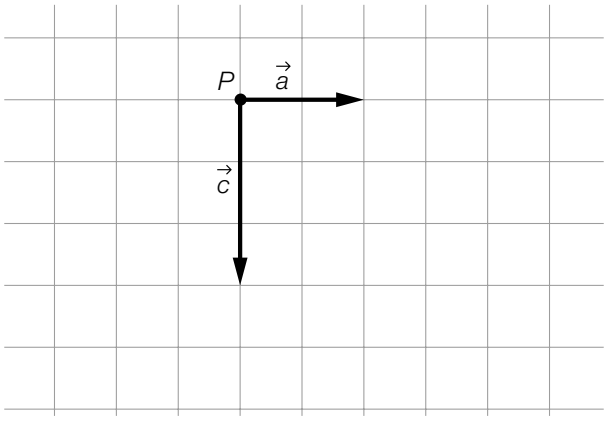
Task 4

Graphical Representation of Vectors

The diagram below shows the two vectors $\vec{a}$ and $\vec{c}$ as arrows starting at point P.

Task:

Starting at point P, draw the vector $\vec{b}$ as an arrow such that $\vec{a} + \vec{b} = \vec{c}$ holds.



[0 / 1 p.]

Task 5

Equations of Lines

The lines g and h with equations $g\colon X = \begin{pmatrix}1\\0\end{pmatrix} + t \cdot \begin{pmatrix}1\\1\end{pmatrix}$ with $t \in \mathbb{R}$ and $h\colon X = \begin{pmatrix}2\\b\end{pmatrix} + s \cdot \begin{pmatrix}a\\2\end{pmatrix}$ with $s \in \mathbb{R}$ are given.
The lines g and h are identical.

Task:

Determine the real numbers a and b.

$a =$ _____

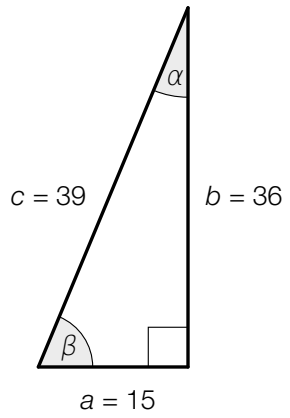
$b =$ _____

[0 / ½ / 1 p.]

Task 6

Triangle

The not-to-scale diagram below shows a right-angled triangle. The angles are measured in degrees and the side lengths in cm.



Task:

Put a cross next to each of the two correct statements. *[2 out of 5]*

$\sin(\alpha) = \frac{5}{13}$	☐
$\cos(\beta) = \frac{5}{12}$	☐
$\tan(\alpha) = \frac{12}{5}$	☐
$\sin(90° - \beta) = \frac{15}{36}$	☐
$\cos(90° - \alpha) = \frac{15}{39}$	☐

[0/1 p.]

Task 7

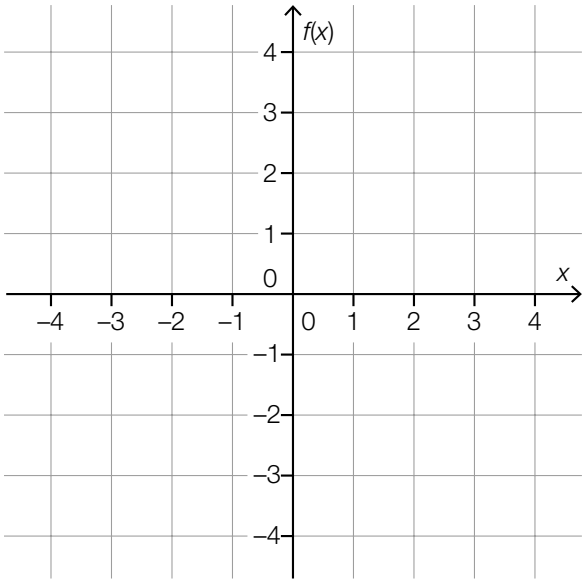
Graph of a Polynomial Function

A fourth-degree polynomial function f has the following properties:

- f has a local maximum when $x = -3$.
- The graph of f is symmetrical about the vertical axis.

Task:

In the coordinate system shown below, sketch the graph of one such polynomial function f in the interval $[-4, 4]$.



[0 / 1 p.]

Task 8

Length of a Candle

A cylindrical candle has a length of 10 cm at time $t = 0$. After burning for 120 min, the candle has a length of 4 cm.
The linear function L models the length of the candle in terms of the burning time t with $0 \leq t \leq 200$ (t in min, $L(t)$ in cm).

Task:

Write down an equation of the function L.

[0 / 1 p.]

Task 9

Parameters of a Quadratic Function

The graph of the quadratic function f with equation $f(x) = a \cdot x^2 + b$ has a local minimum at point $S = (0, -2)$ and goes through the point $P = (1, 0)$.

Task:

Determine the real parameters a and b.

$a =$ _____

$b =$ _____

[0 / ½ / 1 p.]

Task 10

Zeros, Maxima, Minima and Points of Inflexion

The number of real zeros, local maxima and minima and points of inflexion of a polynomial function depends on the degree of the function, among other things.

Task:

Put a cross next to each of the two correct statements. *[2 out of 5]*

Every 1st degree polynomial function has exactly 1 local maximum or minimum.	☐
Every 2nd degree polynomial function has at least 1 real zero.	☐
Every 3rd degree polynomial function has at least 1 real zero.	☐
Every 4th degree polynomial function has exactly 3 local maxima or minima.	☐
Every 5th degree polynomial function has at least 1 point of inflexion.	☐

[0 / 1 p.]

Task 11

Annual Interest Rate

The capital K_0 grows exponentially under a consistent annual interest rate i.
After n years, the capital reaches the value K_n, which can be calculated with the formula shown below.

$K_n = K_0 \cdot (1 + i)^n \quad \text{with} \quad n \in \mathbb{N}$

After 6 years, the capital K_0 has increased by a total of 8.62 %.

Task:

Determine the annual interest rate i.

[0 / 1 p.]

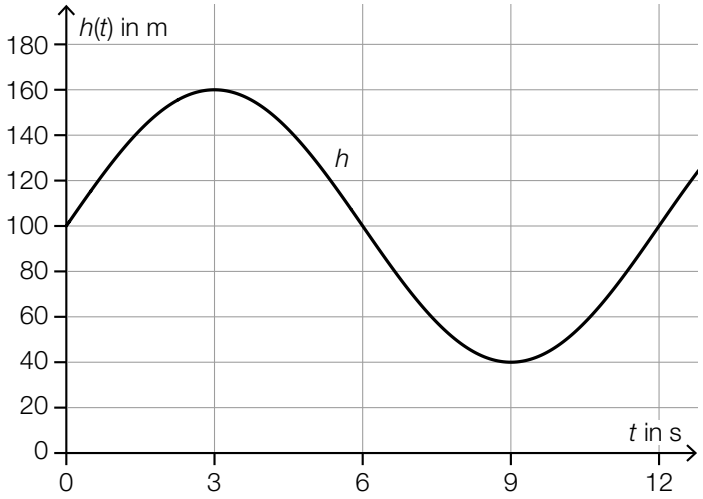
Task 12

Windmill

The tips of the rotor blades of windmills move in a circle whose diameter is known as the *rotor diameter*.

The function $h\colon \mathbb{R} \to \mathbb{R},\ t \mapsto h(t)$ models the height of the tip of one of the rotor blades of a particular windmill above the ground in terms of the time t (t in s, $h(t)$ in m).

The graph of the function h is shown in the diagram below.



Task:

Using the diagram above, write down the rotor diameter as well as the time that a rotor blade requires for a full revolution.

rotor diameter: _____ m

time for a full revolution: _____ s

[0/½/1 p.]

Task 13

Gradient of a Tangent

The function $f: \mathbb{R} \to \mathbb{R}$ is an n^{th} degree polynomial function with $n \geq 2$.

Task:

Put a cross next to each of the two limits that, in all cases, correspond to the gradient of the tangent of the graph of the function f at $x = 5$. *[2 out of 5]*

$\lim_{x_1 \to 5} \frac{f(x_1) - f(5)}{5 - x_1}$	☐
$\lim_{h \to 0} \frac{f(5 + h) - f(5)}{5 + h}$	☐
$\lim_{h \to 5} \frac{f(5 + h) - f(5)}{h}$	☐
$\lim_{x_1 \to 5} \frac{f(x_1) - f(5)}{x_1 - 5}$	☐
$\lim_{h \to 0} \frac{f(5 + h) - f(5)}{h}$	☐

[0/1 p.]

Task 14

Cyclist

The differentiable function $v\colon \mathbb{R}_0^+ \to \mathbb{R}_0^+,\ t \mapsto v(t)$ models the velocity of a cyclist on her journey to school in terms of the time (t in s, $v(t)$ in m/s).

For all $t \in [0, 6]$, $v'(t) > 0$ holds.

Task:

Describe the meaning of the inequality shown in the given context.

[0/1 p.]

Task 15

Production Costs

The monthly fixed costs of a business for the production of soft drinks are € 200,000.
The function K models the total monthly costs for this production (in euros) in terms of the amount produced x.
The marginal costs for this production are described by the function K'.

$K'(x) = 0.003 \cdot x^2 - 6 \cdot x + 3500$

x ... amount produced in units of quantity
$K'(x)$ ... marginal costs for the amount produced x in euros per unit of quantity

Task:

Write down an equation of the function K.

$K(x) =$ _____

[0 / 1 p.]

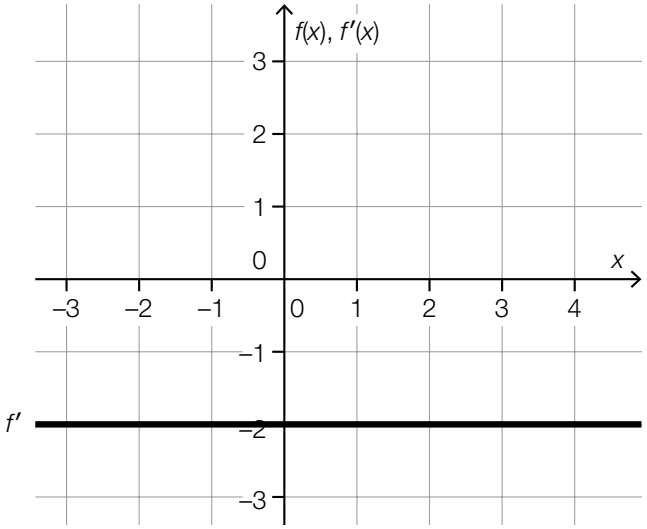
Task 16

Derivative

The diagram below shows the graph of the constant derivative function f' of a function f. For this function f, the statement $f(0) = 2$ holds.

Task:

Draw the graph of the function f on the diagram below.

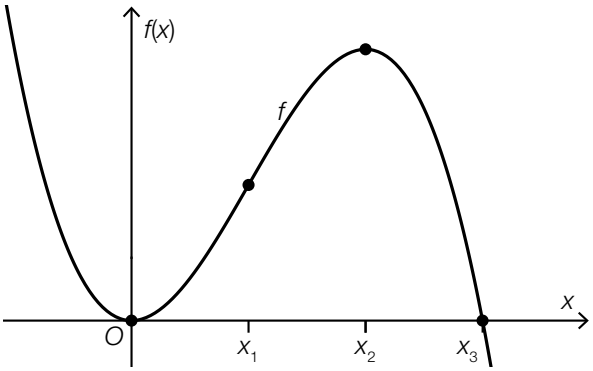


[0/1 p.]

Task 17

Points on a Graph

The graph of a 3rd degree polynomial function f is shown below.
Four points with x-coordinates 0, x_1, x_2 and x_3 are also displayed. These four points are characteristic points of the graph (points of intersection with the axes, maxima or minima, point of inflexion).



Task:

Match each of the four x-coordinates 0, x_1, x_2 and x_3 to the corresponding statement from A to F.

0	
x_1	
x_2	
x_3	

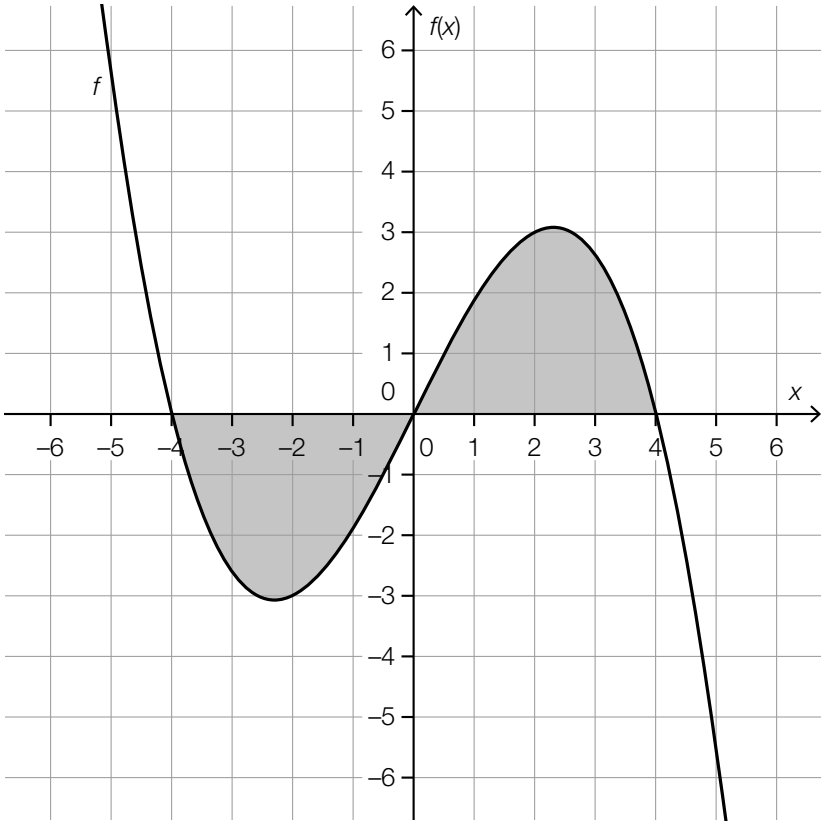
A	At the point with this x-coordinate, the first derivative is zero and the second derivative is negative.
B	At the point with this x-coordinate, the first and the second derivatives are negative.
C	At the point with this x-coordinate, the first derivative is zero and the second derivative is positive.
D	At the point with this x-coordinate, the first and the second derivatives are positive.
E	At the point with this x-coordinate, the first and the second derivatives are zero.
F	At the point with this x-coordinate, the first derivative is positive and the second derivative is zero.

[0 / ½ / 1 p.]

Task 18

Area

The graph of the function $f\colon \mathbb{R} \to \mathbb{R}$ with integer zeros is shown below.



The areas of each of the two regions shaded grey are of equal size.

Task:

Put a cross next to each of the two expressions with which the total area of the regions shaded grey can be calculated. *[2 out of 5]*

$2 \cdot \int_0^4 f(x)\,dx$	☐
$\int_0^4 f(x)\,dx - \int_{-4}^0 f(x)\,dx$	☐
$2 \cdot \int_{-4}^0 f(x)\,dx$	☐
$\int_{-4}^0 f(x)\,dx - \int_0^4 f(x)\,dx$	☐
$\left\lvert \int_{-4}^4 f(x)\,dx \right\rvert$	☐

[0 / 1 p.]

Task 19

Monthly Wages

A particular company has two departments.

In the first department, there are 14 employees, and in the second department there are 26 employees.

The following information is known about the monthly wages of the employees:

- The mean of the monthly wages of all 40 employees is € 2,280.50.
- The mean of the monthly wages of the employees of the second department is € 2,200.00.

Task:

Determine the mean $\bar{x}$ of the monthly wages of the employees of the first department.

$\bar{x}$ = € _____

[0 / 1 p.]

Task 20

Random Experiment

A particular random experiment results in either “success” or “failure”. The random variable X gives the number of times the experiment results in “success” if the random experiment is conducted 7 times.

Task:

Match each of the four probabilities to the corresponding equal probability from A to F.

$P(X < 3)$	
$P(X \leq 3)$	
$P(X \geq 3)$	
$P(X > 3)$	

A	$P(X > 2)$
B	$1 - P(X \leq 4)$
C	$P(X \leq 2)$
D	$P(X = 3) + P(X > 4)$
E	$P(X = 4) + P(X \geq 5)$
F	$1 - P(X > 3)$

[0 / ½ / 1 p.]

Task 21

Card Game

The following statements hold for the 8 cards of a card game:

- 3 cards are labelled with “1”.
- 3 cards are labelled with “2”.
- 2 cards are labelled with “3”.

These 8 cards are shuffled. Then, 2 cards are dealt.

Task:

Determine the probability that at least 1 of the 2 cards dealt is labelled with an odd number.

[0 / 1 p.]

Task 22

Bit Combinations

A computer calculates with so-called *bits*. A bit can either take the value 0 or the value 1. An arbitrary array of eight bits is also known as a *byte*.

Task:

Put a cross next to the correct interpretation of $\binom{8}{3}$ in the given context. *[1 out of 6]*

$\binom{8}{3}$ gives the probability that the first three bits of a byte are all 1s.	☐
$\binom{8}{3}$ gives the probability that exactly three 1s appear in a row in a byte.	☐
$\binom{8}{3}$ gives the probability that there are exactly three 1s in a byte.	☐
$\binom{8}{3}$ gives the number of possibilities for there being exactly three 1s in a byte.	☐
$\binom{8}{3}$ gives the number of possibilities for exactly three 1s to appear in a row in a byte.	☐
$\binom{8}{3}$ gives the number of possibilities for the first three bits of a byte to all be 1s.	☐

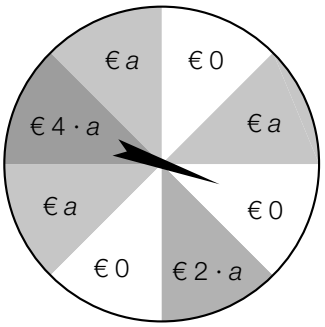
[0/1 p.]

Task 23

Wheel of Fortune

A pointer is mounted in the middle of the wheel of fortune shown below. For each revolution of the pointer, the following statement holds:
The pointer lands in each sector with probability $\frac{1}{8}$.

The winning sums to be paid out when the pointer lands in the various sectors are shown on the wheel of fortune below ($a \in \mathbb{R}^+$).



The pointer is spun once.

The random variable X gives the amount of the winning sum to be paid out.
The expectation value in euros is given as: $E(X) = 4.5$

Task:

Determine a.

[0 / 1 p.]

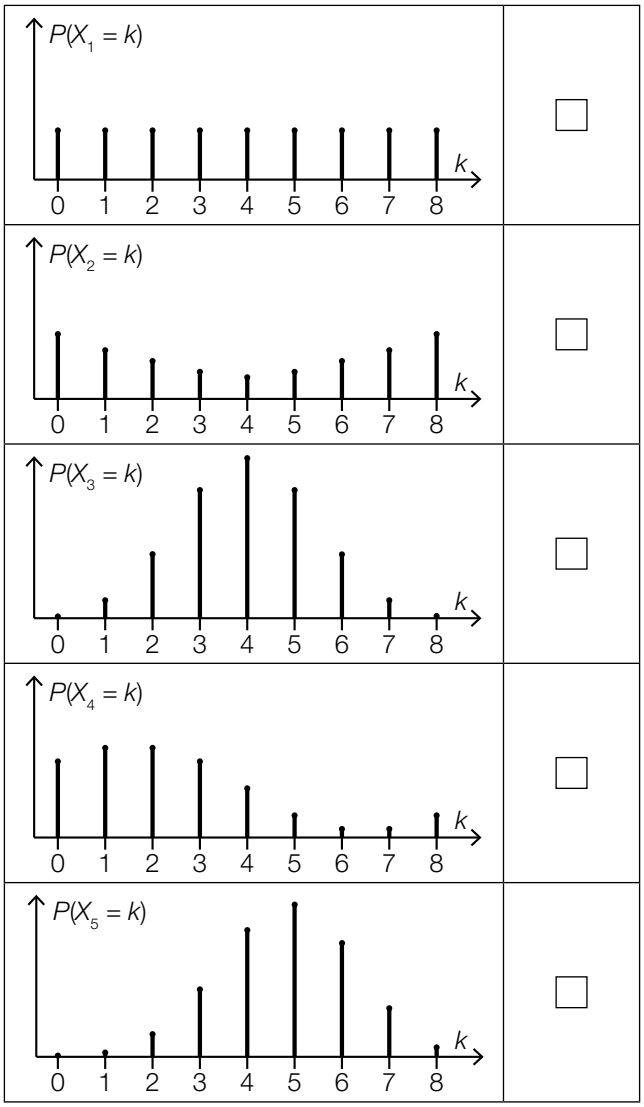
Task 24

Binomial Distribution

The five random variables X_1, X_2, X_3, X_4 and X_5 can only take integer values between 0 and 8. Their probability distributions are shown in the diagrams below.

Task:

Put a cross next to the two diagrams that could show a binomial distribution.
[2 out of 5]



[0/1 p.]

Task 25 (Part 2)

Swimming Pools

There are various pools in a swimming centre.

Task:

a) The volume of a particular cuboid-shaped swimming pool can be described by the equation $V = a^2 \cdot h$.

a ... side length of the square base
h ... depth of the swimming pool

The function $V: \mathbb{R}^+ \to \mathbb{R}^+, a \mapsto V(a)$ with constant h and the function $h: \mathbb{R}^+ \to \mathbb{R}^+, V \mapsto h(V)$ with constant a are to be considered.

1) Complete the gaps in the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement. *[0 / ½ / 1 p.]*

The function V is a _____①_____; the function h is a _____②_____.

①	
linear function	☐
quadratic function	☐
square root function	☐

②	
linear function	☐
quadratic function	☐
square root function	☐

b) In order to fill another swimming pool, p pumps are used, which each pump the same amount of water into the swimming pool per hour. For $p = 2$, the filling time is 19 h.

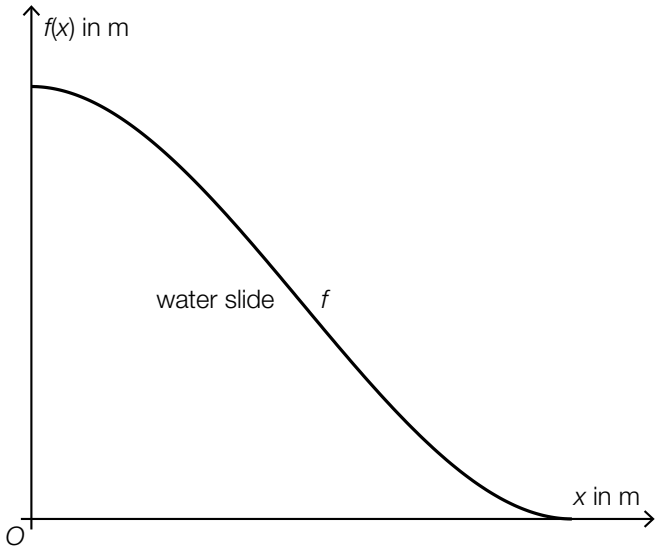
1) Using the number p of pumps, write down a formula that can be used to calculate the filling time T (in h).

$T =$ _____ *[0 / 1 p.]*

The amount of water in this swimming pool decreases due to evaporation and operational causes. The function $W: [0, 10] \to \mathbb{R}$ with $W(t) = -\frac{1}{96} \cdot t^3 + \frac{1}{4} \cdot t^2 - \frac{35}{24} \cdot t$ models the instantaneous rate of change of the amount of water at time t on a particular day (t in h, $W(t)$ in m^3/h).

2) Determine the reduction in the amount of water (in m^3) in the time interval [0, 6]. *[0 / 1 p.]*

c) The diagram below models the side-on profile of a particular water slide.



The side-on profile of the water slide is given by the function $f: [0, 5] \to \mathbb{R}$ with $f(x) = \frac{8}{125} \cdot x^3 - \frac{12}{25} \cdot x^2 + 4$ (x in m, $f(x)$ in m).

1) Determine the x-coordinate x_1 of the point at which the water slide is decreasing most steeply. *[0 / 1 p.]*

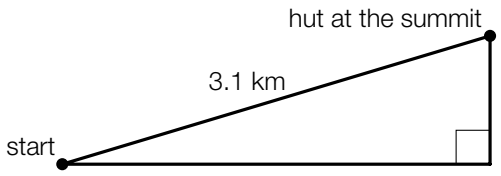
Task 26 (Part 2, Best-of Assessment)

Fitness Watches

Fitness watches are wristwatches that can be used during sports activities.

Task:

a) A 3.1 km mountain hike goes from the start at 680 m above sea level to a hut at the summit at 1 820 m above sea level. The distance covered is modelled as a straight line with a constant gradient and is shown in the (not-to-scale) sketch below.



The path of the mountain hike has a gradient of a %.

1) Determine a.

$a =$ _____ % *[0/1 p.]*

b) The fitness watch *Sporty* is particularly popular.
The probability that a randomly chosen person in Austria possesses a *Sporty* fitness watch is p.

In the course of a study, 160 randomly chosen people in Austria are surveyed.

The binomially distributed random variable X gives the number of people out of the 160 people surveyed who possess a *Sporty* fitness watch.

1) Complete the gaps in the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement. *[0/½/1 p.]*

The probability that none of the 160 people surveyed possesses a *Sporty* fitness watch is _____①_____; the probability that at least 2 of the 160 people surveyed possess a *Sporty* fitness watch is _____②_____.

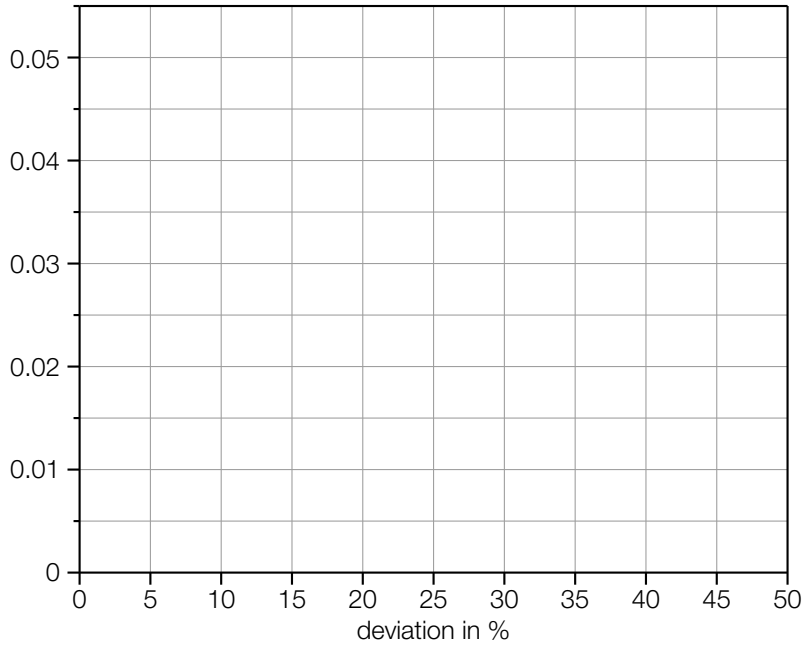
①	
$1 - p$	☐
p^{160}	☐
$(1 - p)^{160}$	☐

②	
$1 - \left[\binom{160}{0} \cdot p^0 \cdot (1-p)^{160} + \binom{160}{1} \cdot p \cdot (1-p)^{159}\right]$	☐
$\binom{160}{0} \cdot p^0 \cdot (1-p)^{160} + \binom{160}{1} \cdot p \cdot (1-p)^{159}$	☐
$\binom{160}{2} \cdot p^2 \cdot (1-p)^{158}$	☐

c) Fitness watches show, among other things, the calories burned during a sports activity. In the course of a study, the percentage deviation of the actual calories burned for a sports activity from the corresponding value measured by the fitness watch was investigated for 60 people. These deviations along with their corresponding absolute frequencies are summarized in classes in the table below.

deviation in %	absolute frequency
[0, 20)	24
[20, 30)	30
[30, 50]	6

1) Construct a histogram that represents the relative frequencies for the three classes shown above as areas of rectangles. *[0/1 p.]*



2) Justify why the median of the list of data (that forms the basis of the table above) has to lie in the interval [20, 30). *[0/1 p.]*

Task 27 (Part 2, Best-of Assessment)

Oxygen Consumption in Mammals

For mammals, there is a relationship between the body mass and the oxygen consumption.

Task:

a) For a mammal that does not move during the observation period, the oxygen consumption can be approximated by a function $S\colon \mathbb{R}^+ \to \mathbb{R}^+,\ m \mapsto S(m)$ in terms of the body mass m (m in kg, $S(m)$ in L/h).

For cats and dogs with a body mass m, the following equation holds:

$S(m) = a \cdot m^{0.75}$

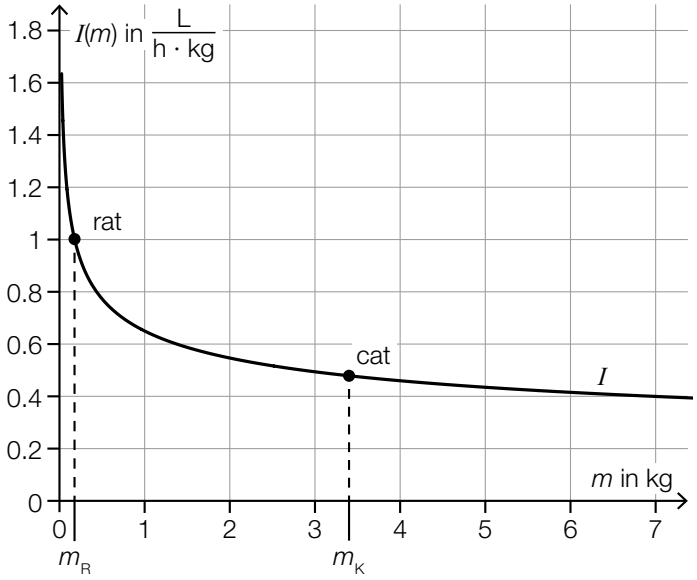
a ... positive constant

The body mass of a particular dog is twice as big as that of a particular cat.

1) Determine the percentage by which the oxygen consumption of this dog is higher than that of this cat. *[0 / 1 p.]*

b) The function $I\colon \mathbb{R}^+ \to \mathbb{R}^+$ describes the metabolic intensity of mammals in terms of their body mass m $\left(m \text{ in kg}, I(m) \text{ in } \frac{\text{L}}{\text{h} \cdot \text{kg}}\right)$.

The graph of I is shown in the diagram below.



Source: Sadava, David E., David M. Hillis et al.: *Purves Biologie*. Edited by Jürgen Markl. 10th ed. Berlin et al.: Springer 2019, p. 1201 (adapted).

The body mass of a rat is given by m_R and that of a cat is given by m_K.
For a particular body mass m_1, $I'(m_1)$ is equal to the average rate of change of I in the interval $[m_R, m_K]$.

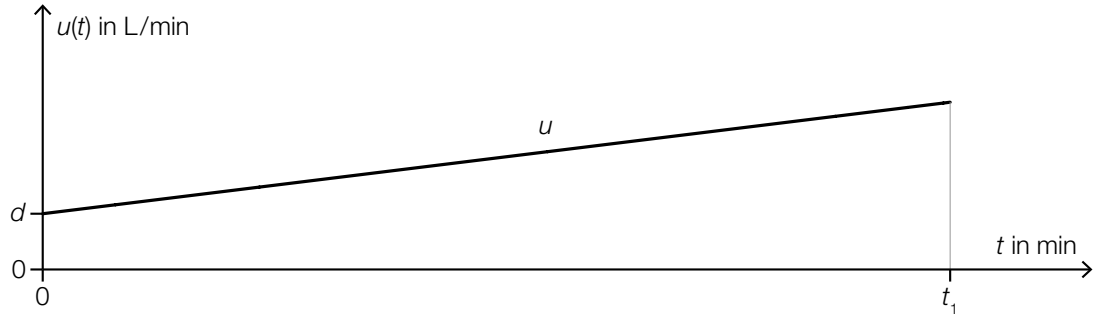
1) Determine m_1 using the diagram above.

$m_1 =$ _____ kg *[0/1 p.]*

c) For a mammal that is moving, the instantaneous rate of change of the oxygen consumption can be approximated in terms of the time t by the linear function u: $[0, t_1] \to \mathbb{R}$ with $t_1 \in \mathbb{R}^+$ (t in min, $u(t)$ in L/min).

$u(0) = d$ with $d \in \mathbb{R}^+$ holds.

The graph of u is shown in the diagram below.



1) Write down a formula that can be used to calculate $\int_0^{t_1} u(t)\,dt$. For this formula, use t_1, $u(t_1)$ and d.

$\int_0^{t_1} u(t)\,dt =$ _____ *[0 / 1 p.]*

2) Interpret $\int_0^{t_1} u(t)\,dt$ in the given context, including the corresponding unit. *[0 / 1 p.]*

Task 28 (Part 2, Best-of Assessment)

Flights

At Austrian airports, data on the number of flights, the number of passengers as well as the routes of the passengers is collected.

Data source: https://www.statistik.at/web_de/statistiken/energie_umwelt_innovation_mobilitaet/verkehr/luftfahrt/personenverkehr/index.html [19.12.2020].

Task:

a) The annual number of all passengers in Austria increased from 0.14 million in the year 1955 to 28.95 million in the year 2017.

This development of the number of passengers in Austria over time can be approximated by the exponential function $N\colon \mathbb{R}_0^+ \to \mathbb{R}^+$ with $N(t) = a \cdot b^t$ with $a, b \in \mathbb{R}^+$ (t in years with $t = 0$ for the year 1955, $N(t)$ in millions of passengers).

1) Determine a and b. *[0 / 1 p.]*

In the year 2018, there were 31.73 million passengers in Austria.

2) Show by calculation that the number of passengers given by N for the year 2018 deviates from the actual number by less than 1 %.

[0 / 1 p.]

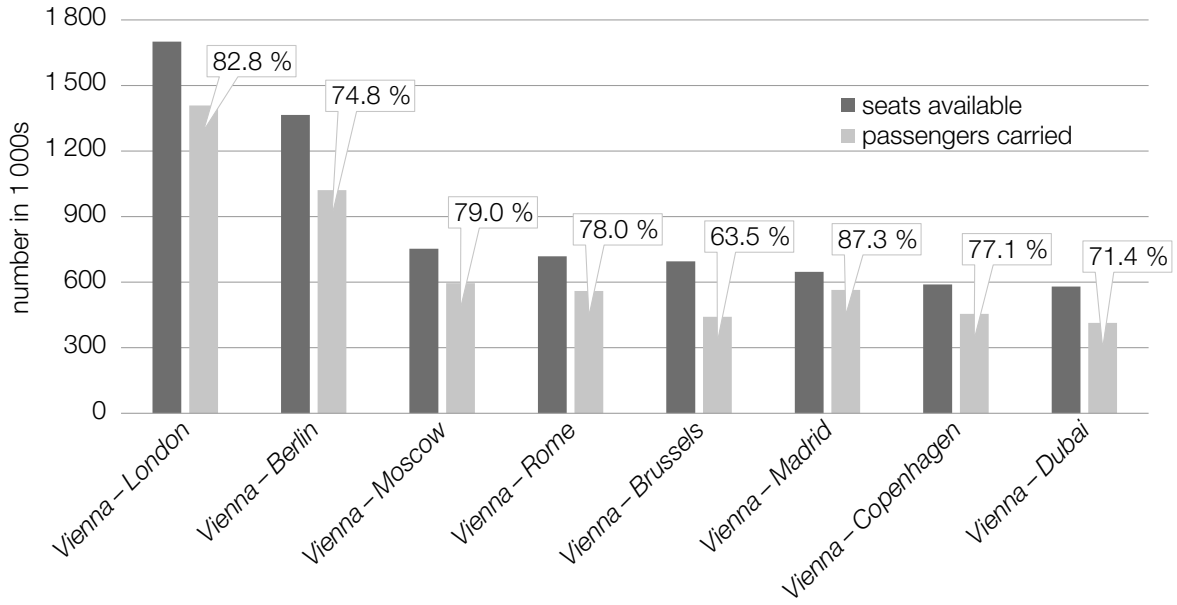
b) The number of flights and passengers in Austria for the years 2018 and 2019 are shown in the table below.

	number of flights	number of passengers
2018	296 852	31 725 019
2019	319 945	36 206 642

The average number of passengers per flight increased by n from 2018 to 2019.

1) Determine n. *[0 / 1 p.]*

c) The diagram below shows the number of seats available as well as the number of passengers for flights to and from Vienna for the year 2019. The percentages give the relative proportion of seats occupied by passengers.



1) Match each of the four statements for the year 2019 to the corresponding route from A to F. *[0/½/1 p.]*

On this route, more than twice as many passengers were carried than on the *Vienna – Moscow* route.	
On this route, the number of unoccupied seats was the smallest.	
On this route, the number of passengers carried was greater than 650 000 and smaller than 1.1 million.	
On this route, more than one third of the seats available were unoccupied.	

A	*Vienna – Berlin*
B	*Vienna – Madrid*
C	*Vienna – Brussels*
D	*Vienna – Copenhagen*
E	*Vienna – London*
F	*Vienna – Rome*