Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

June 2024

Mathematics

Supplementary Examination 1
Examiner's Version

Bundesministerium
Bildung, Wissenschaft
und Forschung

Instructions for the standardized implementation of the supplementary examination

The following supplementary examination booklet contains four tasks that can be completed independently of one another as well as the corresponding solutions.

Each task comprises three competencies to be demonstrated.

The preparation time is to be at least 30 minutes; the examination time is at most 25 minutes.

The use of the official formula booklet that has been approved by the relevant government authority for use in the standardized school-leaving examination in mathematics is allowed. Furthermore, the use of electronic devices (e.g. graphic display calculators or other appropriate technology) is allowed provided there is no possibility to communicate (e.g. via the internet, intranet, Bluetooth, mobile networks etc.) and there is no access to an individual's data stored on the device.

After the examination, all materials (tasks, extra sheets of paper etc.) from the candidates are to be collected in. The examination materials (tasks, extra sheets of paper, data that has been produced digitally etc.) may only be made public after the time period allocated for the examination has passed.

Evaluation grid for the supplementary examination

The evaluation grid below may be used to assist in assessing the candidates' performances.

	Candidate 1			Candidate 2			Candidate 3			Candidate 4			Candidate 5		
Task 1															
Task 2															
Task 3															
Task 4															
Total															

Explanatory notes on assessment

Each task can be awarded zero, one, two or three points. A maximum of twelve points can be achieved.

Assessment scale for the supplementary examination

Total number of competencies demonstrated	Assessment of the oral supplementary examination
12	Very good
10–11	Good
8–9	Satisfactory
6–7	Pass
0–5	Fail

Task 1

Railways

a) The rail network of the Austrian Federal Railways comprises a length of around 3686 km of single-track routes. This length corresponds to 65.37 % of the total length of all routes of the Austrian Federal Railways.

1) Determine the total length of all routes of the Austrian Federal Railways.

b) The Mittenwald Railway has a gradient of 3.8 % at its steepest point.

1) Show that if the gradient of 3.8 % is doubled, then this means that the angle of the slope will also be approximately doubled.

c) A group of 9 people buys tickets for a train trip. This group comprises 3 adults, 2 seniors and 4 children; they pay a total of g euros.
The price of 1 adult ticket is twice the price of 1 child ticket.
The price of 1 senior ticket is 25 % less than the price of 1 adult ticket.

a ... price of 1 adult ticket in euros
s ... price of 1 senior ticket in euros
c ... price of 1 child ticket in euros

1) Using g, write down a system of equations that can be used to determine a, s and c.

Solution to Task 1

Railways

a1) $\frac{3686}{0.6537} = 5638.6...$

The total length of all routes of the Austrian Federal Railways is around 5639 km.

b1) angle of the slope for a gradient of 3.8 %:

$\alpha = \arctan(0.038) = 2.17...°$

angle of the slope for a doubled gradient of 7.6 %:

$\beta = \arctan(0.076) = 4.34...°$

Therefore $\beta \approx 2 \cdot \alpha$ holds.

c1) $3 \cdot a + 2 \cdot s + 4 \cdot c = g$
$a = 2 \cdot c$
$s = 0.75 \cdot a$

Task 2

Rare Breed

In a particular area, animals of a rare breed are observed over a certain time period.

The table below shows the number of animals for the years 2010 and 2020.

year	number of animals
2010	600
2020	300

a) It can be assumed that the number of animals decayed exponentially in the time period from 2010 to 2020.
The exponential function f models the number of animals in terms of time.

t ... time in years with $t = 0$ for the year 2010
$f(t)$ ... number of animals at time t

1) Write down an equation of the exponential function f.

b) 1) Interpret the result of the calculation shown below in the given context.

$$\frac{300 - 600}{2020 - 2010} = -30$$

c) In another model, the number of animals in the time period from 2010 to 2020 is given by the function g.

$g(t) = \frac{c}{t}$ with $10 \leq t \leq 20$
t ... time in years with $t = 0$ for the year 2000
$g(t)$ ... number of animals at time t
c ... positive parameter

1) Determine the number of animals in the year 2015 according to this model.

Solution to Task 2

Rare Breed

a1) $f(t) = a \cdot b^t$ *or:* $f(t) = a \cdot e^{\lambda \cdot t}$

$f(0) = 600$ $f(0) = 600$

$f(10) = 300$ $f(10) = 300$

$a = 600$ $a = 600$

$b = \sqrt[10]{\frac{300}{600}} = 0.9330...$ $\lambda = \ln(0.9330...) = -0.0693...$

$f(t) = 600 \cdot 0.933^t$ $f(t) = 600 \cdot e^{-0.0693... \cdot t}$

b1) In the time period from 2010 to 2020, the number of animals reduced by an average of 30 animals per year.

c1) $g(10) = 600$

$c = 10 \cdot 600 = 6000$

$g(15) = \frac{6000}{15} = 400$

In the year 2015, the number of animals according to this model was 400.

Task 3

Wine Cellar

a) The air temperature in a wine cellar is measured regularly (see table below).

time in days	0	60	100
air temperature in °C	8	13	17

1) Show by calculation that the three pairs of values shown in the table above are points that do not lie on a line.

b) The temperature over time in a different cellar can be modelled by the function T.

$T(t) = 0.0005 \cdot t^3 - 0.02 \cdot t^2 + 0.23 \cdot t + 8 \quad \text{with} \quad 0 \leq t \leq 24$

t ... time in h with $t = 0$ for the start of the measurements
$T(t)$ ... temperature at time t in °C

The average temperature in a time interval $[t_1, t_2]$ can be calculated using the expression below.

$$\frac{1}{t_2 - t_1} \cdot \int_{t_1}^{t_2} T(t)\,\mathrm{d}t$$

1) Determine the average temperature in this cellar in the time interval [0, 24].

c) In a wine cellar, there is a dehumidifier that collects water from the air in the form of condensation.

At an air temperature of 10°C, the volume of condensation collected per day is 5 l. At an air temperature of 20°C, the volume of condensation collected per day is 7 l.
At an air temperature of 11.25°C, the volume of condensation collected per day is lowest.
The volume of condensation collected per day in terms of the air temperature is to be modelled by the quadratic function V.

$V(T) = a \cdot T^2 + b \cdot T + c$

T ... air temperature in °C
$V(T)$ ... volume of condensation collected per day at an air temperature of T in l

1) Write down a system of linear equations that can be used to determine the coefficients a, b and c.

Solution to Task 3

Wine Cellar

a1) $\frac{13-8}{60-0} = 0.083...$

$\frac{17-13}{100-60} = 0.1$

$\frac{17-8}{100-0} = 0.09$

As the difference quotients are not equal, the three points do not lie on a line.

For the point to be awarded, it is not necessary for all 3 difference quotients to be calculated. A justification that uses the reciprocal values of the difference quotients given is also correct.

b1) $\frac{1}{24-0} \cdot \int_0^{24} T(t)\,\mathrm{d}t = 8.648$

The average temperature in this cellar in the time interval [0, 24] is around 8.65 °C.

c1) $V'(T) = 2 \cdot a \cdot T + b$

I: $V(10) = 5$
II: $V(20) = 7$
III: $V'(11.25) = 0$

or:

I: $100 \cdot a + 10 \cdot b + c = 5$
II: $400 \cdot a + 20 \cdot b + c = 7$
III: $22.5 \cdot a + b = 0$

Task 4

Cinema

7 friends are going to the cinema to watch a film together.

a) At this cinema, there are reduced ticket prices for members of the bonus club and for school pupils.

All of the prices are shown in the table below.

	price per cinema ticket in €
normal price	15
member of the bonus club	13.50
school pupil	12

The 7 friends buy 2 tickets at the normal price, 1 ticket for a member of the bonus club and 4 tickets at the school pupil price.

1) Interpret the result of the calculation shown below in the given context.

$$\frac{2 \cdot 15 + 13.50 + 4 \cdot 12}{7} \approx 13.07...$$

b) At this cinema, there is the opportunity to win vouchers. Every person receives exactly one chance. The probability of winning a voucher is the same for each person.

The binomially distributed random variable X describes how many of the 7 friends win exactly one voucher.

$P(X = 0) = 0.3206$ holds.

1) Determine the probability that at least 3 out of the 7 friends win exactly one voucher.

c) The number of the 7 friends who enjoyed the film is given by a.
After going to the cinema, 2 out of the 7 friends are chosen at random to participate in a questionnaire.
The event that these 2 friends enjoyed the film is given by E.

1) Using a, write down a formula that can be used to calculate the probability $P(E)$.

$P(E) =$ ____

Solution to Task 4

Cinema

a1) The 7 friends spend on average around 13.07 euros on a ticket.

or:

The mean of the prices of the tickets for these 7 friends is around 13.07 euros.

b1) $\binom{7}{0} \cdot p^0 \cdot (1 - p)^7 = (1 - p)^7 = 0.3206$
$p = 1 - \sqrt[7]{0.3206}$
$p = 0.1499...$

calculation using technology:

$P(X \geq 3) = 0.0737...$

The probability that at least 3 out of the 7 friends win exactly one voucher is around 7.4 %.

c1) $P(E) = \frac{a}{7} \cdot \frac{a - 1}{6}$

Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

May / June 2023

Mathematics

Supplementary Examination 1
Examiner's Version

Bundesministerium
Bildung, Wissenschaft
und Forschung

Instructions for the standardized implementation of the supplementary examination

The following supplementary examination booklet contains four tasks that can be completed independently of one another as well as the corresponding solutions.

Each task comprises three competencies to be demonstrated.

The preparation time is to be at least 30 minutes; the examination time is at most 25 minutes.

The use of the official formula booklet that has been approved by the relevant government authority for use in the standardized school-leaving examination in mathematics is allowed. Furthermore, the use of electronic devices (e.g. graphic display calculators or other appropriate technology) is allowed provided there is no possibility to communicate (e.g. via the internet, intranet, Bluetooth, mobile networks etc.) and there is no access to an individual's data stored on the device.

After the examination, all materials (tasks, extra sheets of paper etc.) from the candidates are to be collected in. The examination materials (tasks, extra sheets of paper, data that has been produced digitally etc.) may only be made public after the time period allocated for the examination has passed.

Evaluation grid for the supplementary examination

The evaluation grid below may be used to assist in assessing the candidates' performances.

	Candidate 1			Candidate 2			Candidate 3			Candidate 4			Candidate 5		
Task 1															
Task 2															
Task 3															
Task 4															
Total															

Explanatory notes on assessment

Each task can be awarded zero, one, two or three points. A maximum of twelve points can be achieved.

Assessment scale for the supplementary examination

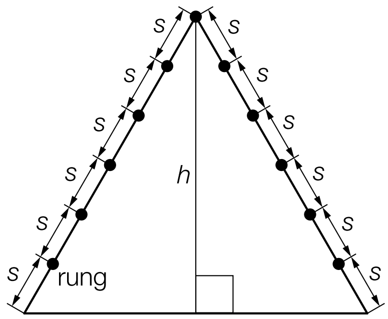
Total number of competencies demonstrated	Assessment of the oral supplementary examination
12	Very good
10–11	Good
8–9	Satisfactory
6–7	Pass
0–5	Fail

Task 1

Climbing Frame

a) The diagrams below show a climbing frame. The side view shows an equilateral triangle. The rungs are represented as points.

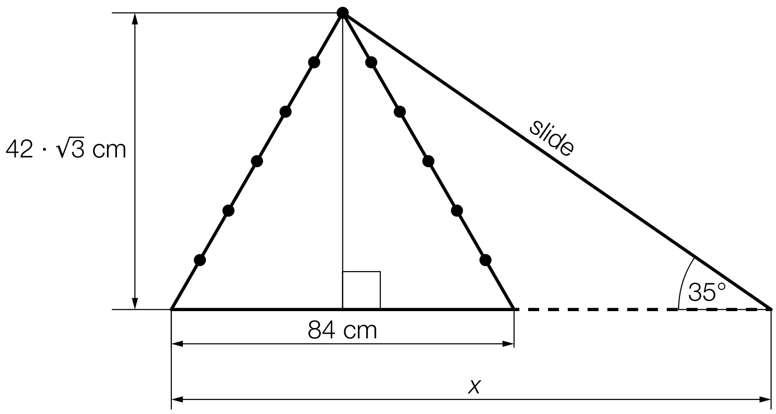
Source: BMBWF



1) Using the distance between the rungs *s*, write down a formula that can be used to calculate the height *h* of this climbing frame.

$h =$ ______________________________

In a toy shop, a climbing frame is also sold together with a straight slide (see the not-to-scale diagram on the right).



2) Determine *x*.

b) A toy shop sells *x* climbing frames without slides and *y* climbing frames with slides in a particular month. In this month, the toy shop takes in a total of € 5760 from the sale of climbing frames with and without slides.

This situation can be described by the system of linear equations shown below.

I: $100 \cdot x + 120 \cdot y = 5760$
II: $x + y = 50$

1) Interpret the values 100, 120 and 50 in the given context.

Solution to Task 1

Climbing Frame

a1) $h = \sqrt{(6 \cdot s)^2 - (3 \cdot s)^2} = \sqrt{27 \cdot s^2} = \sqrt{27} \cdot s$ *or* $h = \frac{6 \cdot s}{2} \cdot \sqrt{3} = 3 \cdot s \cdot \sqrt{3}$

a2) $\tan(35°) = \frac{42 \cdot \sqrt{3}}{x - 42}$

$x = 145.89...$ cm

b1) The price of one climbing frame without a slide is € 100.
The price of one climbing frame with a slide is € 120.
This toy shop sold a total of 50 climbing frames in this month.

Task 2

Play Equipment

A company produces and sells play equipment.
In order to plan financially, the costs, revenue and profit are investigated.

a) The costs can be approximated by the quadratic function K.

$K(x) = a \cdot x^2 + b \cdot x + c$

x ... number of play equipment units produced in ME
$K(x)$ ... the cost of producing x play equipment units in monetary units, GE

The following statements hold:

The fixed costs are 22 GE.
The cost of producing 20 ME is 40 GE.
The instantaneous rate of change of the costs when 20 ME are produced is 1.5 GE/ME.

1) Write down a system of equations that can be used to calculate the coefficients of K.

b) The profit can be approximated by the function G.

$G(x) = -\frac{11}{300} \cdot (x^2 - 70 \cdot x + 600)$

x ... number of play equipment units sold in ME
$G(x)$ ... the profit from selling x play equipment units in units of currency, GE

1) Determine the zeros of the function G.

c) For a particular x_0, the following statements hold:

$E'(x_0) = 0$
$E''(x_0) < 0$

x ... number of play equipment units sold in ME
$E(x)$ ... the revenue from selling x play equipment units in units of currency, GE

1) Interpret the meaning of x_0 in the given context.

Solution to Task 2

Play Equipment

a1) $K'(x) = 2 \cdot a \cdot x + b$

I: $K(0) = 22$
II: $K(20) = 40$
III: $K'(20) = 1.5$

or:

I: $a \cdot 0^2 + b \cdot 0 + c = 22$
II: $a \cdot 20^2 + b \cdot 20 + c = 40$
III: $2 \cdot a \cdot 20 + b = 1.5$

b1) $G(x) = 0$

calculation using technology:

$x_1 = 10$, $x_2 = 60$

c1) The maximum revenue is obtained for x_0 play equipment units (in ME).

Task 3

Internet Platform

a) The function N models the number of people who use an internet platform in terms of the time t.

$N(t) = 3000 \cdot 1.22^t$

t ... time in years since the start of the observations
$N(t)$ ... number of people who use this internet platform at time t

1) Determine the doubling time for the number of people who use this internet platform.
2) Write down the equation of the function N in the form $N(t) = a \cdot e^{\lambda \cdot t}$.

The expression below can be used to calculate the average rate of change of the number of people who use this internet platform in the first 6 years.

$$\frac{3000 \cdot 1.22^{\boxed{}} - \boxed{}}{\boxed{} - 0}$$

3) Complete the expression by writing the missing numbers in the boxes provided.

Solution to Task 3

Internet Platform

a1) $6\,000 = 3\,000 \cdot 1.22^t$

calculation using technology:

$t = 3.48...$

The doubling time is around 3.5 years.

a2) $\ln(1.22) = 0.1988...$

$N(t) = 3\,000 \cdot e^{0.199 \cdot t}$ *(coefficient rounded)*

a3) $\dfrac{3\,000 \cdot 1.22^{\boxed{6}} - \boxed{3\,000}}{\boxed{6} - 0}$

Task 4

Blood Groups

The table below shows the distribution of blood groups (in Austria).

blood group	0	A	B	AB
frequency	36 %	44 %	14 %	6 %

a) For a study, *n* people from Austria are selected at random and their blood group is determined.

1) Complete the formula below that can be used to calculate the probability that exactly 5 people have the blood group AB.

$P(\text{“exactly 5 people have the blood group AB”}) = \binom{n}{5} \cdot \boxed{}^{5} \cdot \boxed{}^{\boxed{}}$

b) For another study, 85 people from Austria are selected at random and their blood group is determined.

1) Determine the probability that the number of people with blood group A is at least 25 and at most 30.

c) In yet another study, 2 people from Austria are selected at random.

1) Write down a possible event E in the given context whose probability can be calculated using the expression below.

$P(E) = 2 \cdot 0.36 \cdot 0.14 \approx 0.10$

Solution to Task 4

Blood Groups

a1) $P(\text{“exactly 5 people have the blood group AB”}) = \binom{n}{5} \cdot \boxed{0.06}^{5} \cdot \boxed{0.94}^{\boxed{n-5}}$

b1) X ... number of people with blood group A

binomial distribution with $n = 85$ and $p = 0.44$

calculation using technology:

$P(25 \leq X \leq 30) = 0.0627...$

The probability is around 6.3 %.

c1) E ... of these two people, exactly 1 person has the blood group 0 and 1 person has the blood group B

Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

October 2023

Mathematics

Supplementary Examination 1
Examiner's Version

Bundesministerium
Bildung, Wissenschaft
und Forschung

Instructions for the standardized implementation of the supplementary examination

The following supplementary examination booklet contains four tasks that can be completed independently of one another as well as the corresponding solutions.

Each task comprises three competencies to be demonstrated.

The preparation time is to be at least 30 minutes; the examination time is at most 25 minutes.

The use of the official formula booklet that has been approved by the relevant government authority for use in the standardized school-leaving examination in mathematics is allowed. Furthermore, the use of electronic devices (e.g. graphic display calculators or other appropriate technology) is allowed provided there is no possibility to communicate (e.g. via the internet, intranet, Bluetooth, mobile networks etc.) and there is no access to an individual's data stored on the device.

After the examination, all materials (tasks, extra sheets of paper etc.) from the candidates are to be collected in. The examination materials (tasks, extra sheets of paper, data that has been produced digitally etc.) may only be made public after the time period allocated for the examination has passed.

Evaluation grid for the supplementary examination

The evaluation grid below may be used to assist in assessing the candidates' performances.

	Candidate 1			Candidate 2			Candidate 3			Candidate 4			Candidate 5		
Task 1															
Task 2															
Task 3															
Task 4															
Total															

Explanatory notes on assessment

Each task can be awarded zero, one, two or three points. A maximum of twelve points can be achieved.

Assessment scale for the supplementary examination

Total number of competencies demonstrated	Assessment of the oral supplementary examination
12	Very good
10–11	Good
8–9	Satisfactory
6–7	Pass
0–5	Fail

Task 1

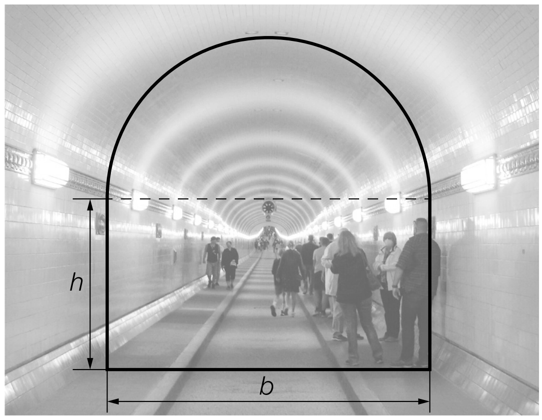
Old Elbe Tunnel

The Old Elbe Tunnel in Hamburg is an underpass for the Elbe.

a) The cross-section of the tunnel can be approximated by a rectangle with a semi-circle on top (see diagram on the right).

b ... width in m
h ... height in m

Daniel would like to calculate the volume of air V in the 426.5 m long Old Elbe Tunnel.

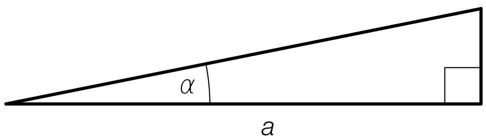


Source: BMBWF

1) Write down a formula that can be used to calculate V in terms of b and h.

$V =$ ______________________________

b) The diagram below shows a model of the gradient of a section of the cycle path in the tunnel.



a ... horizontal length of this section of the tunnel in m
α ... angle of elevation of this section of the tunnel

A cyclist cycles along this section of the tunnel with velocity v in m/s.

The following statement holds: $\dfrac{\frac{a}{\cos(\alpha)}}{v} = 12.5$

1) Interpret the value 12.5 in the given context including the corresponding unit.

c) In the first year after opening, 20 million people used the Old Elbe Tunnel. The number of people who used the Old Elbe Tunnel each year reduced by 97.5 % by 1985 and then increased again. In the year 2008, 40 % more people used the Old Elbe Tunnel than in the year 1985.

1) Determine the number of people who used the Old Elbe Tunnel in the year 2008.

Solution to Task 1

Old Elbe Tunnel

a1) $V = 426.5 \cdot \left(b \cdot h + \frac{1}{2} \cdot \left(\frac{b}{2}\right)^2 \cdot \pi\right)$

or:

$V = 426.5 \cdot \left(b \cdot h + \frac{b^2}{8} \cdot \pi\right)$

b1) The cyclist requires 12.5 s for this section of the tunnel.

c1) $20\,000\,000 \cdot 0.025 \cdot 1.4 = 700\,000$

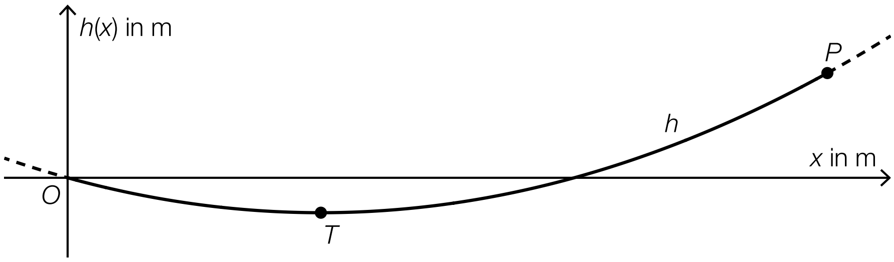
700 000 people used the Old Elbe Tunnel in the year 2008.

Task 2

Suspension Bridge

The shape of a particular suspension bridge for pedestrians can be modelled by quadratic functions.

a) In one model, the shape of the suspension bridge is described by the function h with $h(x) = a \cdot x^2 + b \cdot x$ (the side view is shown in the diagram below).



The graph of h goes through the point $P = (120, 6)$. The lowest point T of the bridge occurs when $x = 40$.

In order to determine the coefficients a and b, the system of equations shown below is created using the information about the points P and T.

1) Write down the missing numbers in the boxes provided.

I: $a \cdot \boxed{}^2 + b \cdot \boxed{} = \boxed{}$

II: $a \cdot \boxed{} + b = \boxed{}$

The following statement about the function h holds: $h(x) = 0.00125 \cdot x^2 - 0.1 \cdot x$

2) Determine the angle of elevation of the tangent to the graph of h at the point P.

In another coordinate system, the shape of the suspension bridge can be described by the function f with $f(x) = a \cdot x^2$.

3) In the diagram above, draw the coordinate axes for the graph of f.

Solution to Task 2

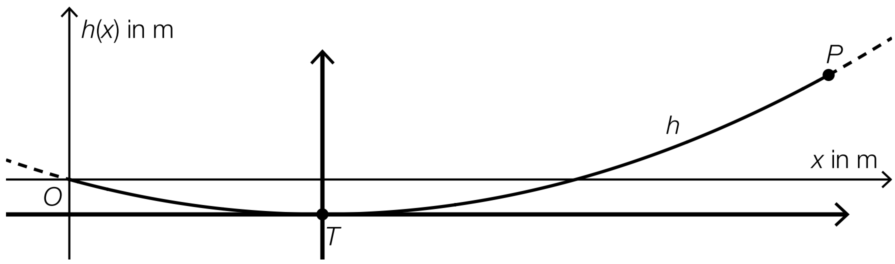
Suspension Bridge

a1) I: $a \cdot \boxed{120}^2 + b \cdot \boxed{120} = \boxed{6}$

II: $a \cdot \boxed{80} + b = \boxed{0}$

a2) $\alpha = \arctan(h'(120)) = \arctan(0.2)$
$\alpha = 11.30...°$

a3)



Task 3

Sporting Goods

a) For a particular item of sporting goods, the derivative K' of the cost function K is given.

$K'(x) = 3 \cdot x^2 - 8 \cdot x + 20$

x ... number of produced units of quantity in ME
$K'(x)$ ... 1st derivative of the cost function K for x ME in GE/ME, where GE are units of currency

The fixed costs are 4200 GE.

1) Write down an equation of the cost function K.

b) For another item of sporting goods, the cost function K_1 and the revenue function E_1 are given.

$K_1(x) = 0.01 \cdot x^2 + 10 \cdot x + 200$
$E_1(x) = -0.25 \cdot x^2 + 50 \cdot x$

x ... number of produced and sold units of quantity in ME
$K_1(x)$ ... total costs for x ME in units of currency, GE
$E_1(x)$ ... revenue for x ME in units of currency, GE

1) Determine the profit when $x = 70$ ME.

c) In a study it has been investigated how many units of a particular item of sporting goods can be sold long-term.
The number of units sold can be modelled by the function A in terms of time.

$A(t) = a - 30 \cdot b^t$ with $0 < b < 1$

t ... time in months with $t = 0$ for the start of sales
$A(t)$ ... number of units sold at time t
a, b ... parameters

1) Using the equation of the function of A, justify why no more than a units can ever be sold according to this model.

Solution to Task 3

Sporting Goods

a1) $K(x) = \int K'(x)\,\mathrm{d}x = 3 \cdot \frac{x^3}{3} - 8 \cdot \frac{x^2}{2} + 20 \cdot x + C = x^3 - 4 \cdot x^2 + 20 \cdot x + C$
$K(0) = 4\,200$
$C = 4\,200$

$K(x) = x^3 - 4 \cdot x^2 + 20 \cdot x + 4\,200$

b1) $G_1(x) = E_1(x) - K_1(x) = -0.26 \cdot x^2 + 40 \cdot x - 200$
$G_1(70) = 1\,326$

The profit for 70 ME is 1 326 GE.

c1) The expression $30 \cdot b^t$ is positive for all values of t and therefore the expression $a - 30 \cdot b^t$ can never take a greater value than a.

Task 4

Dice

In a particular game, players roll fair six-sided dice. The faces of these dice are labelled with the digits 1, 2, 3, … , 6.

a) Andrea rolls a dice multiple times.

1) Write down a formula that can be used to calculate the probability P shown below.

   P("Andrea does not roll a single 6 in n rolls") = ____________________

b) Ferdinand rolls 2 dice once.

He claims: "The probability that the sum of the faces is 5 is greater than the probability that the sum of the faces is 4."

1) Justify by calculation that Ferdinand's claim is correct.

c) Sabrina rolls 5 dice once.

1) Determine the probability that exactly 4 of the 5 dice show the same digit.

Solution to Task 4

Dice

a1) $P(E) = \left(\frac{5}{6}\right)^n$

b1) sum of the faces is 5: 1 + 4 *or* 2 + 3 *or* 3 + 2 *or* 4 + 1
sum of the faces is 4: 1 + 3 *or* 2 + 2 *or* 3 + 1
Therefore:
$P(X = 5) = \frac{4}{36}$
$P(X = 4) = \frac{3}{36}$
$\frac{4}{36} > \frac{3}{36}$

c1) $6 \cdot \binom{5}{4} \cdot \left(\frac{1}{6}\right)^4 \cdot \left(\frac{5}{6}\right) = 0.0192...$
The probability is around 1.9 %.

Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

January 2024

Mathematics

Supplementary Examination 1
Examiner's Version

Bundesministerium
Bildung, Wissenschaft
und Forschung

Instructions for the standardized implementation of the supplementary examination

The following supplementary examination booklet contains four tasks that can be completed independently of one another as well as the corresponding solutions.

Each task comprises three competencies to be demonstrated.

The preparation time is to be at least 30 minutes; the examination time is at most 25 minutes.

The use of the official formula booklet that has been approved by the relevant government authority for use in the standardized school-leaving examination in mathematics is allowed. Furthermore, the use of electronic devices (e.g. graphic display calculators or other appropriate technology) is allowed provided there is no possibility to communicate (e.g. via the internet, intranet, Bluetooth, mobile networks etc.) and there is no access to an individual's data stored on the device.

After the examination, all materials (tasks, extra sheets of paper etc.) from the candidates are to be collected in. The examination materials (tasks, extra sheets of paper, data that has been produced digitally etc.) may only be made public after the time period allocated for the examination has passed.

Evaluation grid for the supplementary examination

The evaluation grid below may be used to assist in assessing the candidates' performances.

	Candidate 1			Candidate 2			Candidate 3			Candidate 4			Candidate 5		
Task 1															
Task 2															
Task 3															
Task 4															
Total															

Explanatory notes on assessment

Each task can be awarded zero, one, two or three points. A maximum of twelve points can be achieved.

Assessment scale for the supplementary examination

Total number of competencies demonstrated	Assessment of the oral supplementary examination
12	Very good
10–11	Good
8–9	Satisfactory
6–7	Pass
0–5	Fail

Task 1

Crude Oil

a) On a particular day, the global consumption of crude oil was 15.1 billion litres.

A unit for the volume of crude oil is the barrel.

1 barrel corresponds to the volume of a cylindrical container with a diameter of 50 cm and a height of 81 cm.

1) Write down 15.1 billion litres in the unit barrels.

b) In the year 2018, 8.4 billion litres of diesel and 2.2 billion litres of petrol were sold in Austria.

The average price of 1 litre of diesel was x euros; the average price for 1 litre of petrol was y euros.
The total income from the sale of diesel and petrol was 13.02 billion euros.
The income from the sale of diesel was 7.476 billion euros higher than the income from the sale of petrol.

1) Write down a system of equations that can be used to determine x and y.

c) A system of equations in the variables x and y with the parameter c is shown below.

I: $c \cdot x + 4 \cdot y = 40$
II: $4 \cdot x + 2 \cdot y = 26$

1) Write down the value of c for which the system of equations has no solution.

$c =$ ____________________

Solution to Task 1

Crude Oil

a1) volume of a barrel in litres:

$V = 2.5^2 \cdot \pi \cdot 8.1 = 159.0...$

$\frac{15.1 \cdot 10^9}{159.0...} = 94.9... \cdot 10^6$

15.1 billion litres correspond to around 95 million barrels.

b1) I: $8.4 \cdot x + 2.2 \cdot y = 13.02$

II: $8.4 \cdot x = 7.476 + 2.2 \cdot y$

c1) $c = 8$

Task 2

Lighting

a) On a particular street in a town, new bulbs are to be installed in 174 streetlights. The town receives the following estimate:
A new bulb costs € 7.90, and exactly 1 bulb will be installed in each streetlight.
The running costs for all 174 streetlights are € 2.86 per hour.

The total costs for the lighting on this street are to be described by the function K in terms of the running time t.

t ... running time in h
$K(t)$ ... cost of the running time t in euros

1) Write down an equation of the function K. The time $t = 0$ corresponds to the time when the new bulbs are installed.

b) For the lighting on another street, a selection of two types of bulb, A and B, is available.
The lighting costs for bulb type A can be described by the function K_A.
The lighting costs for bulb type B can be described by the function K_B.

$K_A(t) = 600 + 429 \cdot t$
$K_B(t) = 1\,050 + 285 \cdot t$

t ... time in years
$K_A(t)$, $K_B(t)$ ... lighting costs after a total of t years in euros

1) Determine after how many years the lighting costs for each bulb type are the same.
2) Interpret the result of the calculation shown below in the given context.

$K_A(10) - K_B(10) = 990$

Solution to Task 2

Lighting

a1) $K(t) = 174 \cdot 7.9 + 2.86 \cdot t$

or:

$K(t) = 1\,374.6 + 2.86 \cdot t$

b1) $1\,050 + 285 \cdot t = 600 + 429 \cdot t$

calculation using technology:

$t = 3.125$

After 3.125 years, the lighting costs for the two bulb types are the same.

b2) After a total of 10 years, the lighting costs for bulb type *A* are 990 euros higher than the lighting costs for bulb type *B*.

Task 3

Storms

In June 2012, there were heavy storms in Austria.

a) During one storm in Graz, the following data was collected:
At the beginning of the storm, the instantaneous precipitation per square metre was 150 ml per min.
The maximum value of the instantaneous precipitation per square metre of 400 ml per min was reached 50 min after the beginning of the storm.

The trend of the instantaneous precipitation per square metre over time can be approximated by the quadratic function f with $f(t) = a \cdot t^2 + b \cdot t + c$.

t ... time since the beginning of the storm in min
$f(t)$ ... instantaneous precipitation per square metre at time t in ml per min

1) Write down a system of equations that can be used to calculate the coefficients a, b and c.

b) In Mürzzuschlag, a storm lasted 2.5 h. For this storm, the instantaneous precipitation per square metre can be approximated by the function N shown below.

$N(t) = -\frac{44}{3} \cdot t^3 + 44 \cdot t^2 - \frac{103}{3} \cdot t + 40 \quad \text{with} \quad 0 \leq t \leq 2.5$
t ... time since the beginning of the storm in h
$N(t)$ ... instantaneous precipitation per square metre at time t in L per h

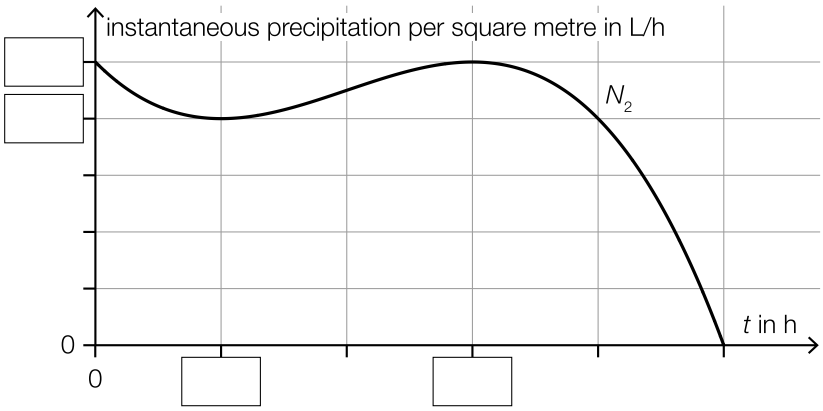
The total amount of precipitation per square metre in the time interval $[t_1, t_2]$ can be calculated using the expression shown below.
$\int_{t_1}^{t_2} N(t)\,\mathrm{d}t$

1) Determine the total amount of precipitation per square metre that fell during these 2.5 hours. Write down the result with the corresponding unit.

c) The instantaneous precipitation per square metre was also measured in a neighbouring town. Using the values collected, the graph of the 3rd degree polynomial function N_2 was created.

- $t_w = 1$ is the x-coordinate of the point of inflexion of the function N_2.
- At the x-coordinate t_m of the minimum of the function N_2, the following statements hold: $f(t_m) = 32$ and $f'(t_m) = 0$

1) In the diagram below, write down the missing numbers in the boxes provided.



Solution to Task 3

Storms

a1) $f(t) = a \cdot t^2 + b \cdot t + c$
$f'(t) = 2 \cdot a \cdot t + b$

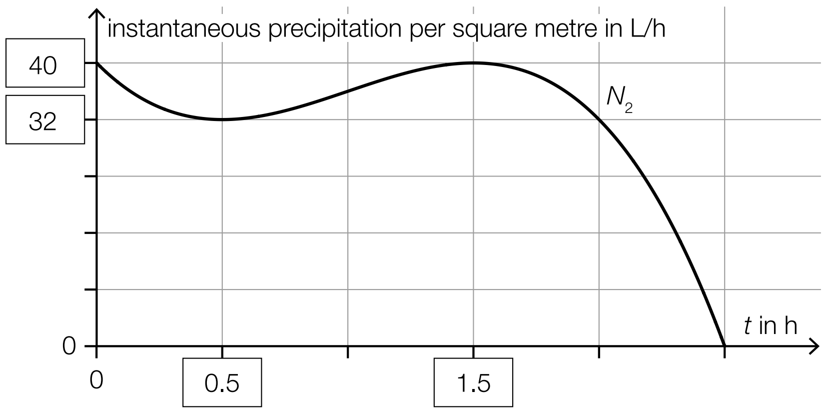
$f(0) = 150$
$f'(50) = 0$
$f(50) = 400$

or:

$c = 150$
$100 \cdot a + b = 0$
$2500 \cdot a + 50 \cdot b + c = 400$

b1) $\int_0^{2.5} N(t)\,\mathrm{d}t = 78.645...$
The total amount of precipitation per square metre was around 78.6 L.

c1)



Task 4

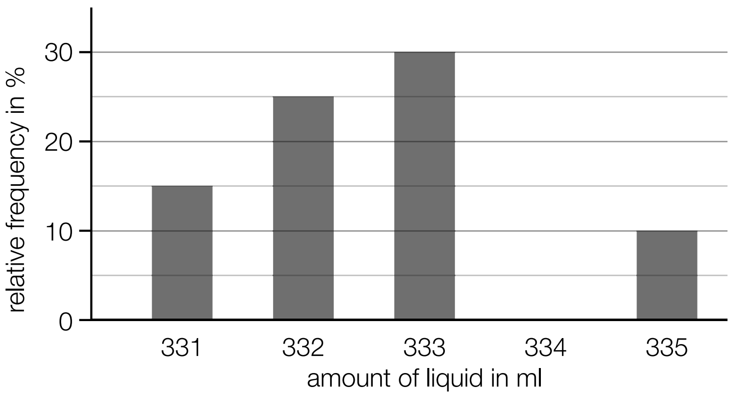
Soft Drinks

Soft drinks are offered in a variety of bottles and cans.

a) Markus investigates the amount of liquid in 5 glass bottles and measures the following values:
amount of liquid in ml: 753, 754, 752, 754, 753

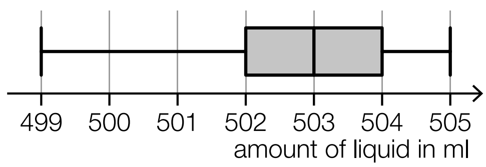
1) Determine the mean and the standard deviation of the amount of liquid in these 5 glass bottles.

b) Using a machine, Nina investigates the amount of liquid in plastic bottles. She measures the following amounts in ml: 331, 332, 333, 334 and 335.
The results of her investigation are represented in the bar chart below. One bar on the chart is missing.



1) Complete the bar chart above by drawing the missing bar.

c) Antonia investigates the amount of liquid in 37 cans. The results of her investigation are shown in the box plot on the right.



1) Put a cross next to each of the two correct statements. *[2 out of 5]*

The range is 2 ml.	☐
The median is 503 ml.	☐
The interquartile range (the difference between the 3rd quartile and the 1st quartile) is 1 ml.	☐
At least 18 cans contain at most 503 ml of liquid.	☐
At least 25 cans contain at least 504 ml of liquid.	☐

Solution to Task 4

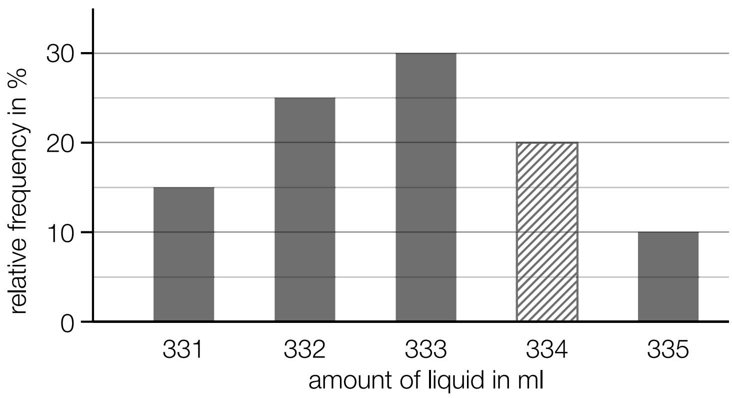
Soft Drinks

a1) calculation using technology:

mean: $\bar{x} = 753.2$ ml
standard deviation: $s = 0.748...$ ml

A standard deviation calculated to be $s_{n-1} = 0.836...$ ml *is also to be considered correct.*

b1)



c1)

The median is 503 ml.	☒
At least 18 cans contain at most 503 ml of liquid.	☒

Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

June 2022

Mathematics

Supplementary Examination 5
Examiner's Version

Bundesministerium
Bildung, Wissenschaft
und Forschung

Instructions for the standardized implementation of the supplementary examination

The following supplementary examination booklet contains four tasks that can be completed independently of one another as well as the corresponding solutions.

Each task comprises three competencies to be demonstrated.

The preparation time is to be at least 30 minutes; the examination time is at most 25 minutes.

The use of the official formula booklet that has been approved by the relevant government authority for use in the standardized school-leaving examination in mathematics is allowed. Furthermore, the use of electronic devices (e.g. graphic display calculators or other appropriate technology) is allowed provided there is no possibility to communicate (e.g. via the internet, intranet, Bluetooth, mobile networks etc.) and there is no access to an individual's data stored on the device.

After the examination, all materials (tasks, extra sheets of paper etc.) from the candidates are to be collected in. The examination materials (tasks, extra sheets of paper, data that has been produced digitally etc.) may only be made public after the time period allocated for the examination has passed.

Evaluation grid for the supplementary examination

The evaluation grid below may be used to assist in assessing the candidates' performances.

	Candidate 1			Candidate 2			Candidate 3			Candidate 4			Candidate 5		
Task 1															
Task 2															
Task 3															
Task 4															
Total															

Explanatory notes on assessment

Each task can be awarded zero, one, two or three points. A maximum of twelve points can be achieved.

Assessment scale for the supplementary examination

Total number of competencies demonstrated	Assessment of the oral supplementary examination
12	Very good
10–11	Good
8–9	Satisfactory
6–7	Pass
0–5	Fail

Task 1

Playground

At a playground, there is an assortment of play equipment.

a) Diagram 1 shows a seesaw. In diagram 2, a model of this seesaw is shown from the side.

Diagram 1

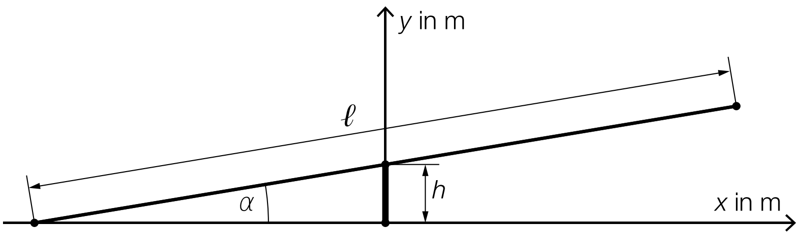


Diagram 2

Image source: Chabe01 – own work, CC BY-SA 4.0, https://commons.wikimedia.org/wiki/File:Aire_Jeux_Rives_Menthon_St_Cyr_Menthon_16.jpg [23.12.2021] (adapted).

The beam has length ℓ and its midpoint has a height h.

1) Using h and ℓ, write down a formula that can be used to calculate the angle α.

α = ____

b) The circular bounce mat of a trampoline has an area of 5 m^2.

1) Determine the diameter of the bounce mat of this trampoline.

c) An old sandpit with a square base with side length a and height h is to be replaced by a new sandpit.

This new sandpit with a square base should have the same height as the old sandpit but the side lengths should be 50 % longer.

1) Show that the volume of the new sandpit is <u>not</u> twice the size of that of the old sandpit.

Solution to Task 1

Playground

a1) $\alpha = \arcsin\left(\frac{h}{\frac{\ell}{2}}\right)$

or:

$\alpha = \arcsin\left(\frac{2 \cdot h}{\ell}\right)$

b1) $d = 2 \cdot \sqrt{\frac{5}{\pi}} = 2.52...$

The bounce mat has a diameter of around 2.5 m.

c1) $V_{\text{old}} = a^2 \cdot h$

$V_{\text{new}} = (1.5 \cdot a)^2 \cdot h = 2.25 \cdot a^2 \cdot h = 2.25 \cdot V_{\text{old}}$

The volume of the new sandpit is therefore not twice the size of that of the old sandpit.

A justification that uses concrete values is also correct.

Task 2

Beer Foam

After a beer has been poured into a glass, the resulting beer foam slowly collapses in upon itself.

a) Thomas measures the height of the beer foam after beer has been poured into a particular glass. The table below shows the results of his measurements.

Time after pouring in s	0	20	60
Height of the beer foam in cm	4	2.5	2

1) Determine the average rate of change of the height of the beer foam for the first 60 seconds after it has been poured. Write down the result with the corresponding unit.

The height of the beer foam is to be described by an exponential function h of the form $h(t) = a \cdot b^t$.

t ... time after pouring in s
$h(t)$ ... height of the beer foam at time t in cm

2) Show that no exponential function h of this form exists whose graph goes through all 3 measurement points.

b) Martin describes the height of the beer foam after pouring beer into a different glass with the function f (see diagrams below).

1) In diagram 2 below, sketch the graph of f'.

Diagram 1

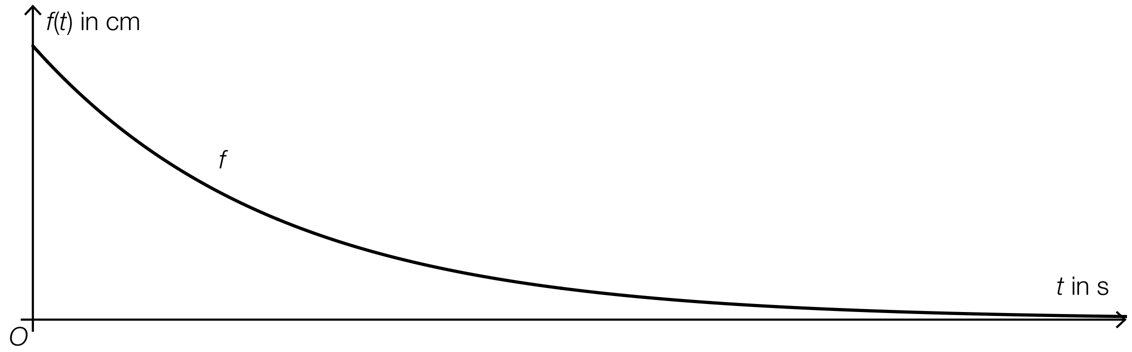


Diagram 2

Solution to Task 2

Beer Foam

a1) $\frac{2-4}{60-0} = -0.0\dot{3}$

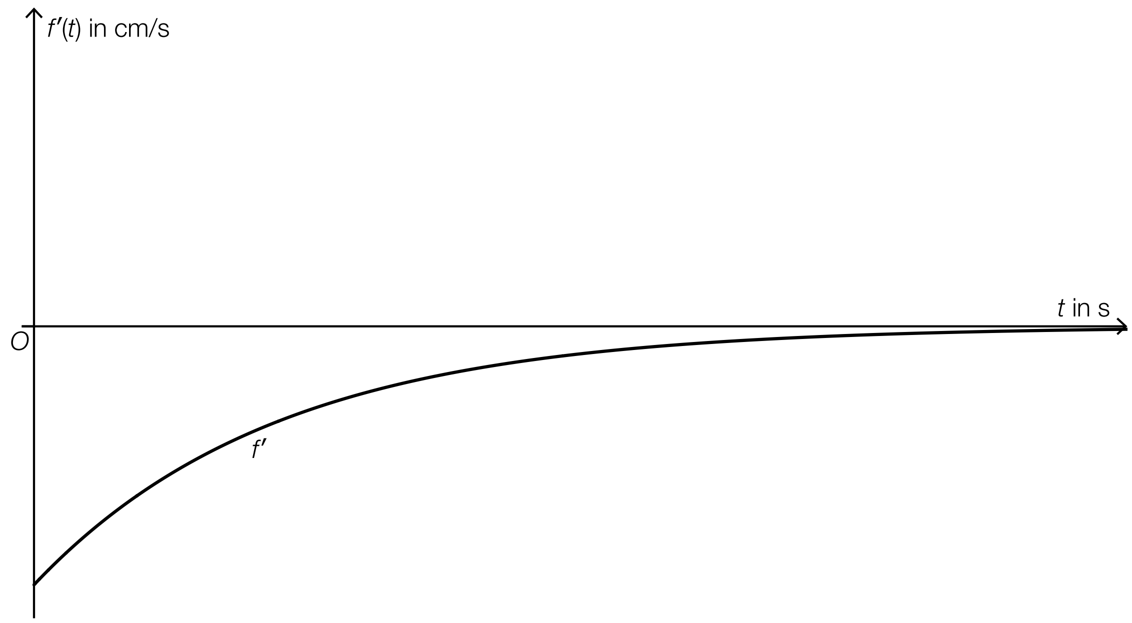
The average rate of change is around –0.03 cm/s.

a2) $4 \cdot b^{20} = 2.5 \quad \Rightarrow \quad b = \sqrt[20]{\frac{2.5}{4}} = 0.976...$

$4 \cdot b^{60} = 2 \quad \Rightarrow \quad b = \sqrt[60]{\frac{2}{4}} = 0.988...$

As the change factors are not the same, there is no exponential function of this form whose graph goes through all 3 measurement points.

b1)



The graph must be monotonically increasing and concave down; it must also approach the horizontal axis asymptotically.

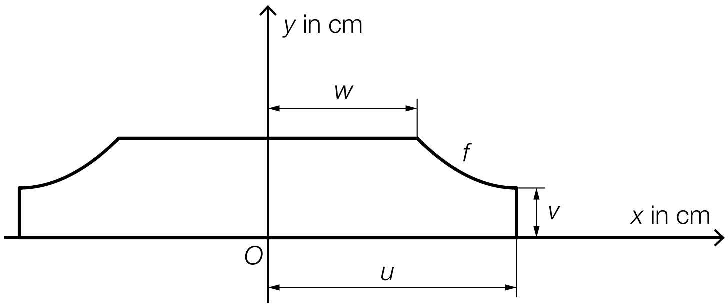
Task 3

Pipe Covering

a) The diagram on the right shows an image of a pipe covering for two heating pipes.

Image source: BMBWF

The diagram below shows a model of the cross-section of this pipe covering from the side.



A part of the boundary line of this cross-section can be modelled by the graph of the quadratic function f with $f(x) = a \cdot x^2 + b \cdot x + c$.

The vertex of the function f has coordinates (u, v).
The angle of the slope when $x = w$ is –45°.

1) Write down a system of equations in terms of u, v and w that can be used to calculate the coefficients a, b and c.
2) On the diagram above, mark the area whose size can be calculated with the expression shown below.
$\int_w^u f(x)\,\mathrm{d}x$

For a particular pipe covering with $u = 5$, the following equation holds:

$f(x) = 0.25 \cdot x^2 - 2.5 \cdot x + 8.75 \quad \text{with} \quad w \leq x \leq u$

3) Determine the length v for this pipe covering.

Solution to Task 3

Pipe Covering

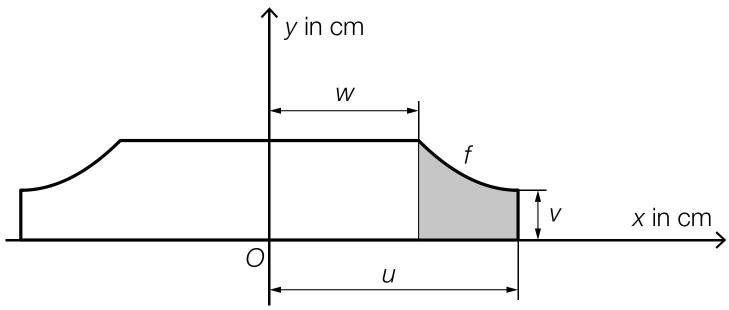
a1) $f(x) = a \cdot x^2 + b \cdot x + c$
$f'(x) = 2 \cdot a \cdot x + b$

I: $f(u) = v$
II: $f'(u) = 0$
III: $f'(w) = -1$

or:

I: $a \cdot u^2 + b \cdot u + c = v$
II: $2 \cdot a \cdot u + b = 0$
III: $2 \cdot a \cdot w + b = -1$

a2)



a3) $f(5) = 2.5$

The length v is 2.5 cm.

Task 4

Parcel Services

Due to the sharp increase in online trade, more and more people are using parcel services.

a) There are complaints offices for registering problems with parcel services.
From long-term observations it is known that 11 % of all complaints at one particular complaints office are due to delivery times that are too long.

On a particular day, a total of 42 independent complaints are received.

1) Determine the probability that exactly 8 of these 42 complaints are received due to delivery times that are too long.

b) For every parcel service, the *first attempt success ratio* is an important metric.
The first attempt success ratio is the probability that a packet selected at random can be delivered on the first attempt. At a particular parcel service, the first attempt success ratio is 90 %.

A delivery driver is to deliver n parcels.

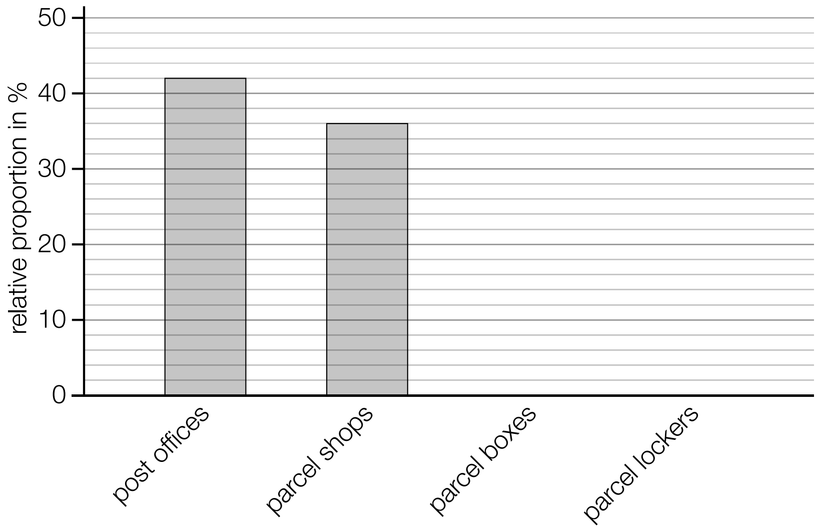
1) Describe the event E whose probability can be calculated with the expression shown below in the given context.

$P(E) = 1 - 0.9^n$

c) In the year 2020, parcels could be sent through a particular parcel service from a total of 31 200 drop-off points.

These 31 200 drop-off points comprised 13 104 post offices, 11 232 parcel shops, 624 parcel boxes and a particular number of parcel lockers.

1) Complete the two missing columns in the bar chart shown below.



Solution to Task 4

Parcel Services

a1) X ... number of complaints due to delivery times that are too long

binomial distribution with $n = 42$, $p = 0.11$

calculation using technology:

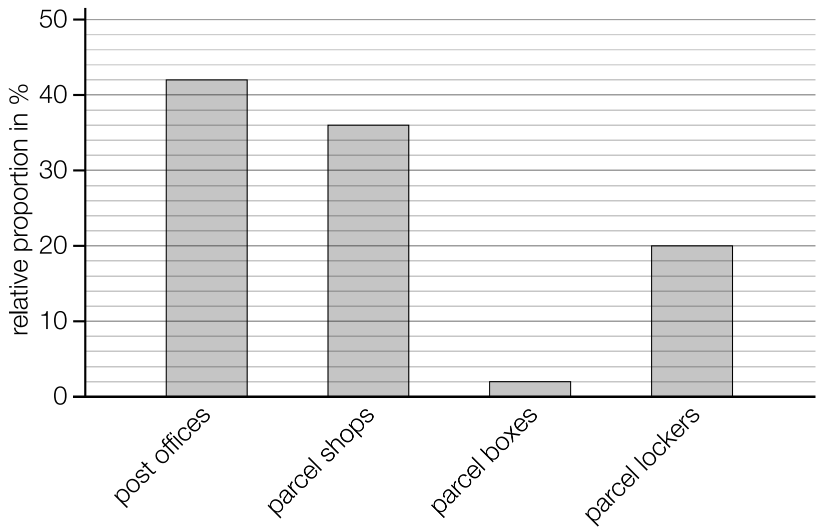
$P(X = 8) = 0.0481...$

The probability is around 4.8 %.

b1) E ... "the delivery driver cannot deliver at least 1 parcel of these n parcels on the first attempt"

c1) $\frac{624}{31\,200} = 0.02$

$\frac{31\,200 - 13\,104 - 11\,232 - 624}{31\,200} = 0.2$



Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

October 2022

Mathematics

Supplementary Examination 1
Examiner's Version

Bundesministerium
Bildung, Wissenschaft
und Forschung

Instructions for the standardized implementation of the supplementary examination

The following supplementary examination booklet contains four tasks that can be completed independently of one another as well as the corresponding solutions.

Each task comprises three competencies to be demonstrated.

The preparation time is to be at least 30 minutes; the examination time is at most 25 minutes.

The use of the official formula booklet that has been approved by the relevant government authority for use in the standardized school-leaving examination in mathematics is allowed. Furthermore, the use of electronic devices (e.g. graphic display calculators or other appropriate technology) is allowed provided there is no possibility to communicate (e.g. via the internet, intranet, Bluetooth, mobile networks etc.) and there is no access to an individual's data stored on the device.

After the examination, all materials (tasks, extra sheets of paper etc.) from the candidates are to be collected in. The examination materials (tasks, extra sheets of paper, data that has been produced digitally etc.) may only be made public after the time period allocated for the examination has passed.

Evaluation grid for the supplementary examination

The evaluation grid below may be used to assist in assessing the candidates' performances.

	Candidate 1			Candidate 2			Candidate 3			Candidate 4			Candidate 5		
Task 1															
Task 2															
Task 3															
Task 4															
Total															

Explanatory notes on assessment

Each task can be awarded zero, one, two or three points. A maximum of twelve points can be achieved.

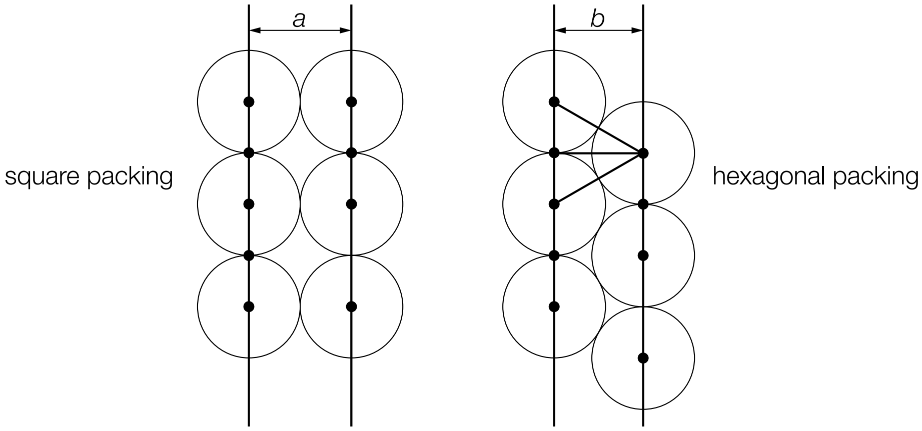
Assessment scale for the supplementary examination

Total number of competencies demonstrated	Assessment of the oral supplementary examination
12	Very good
10–11	Good
8–9	Satisfactory
6–7	Pass
0–5	Fail

Task 1

Plant Pots

Cylindrical plant pots can be arranged in so-called *square packing* or *hexagonal packing* (see the model bird's-eye view diagrams shown below).



a) The distance b in hexagonal packing is smaller than the distance a in square packing.

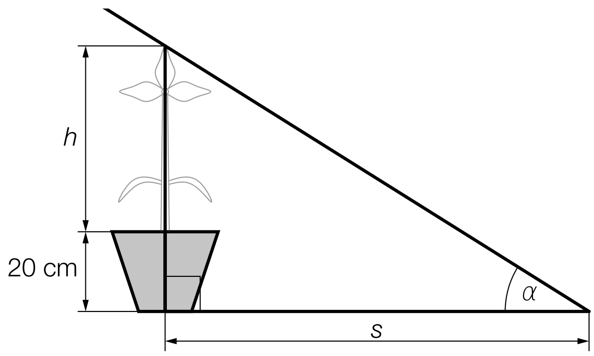
1) Determine the difference $a - b$ for the case when the diameter of the plant pots is 40 cm.

b) Two cylindrical plant pots with a circular base are compared to each other.

Plant pot A has radius r and height h.
The volume of the plant pot is V_A.
Plant pot B has the same height h but a radius that is 10 % larger than plant pot A.

1) Show that the volume V_B of plant pot B is 21 % greater than V_A.

c) In a plant pot with a height of 20 cm, there is a plant with height h (in cm). The rays of sunlight that fall on the plant make an angle of α with the horizontal (see diagram on the right).



1) Write down a formula in terms of h and α that can be used to determine the length s (in cm) of the shadow.

$s =$ ____

Solution to Task 1

Plant Pots

a1) $a = 40$

b is the height of an equilateral triangle with side length 40.

$b = \frac{a}{2} \cdot \sqrt{3} = 34.64...$

or:

$b = \sqrt{40^2 - 20^2} = 34.64...$

$a - b = 40 - 34.64...$
$a - b = 5.35...$ cm

b1) $V_A = r^2 \cdot \pi \cdot h$
$V_B = (1.1 \cdot r)^2 \cdot \pi \cdot h = 1.21 \cdot V_A$

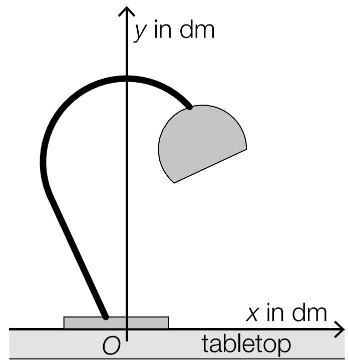
c1) $s = \frac{h + 20}{\tan(\alpha)}$

Task 2

Desk Lamps

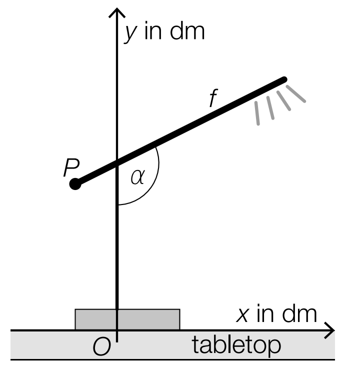
Various models of desk lamp are available. The light source is suspended in different ways depending on design of the model. The suspension methods are modelled by a thick black line in the diagrams below.

a) The suspension of the light source in model *A* is shown in the diagram on the right.



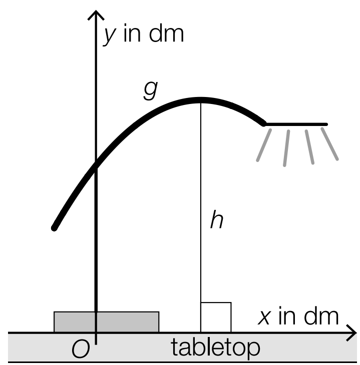
1) Justify why this method of suspension cannot be described by the graph of a single function (y in terms of x).

b) The suspension of the light source in model *B* can be described by the graph of the linear function f (see diagram on the right).



1) Using $P = (-1, 3.5)$ and $\alpha = 116.56°$, write down an equation of the function f.

c) The suspension of the light source in model *C* can be described by the graph of the quadratic function g (see diagram on the right).



The following statement holds: $g(x) = -0.25 \cdot x^2 + 1.25 \cdot x + 4$

1) Determine the maximum height h of the lamp above the tabletop.

Solution to Task 2

Desk Lamps

a1) A function assigns each x-value to exactly one y-value. As there is a region for which 2 points of the desk lamp are directly above each other, this method of suspension cannot be described by the graph of a single function.

b1) $f(x) = k \cdot x + d$

$k = \tan(116{,}56° - 90°) = 0.499...$
$-1 \cdot 0.499... + d = 3.5$
$d = 3.99...$

$f(x) = 0.5 \cdot x + 4$ *(coefficients rounded)*

c1) $g'(x) = 0$ *or* $-0.5 \cdot x + 1.25 = 0$

Calculation using technology:

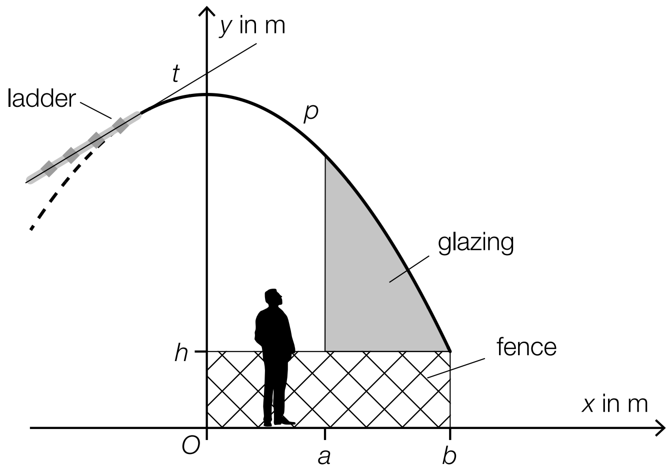
$x = 2.5$
$g(2.5) = 5.56...$

The maximum height h of the desk lamp above the tabletop is around 5.6 dm.

Task 3

Viewing Platform

The diagram below shows a covered viewing platform shown from the side.



a) The roof is modelled by the graph of the quadratic function p.

$p(x) = -0.302 \cdot x^2 + 4.8$

x, $p(x)$ ... coordinates in m

For cleaning purposes, a ladder is mounted on the roof. The ladder runs along the tangent t to the graph p when $x = -1$.

1) Determine the angle of elevation of the tangent t.

b) The platform is to be glazed at the side. The glazing will cover the space between the top of the fence and the roof (see diagram above).

1) Write down a formula that can be used to calculate the size A of the area shaded grey.

$A =$ ____

c) For safety reasons, the roof requires a supporting beam of length $\ell = p(a) - h$.

1) Draw the length ℓ in the diagram above.

Solution to Task 3

Viewing Platform

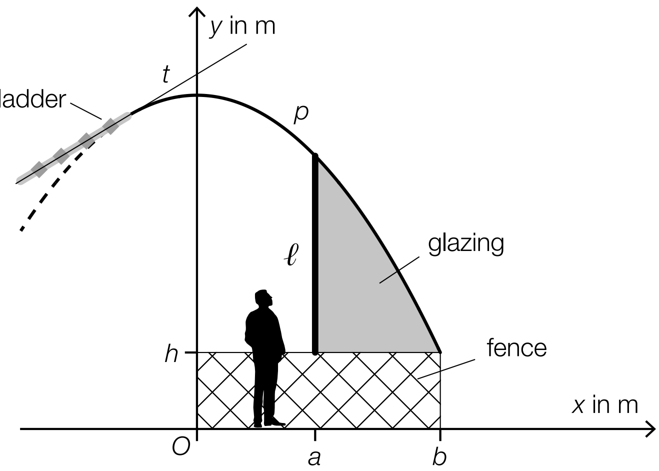
a1) $p'(x) = -0.604 \cdot x$

$p'(-1) = 0.604$
$\alpha = \arctan(0.604) = 31.13...°$

The angle of elevation of the tangent *t* is around 31.1°.

b1) $A = \int_a^b (p(x) - h)\,\mathrm{d}x$ *or* $A = \int_a^b p(x)\,\mathrm{d}x - (b - a) \cdot h$

c1)



Task 4

Cigarettes

Many of the substances contained in cigarette smoke are hazardous to health.

a) The amount in mg of substances contained in cigarette smoke in the cigarettes of 100 smokers is investigated. These have been sorted into 3 categories (see table below).

class	amount in mg of substances per cigarette	class midpoint	absolute frequency
1	[0, 10[	5	55
2	[10, 30[	20	40
3	[30, 50[	40	5

An estimate of the mean of the amount of substances is to be calculated. The respective class midpoints are determined for this purpose.

1) Determine the mean of the amount of substances.
2) Explain why the median of the amount of substances is in class 1.

b) The probability that a randomly chosen smoker smokes more than one cigarette per day is p.

The probability that exactly 5 out of 100 smokers smoke more than one cigarette per day is to be calculated.

1) Write down a formula that can be used to determine this probability.

Solution to Task 4

Cigarettes

a1) $\frac{5 \cdot 55 + 20 \cdot 40 + 40 \cdot 5}{100} = 12.75$

The mean of the amount of substances is 12.75 mg.

a2) The median of an ordered list always lies in the middle of all values. Of the given 100 values, 55, so more than half, are in class 1. Therefore, the median must also lie in this class.

b1) X ... The number of smokers that smoke more than one cigarette per day

Binomial distribution with $n = 100$ and p

$P(X = 5) = \binom{100}{5} \cdot p^5 \cdot (1 - p)^{95}$

Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

February 2023

Mathematics

Supplementary Examination 1
Examiner's Version

Bundesministerium
Bildung, Wissenschaft
und Forschung

Instructions for the standardized implementation of the supplementary examination

The following supplementary examination booklet contains four tasks that can be completed independently of one another as well as the corresponding solutions.

Each task comprises three competencies to be demonstrated.

The preparation time is to be at least 30 minutes; the examination time is at most 25 minutes.

The use of the official formula booklet that has been approved by the relevant government authority for use in the standardized school-leaving examination in mathematics is allowed. Furthermore, the use of electronic devices (e.g. graphic display calculators or other appropriate technology) is allowed provided there is no possibility to communicate (e.g. via the internet, intranet, Bluetooth, mobile networks etc.) and there is no access to an individual's data stored on the device.

After the examination, all materials (tasks, extra sheets of paper etc.) from the candidates are to be collected in. The examination materials (tasks, extra sheets of paper, data that has been produced digitally etc.) may only be made public after the time period allocated for the examination has passed.

Evaluation grid for the supplementary examination

The evaluation grid below may be used to assist in assessing the candidates' performances.

	Candidate 1			Candidate 2			Candidate 3			Candidate 4			Candidate 5		
Task 1															
Task 2															
Task 3															
Task 4															
Total															

Explanatory notes on assessment

Each task can be awarded zero, one, two or three points. A maximum of twelve points can be achieved.

Assessment scale for the supplementary examination

Total number of competencies demonstrated	Assessment of the oral supplementary examination
12	Very good
10–11	Good
8–9	Satisfactory
6–7	Pass
0–5	Fail

Task 1

The Number π

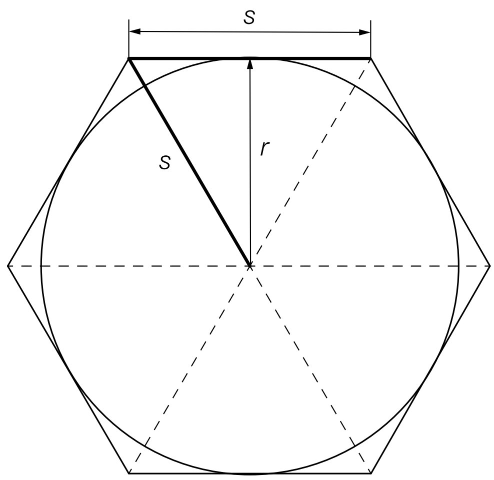
Over the course of history, various methods have been used to determine the value of the number $\pi = 3.141\ldots$ as accurately as possible.

a) In the oldest known book of calculations (*Rhind Mathematical Papyrus*), the following approximation π_N is given:

$$\pi_N = \left(\frac{16}{9}\right)^2$$

1) Determine in how far the approximation π_N differs from π as a percentage.

b) In another method, the circumference of a circle with radius r is approximated by the perimeter of a circumscribed hexagon (see diagram on the right).

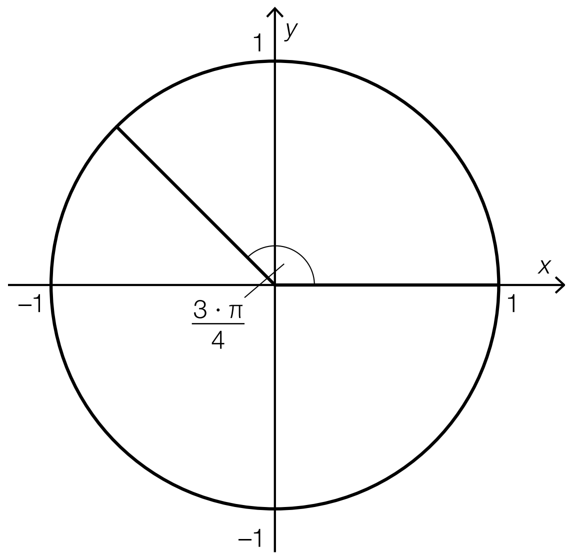


1) Write down a formula in terms of r that can be used to determine the perimeter u of the circumscribed hexagon.

$u =$ ______________________________

c) 1) On the unit circle shown below, sketch the angle α with $\alpha \neq \frac{3 \cdot \pi}{4}$ for which the following statement holds:

$$\sin\left(\frac{3 \cdot \pi}{4}\right) = \sin(\alpha)$$



Solution to Task 1

The Number π

a1) $\frac{\pi_N - \pi}{\pi} = \frac{0.0189...}{3.1415...} = 0.0060...$

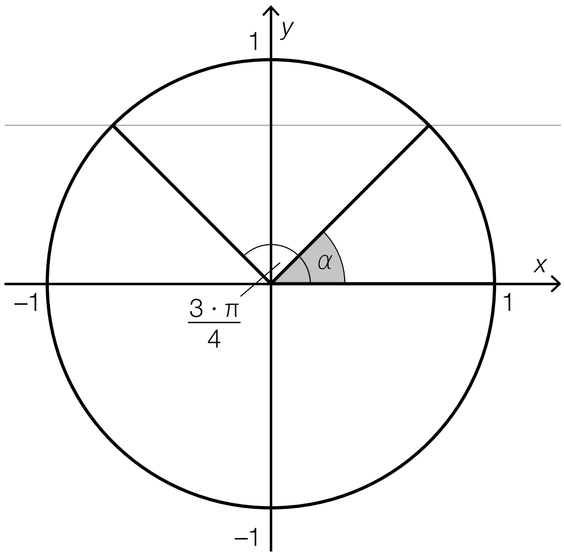
The approximation differs from the number π by around 0.6 %.

b1) $s^2 = \left(\frac{s}{2}\right)^2 + r^2$

$s = \frac{2 \cdot r}{\sqrt{3}}$

$u = 6 \cdot s = \frac{12 \cdot r}{\sqrt{3}}$

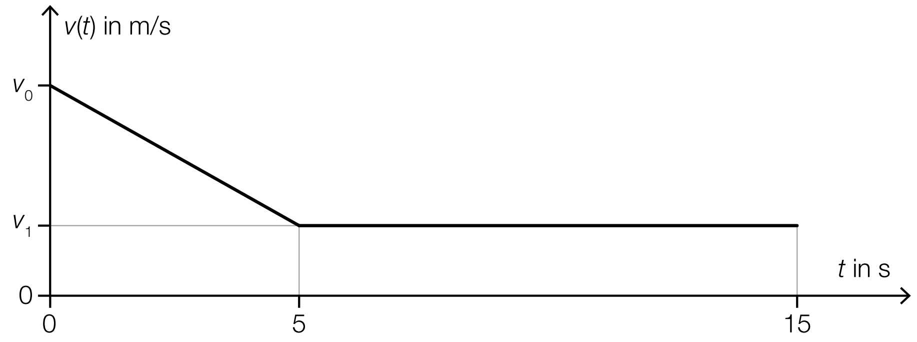
c1)



Task 2

Car Journey

a) The diagram below shows the velocity in terms of time for a particular car journey for a time period of 15 s.



The distance covered in the first 5 s is the same as the distance covered in the following 10 s.

1) Write down an equation in terms of v_0 and v_1 that describes this situation correctly.

b) For another car journey, the velocity can be approximated by the function v_A.

$v_A(t) = 70 \cdot t^3 - 260 \cdot t^2 + 230 \cdot t + 80 \quad \text{with} \quad 0 \leq t \leq 1.5$

t ... time in h

$v_A(t)$ ... velocity at time t in km/h

1) Determine the maximum velocity for this car journey.

2) Interpret the result of the calculation shown below in the given context.

$v_A'(0) = 230$

Solution to Task 2

Car Journey

a1) $\frac{(v_0 + v_1) \cdot 5}{2} = 10 \cdot v_1$

b1) $v_A'(t) = 210 \cdot t^2 - 520 \cdot t + 230$
$v_A'(t) = 0$ *or* $210 \cdot t^2 - 520 \cdot t + 230 = 0$

Calculation using technology:

$t_1 = 0.57...$ ($t_2 = 1.89...$)
$v_A(t_1) = 139.59...$

The maximum velocity is around 140 km/h.

b2) The instantaneous rate of change of the velocity (acceleration) at time $t = 0$ is 230 km/h^2.

Task 3

Bacteria

A sample of bacteria is investigated in a laboratory.

The number of bacteria is 1 200 when the observations begin.
After 6 days, the number of bacteria is 1 800.

a) The development of the number of bacteria over time can be modelled by the linear function f.

t ... time in days with $t = 0$ corresponding to the beginning of the observations
$f(t)$ ... number of bacteria at time t

1) Write down an equation of the linear function f.

b) In a different model, the development of the number of these bacteria over time is described by the exponential function g.

t ... time in days with $t = 0$ corresponding to the beginning of the observations
$g(t)$ ... number of bacteria at time t

1) Complete the boxes below with the missing symbol (<, > or =).

$g'(t) \boxed{} g'(t + 1)$

$g''(t) \boxed{} 0$

2) Determine the time at which the number of bacteria reaches 6 000 according to this model.

Solution to Task 3

Bacteria

a1) $f(t) = 100 \cdot t + 1\,200$

or:

$f(t) = \frac{600}{6} \cdot t + 1\,200$

b1) $g'(t) \boxed{<} g'(t+1)$

$g''(t) \boxed{>} 0$

b2) $g(t) = a \cdot b^t$
$g(0) = 1\,200$
$g(6) = 1\,800$

Calculation using technology:

$g(t) = 1\,200 \cdot 1.0699^t$ *(coefficient rounded)*

$g(t) = 6\,000$
Calculation using technology:

$t = 23.8...$

The number of bacteria reaches 6 000 around 24 days after the beginning of the observations.

Task 4

Dice

a) Two fair six-sided dice with faces showing the values 1, 2, 3, 4, 5 and 6 are rolled simultaneously. The sum of the values shown is determined.

The random variable X describes the sum of the values shown on the pair of dice.

1) Justify why the following statement holds: $P(X = 11) = 2 \cdot P(X = 12)$.

b) Alex participates in a competition. In this competition, a fair six-sided dice with faces showing the values 1, 2, 2, 3, 3 and 3 is rolled once.

Before Alex rolls the dice, the game master collects an amount of e euros.

If the dice shows the value 1, the game master will pay Alex x euros.
If the dice shows the value 2, the game master will pay Alex 2 euros.
If the dice shows the value 3, the game master will not pay Alex any money.

From experience, the game master knows that he can expect a profit of 0.50 euros per roll of the dice.

1) Write down an equation in terms of e that can be used to calculate x.

c) In the production of a particular six-sided dice with faces showing the values 1, 2, 3, 4, 5 and 6, inaccuracies have occurred. This has resulted in one particular face being rolled with a different probability from the others.

The dice is rolled 500 times. The results are shown in the table below.

value shown on the face	1	2	3	4	5	6
absolute frequency of the value shown on the face	75	98	65	110	80	72

1) Based on these results, determine an estimate for the probability shown below.

P(“the value shown on the face is greater than 3 when the dice has been rolled once”) = ________________

Solution to Task 4

Dice

a1) The sum 12 is possible in one outcome of the experiment: {(6, 6)}
The sum 11 is possible in two outcomes of the experiment: {(5, 6), (6, 5)}

All of the possible outcomes of the experiment have the same probability, therefore:
$P(X = 11) = 2 \cdot P(X = 12)$

b1) $e - \left(x \cdot \frac{1}{6} + 2 \cdot \frac{1}{3} + 0 \cdot \frac{1}{2}\right) = 0.5$

c1) $P(\text{“the value shown on the face is greater than 3 when the dice has been rolled once”}) = \frac{110 + 80 + 72}{500} = 0.524$