Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

May/June 2023

Mathematics

Supplementary Examination 1
Examiner's Version

Bundesministerium
Bildung, Wissenschaft
und Forschung

Instructions for the standardized implementation of the supplementary examination

The following supplementary examination booklet contains four tasks that can be completed independently of one another as well as the corresponding solutions.

Each task comprises three competencies to be demonstrated.

The preparation time is to be at least 30 minutes; the examination time is at most 25 minutes.

The use of the official formula booklet that has been approved by the relevant government authority for use in the standardized school-leaving examination in mathematics is allowed. Furthermore, the use of electronic devices (e.g. graphic display calculators or other appropriate technology) is allowed provided there is no possibility to communicate (e.g. via the internet, intranet, Bluetooth, mobile networks etc.) and there is no access to an individual's data stored on the device.

After the examination, all materials (tasks, extra sheets of paper etc.) from the candidates are to be collected in. The examination materials (tasks, extra sheets of paper, data that has been produced digitally etc.) may only be made public after the time period allocated for the examination has passed.

Evaluation grid for the supplementary examination

The evaluation grid below may be used to assist in assessing the candidates' performances.

	Candidate 1			Candidate 2			Candidate 3			Candidate 4			Candidate 5		
Task 1															
Task 2															
Task 3															
Task 4															
Total															

Explanatory notes on assessment

Each task can be awarded zero, one, two or three points. A maximum of twelve points can be achieved.

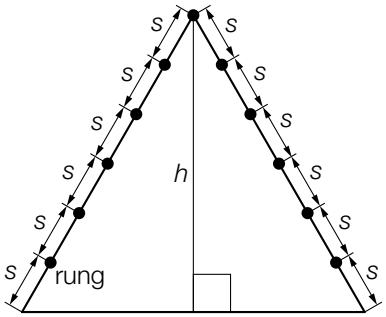
Assessment scale for the supplementary examination

Total number of competencies demonstrated	Assessment of the oral supplementary examination
12	Very good
10–11	Good
8–9	Satisfactory
6–7	Pass
0–5	Fail

Task 1

Climbing Frame

a) The diagrams below show a climbing frame. The side view shows an equilateral triangle. The rungs are represented as points.

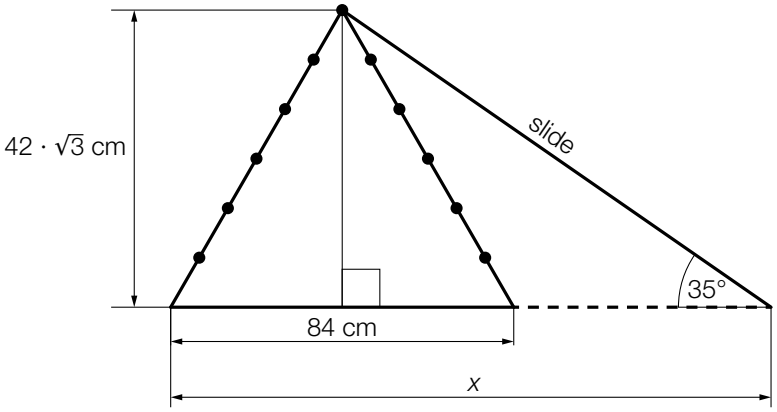


Source: BMBWF

1) Using the distance between the rungs s, write down a formula that can be used to calculate the height h of this climbing frame.

$h =$ _____

In a toy shop, a climbing frame is also sold together with a straight slide (see the not-to-scale diagram on the right).



2) Determine x.

b) A toy shop sells x climbing frames without slides and y climbing frames with slides in a particular month. In this month, the toy shop takes in a total of € 5 760 from the sale of climbing frames with and without slides.

This situation can be described by the system of linear equations shown below.

I: $100 \cdot x + 120 \cdot y = 5\,760$
II: $x + y = 50$

1) Interpret the values 100, 120 and 50 in the given context.

Solution to Task 1

Climbing Frame

a1) $h = \sqrt{(6 \cdot s)^2 - (3 \cdot s)^2} = \sqrt{27 \cdot s^2} = \sqrt{27} \cdot s$ *or* $h = \frac{6 \cdot s}{2} \cdot \sqrt{3} = 3 \cdot s \cdot \sqrt{3}$

a2) $\tan(35°) = \frac{42 \cdot \sqrt{3}}{x - 42}$

$x = 145.89...$ cm

b1) The price of one climbing frame without a slide is € 100.
The price of one climbing frame with a slide is € 120.
This toy shop sold a total of 50 climbing frames in this month.

Task 2

Play Equipment

A company produces and sells play equipment.
In order to plan financially, the costs, revenue and profit are investigated.

a) The costs can be approximated by the quadratic function K.

$K(x) = a \cdot x^2 + b \cdot x + c$

x ... number of play equipment units produced in ME
$K(x)$ ... the cost of producing x play equipment units in monetary units, GE

The following statements hold:

The fixed costs are 22 GE.
The cost of producing 20 ME is 40 GE.
The instantaneous rate of change of the costs when 20 ME are produced is 1.5 GE/ME.

1) Write down a system of equations that can be used to calculate the coefficients of K.

b) The profit can be approximated by the function G.

$G(x) = -\frac{11}{300} \cdot (x^2 - 70 \cdot x + 600)$
x ... number of play equipment units sold in ME
$G(x)$ ... the profit from selling x play equipment units in units of currency, GE

1) Determine the zeros of the function G.

c) For a particular x_0, the following statements hold:

$E'(x_0) = 0$
$E''(x_0) < 0$

x ... number of play equipment units sold in ME
$E(x)$ ... the revenue from selling x play equipment units in units of currency, GE

1) Interpret the meaning of x_0 in the given context.

Solution to Task 2

Play Equipment

a1) $K'(x) = 2 \cdot a \cdot x + b$

I: $K(0) = 22$
II: $K(20) = 40$
III: $K'(20) = 1.5$

or:

I: $a \cdot 0^2 + b \cdot 0 + c = 22$
II: $a \cdot 20^2 + b \cdot 20 + c = 40$
III: $2 \cdot a \cdot 20 + b = 1.5$

b1) $G(x) = 0$

calculation using technology:

$x_1 = 10$, $x_2 = 60$

c1) The maximum revenue is obtained for x_0 play equipment units (in ME).

Task 3

Internet Platform

a) The function N models the number of people who use an internet platform in terms of the time t.

$N(t) = 3000 \cdot 1.22^t$

t ... time in years since the start of the observations
$N(t)$ ... number of people who use this internet platform at time t

1) Determine the doubling time for the number of people who use this internet platform.

2) Write down the equation of the function N in the form $N(t) = a \cdot e^{\lambda \cdot t}$.

The expression below can be used to calculate the average rate of change of the number of people who use this internet platform in the first 6 years.

$$\frac{3000 \cdot 1.22^{\square} - \square}{\square - 0}$$

3) Complete the expression by writing the missing numbers in the boxes provided.

Solution to Task 3

Internet Platform

a1) $6\,000 = 3\,000 \cdot 1.22^{t}$

calculation using technology:

$t = 3.48...$

The doubling time is around 3.5 years.

a2) $\ln(1.22) = 0.1988...$

$N(t) = 3\,000 \cdot e^{0.199 \cdot t}$ *(coefficient rounded)*

a3) $\dfrac{3\,000 \cdot 1.22^{\boxed{6}} - \boxed{3\,000}}{\boxed{6} - 0}$

Task 4

Blood Groups

The table below shows the distribution of blood groups (in Austria).

blood group	0	A	B	AB
frequency	36 %	44 %	14 %	6 %

a) For a study, n people from Austria are selected at random and their blood group is determined.

1) Complete the formula below that can be used to calculate the probability that exactly 5 people have the blood group AB.

$P(\text{“exactly 5 people have the blood group AB”}) = \binom{n}{5} \cdot \square^{5} \cdot \square^{\square}$

b) For another study, 85 people from Austria are selected at random and their blood group is determined.

1) Determine the probability that the number of people with blood group A is at least 25 and at most 30.

c) In yet another study, 2 people from Austria are selected at random.

1) Write down a possible event E in the given context whose probability can be calculated using the expression below.

$P(E) = 2 \cdot 0.36 \cdot 0.14 \approx 0.10$

Solution to Task 4

Blood Groups

a1) P("exactly 5 people have the blood group AB") = $\binom{n}{5} \cdot \boxed{0.06}^{5} \cdot \boxed{0.94}^{\boxed{n-5}}$

b1) X ... number of people with blood group A

binomial distribution with $n = 85$ and $p = 0.44$

calculation using technology:

$P(25 \leq X \leq 30) = 0.0627...$

The probability is around 6.3 %.

c1) E ... of these two people, exactly 1 person has the blood group 0 and 1 person has the blood group B