Name:	
Class:	

Standardised Competence-Oriented
Written School-Leaving Examination

AHS

7th May 2024

Mathematics

Bundesministerium
Bildung, Wissenschaft
und Forschung

Advice for Completing the Tasks

Dear candidate,

The following booklet contains Part 1 and Part 2 tasks (divided into sub-tasks). The tasks can be completed independently of one another. You have a total of *270 minutes* available in which to work through this booklet.

Please do all of your working out solely in this booklet and on the paper provided to you. Write your name and that of your class on the cover page of the booklet in the spaces provided. Please also write your name on any separate sheets of paper used and number these pages consecutively. When responding to the instructions of each task, write the task reference (e.g. 25a1) on your sheet.

In the assessment of your work, everything that is not crossed out will be considered.

The use of the official formula booklet for this examination that has been approved by the relevant government authority is permitted. Furthermore, the use of electronic device(s) (e.g. graphic display calculators or other appropriate technology) is allowed provided there is no possibility of communicating via the internet, Bluetooth, mobile networks etc. and there is no access to your own data stored on the device.

An explanation of the task types is displayed in the examination room.

Instructions for Completing the Tasks

– Solutions must be unambiguous and clearly recognisable.

– Solutions must be given alongside their corresponding units if this has been explicitly required in the task instructions.

For tasks with open answer formats, evidence of the targeted core competency is required for the award of the point. When completing tasks with open answer formats, it is recommended that you:

– document how the solution was reached, even if electronic devices were used,

– explain any variables you have chosen yourself and give their corresponding units,

– avoid rounding prematurely,

– label diagrams or sketches.

Changing an answer for a task that requires a cross:

1. Fill in the box that contains the cross.
2. Put a cross in the box next to your new answer.

In this instance, the answer “5 + 5 = 9” was originally chosen. The answer was later changed to be “2 + 2 = 4”.

1 + 1 = 3	☐
2 + 2 = 4	☒
3 + 3 = 5	☐
4 + 4 = 4	☐
5 + 5 = 9	■
6 + 6 = 10	☐

Selecting an item that has been filled in:

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

In this instance, the answer “2 + 2 = 4” was filled in and then selected again.

1 + 1 = 3	☐
2 + 2 = 4	■
3 + 3 = 5	☐
4 + 4 = 4	■
5 + 5 = 9	☐
6 + 6 = 10	☐

Grading System

points awarded	grade
32–36 points	very good
27–31.5 points	good
22–26.5 points	satisfactory
17–21.5 points	pass
0–16.5 points	fail

Best-of Assessment: A best-of assessment approach will be applied to tasks 26, 27 and 28. Of these three Part 2 tasks, the task with the lowest point score will not be included in the total point score.

Good luck!

Task 1

Comparison of Two Sets

The set $A = \{x \in \mathbb{N} \mid 1 < x < 8\}$ is a subset of the natural numbers,
and the set $B = \{x \in \mathbb{Q} \mid 1 < x < 8\}$ is a subset of the rational numbers.

Task:

Put a cross next to each of the two correct statements. *[2 out of 5]*

Both sets A and B contain rational numbers.	☐
Set B is a subset of set A.	☐
The two sets A and B contain exactly the same number of numbers.	☐
Set A contains exactly 6 numbers that are also contained in set B.	☐
Both sets A and B contain numbers that are greater than 7.	☐

[0/1 p.]

Task 2

Apples and Apricots

A particular fruit seller sells apples and apricots.
The price for 1 kg of apples is *a* euros; the price for 1 kg of apricots is *m* euros ($a, m \in \mathbb{R}^+$).

The following statements hold:

- 1 kg of apricots costs 80 % more than 1 kg of apples.
- 1 kg of apricots costs 1.40 euro more than 1 kg of apples.

Task:

Put a cross next to each of the two correct equations. *[2 out of 5]*

$a \cdot 0.8 = m$	☐
$a + 1.8 = m$	☐
$a = m - 1.4$	☐
$a = \frac{m}{1.4}$	☐
$\frac{m}{a} = 1.8$	☐

[0/1 p.]

Task 3

System of Equations

A system of equations in x and y with $a, c \in \mathbb{R}$ is shown below.

I: $2 \cdot x - y = 3$
II: $a \cdot x + 2 \cdot y = c$

This system of equations has no solution.

Task:

Write down a value for each of a and c.

a = __________

c = __________

[0/1 p.]

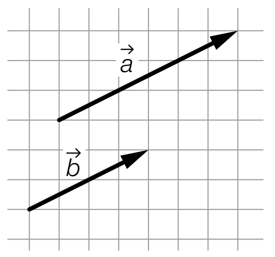
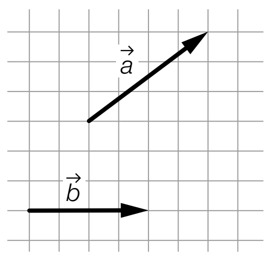
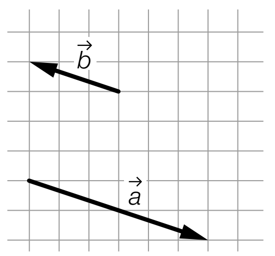
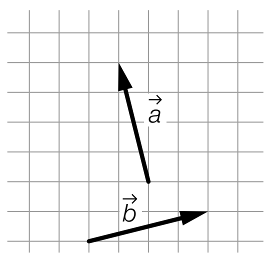
Task 4

Vectors

Let $\vec{a}, \vec{b} \in \mathbb{R}^2$ be vectors.

Task:

Match each of the four diagrams to the statement from A to F that corresponds to the vectors $\vec{a}$ and $\vec{b}$ shown.

	☐
	☐
	☐
	☐

A	$(\vec{a} - \vec{b}) \perp \vec{b}$
B	$\vec{a} \cdot \vec{b} = 0$
C	$\vec{b} = \frac{3}{2} \cdot \vec{a}$
D	$\vec{a} = -2 \cdot \vec{b}$
E	$(\vec{a} - \vec{b}) \perp \vec{a}$
F	$\vec{b} = \frac{2}{3} \cdot \vec{a}$

[0/½/1 p.]

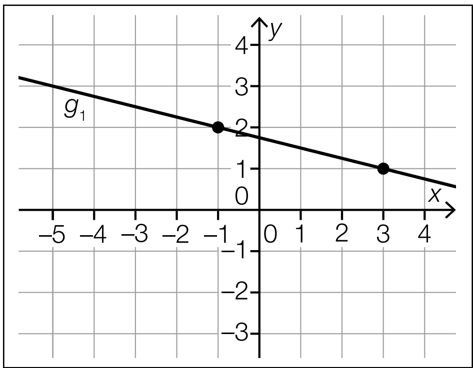
Task 5

Vector Equation of a Line

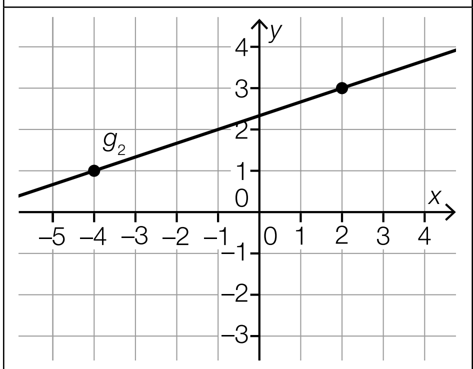
Six lines $g_1, g_2, \dots, g_6$ are represented graphically below. The points shown on the lines have integer coordinates.

Task:

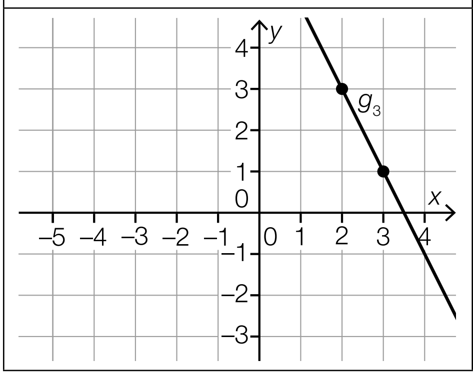
Put a cross next to the line whose vector equation has the form $X = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + t \cdot \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ with $t \in \mathbb{R}$ and $a_1, a_2 \in \mathbb{Z}$. *[1 out of 6]*



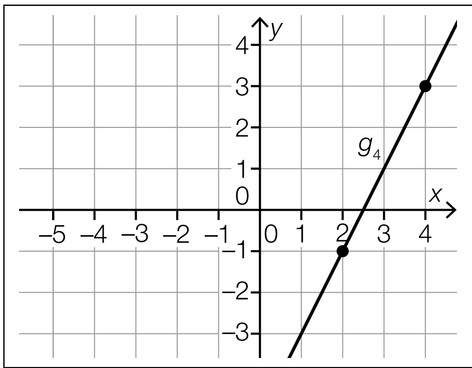
☐



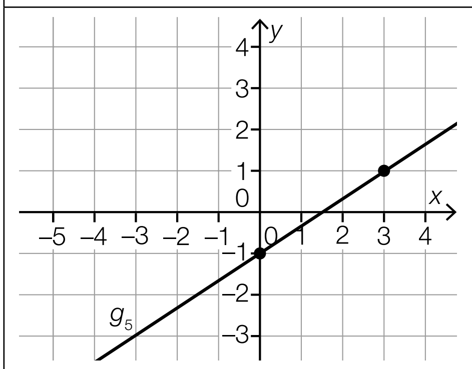
☐



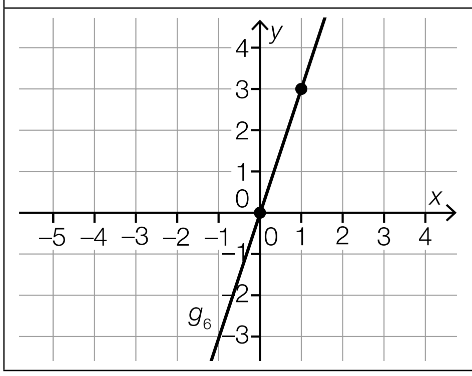
☐



☐



☐



☐

[0/1 p.]

Task 6

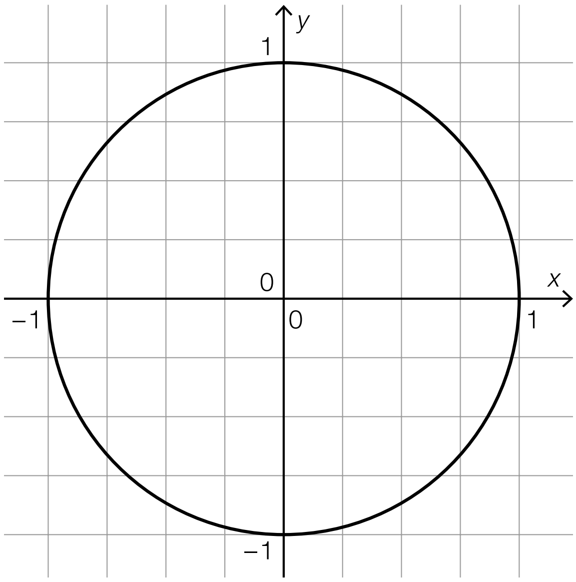
Unit Circle

For the angle $\alpha \in [0°, 360°)$, the following statements hold:
$\sin(\alpha) = -0.5$ and $\cos(\alpha) < 0$

A unit circle is shown in the coordinate system below.

Task:

On this coordinate system, draw the point $P = (\cos(\alpha), \sin(\alpha))$.



[0/1 p.]

Task 7

Acceleration

A body moves along a straight line with constant acceleration in the time interval [0, 5] and comes to rest at the time $t = 5$.

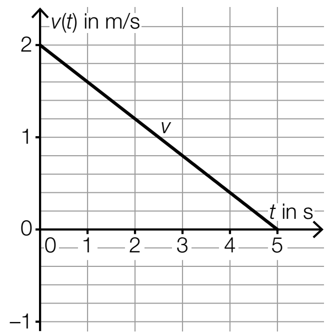
The following statement holds for the acceleration: $a(t) = -0.4$

t ... time in s
$a(t)$ ... acceleration at time t in m/s^2
$v(t)$ ... velocity at time t in m/s
$s(t)$ ... displacement at time t in m

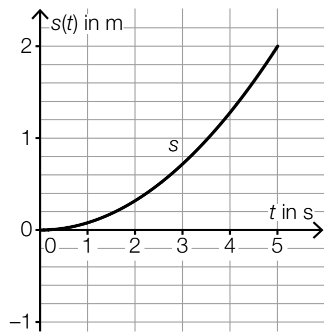
The movement of the body is represented correctly in two of the diagrams shown below.

Task:

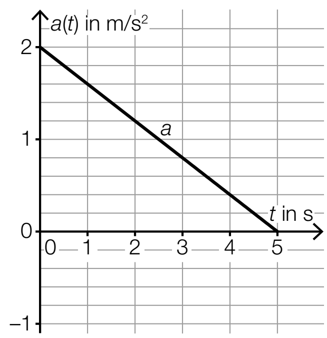
Put a cross next to each of the two correct diagrams. *[2 out of 5]*



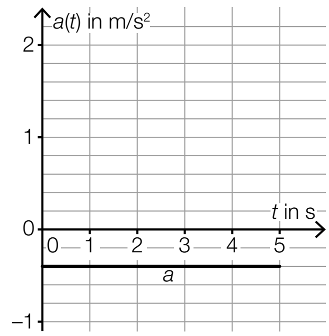
☐



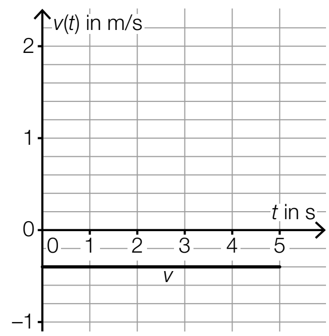
☐



☐



☐



☐

[0/1 p.]

Task 8

Racing Bike

The following values are given in the handbook of a racing bike:

number of crank turns per minute	velocity in km/h
60	28.8
85	40.8

The velocity can be modelled in terms of the number of crank turns by the linear function v.

x ... number of crank turns per minute
$v(x)$ ... velocity at x crank turns per minute in km/h

Task:

Write down an equation of v.

$v(x) =$ ____

[0 / 1 p.]

Task 9

Power Function

Let $f\colon \mathbb{R}\setminus\{0\} \to \mathbb{R}$ be a power function with $f(x) = a \cdot x^z$ with $a \in \mathbb{R}\setminus\{0\}$ and $z \in \mathbb{Z}$.

The following statements hold:

- If the value of the argument x is doubled, then the corresponding value of the function reduces to a quarter of the original value of the function.
- The point (2, 2) lies on the graph of f.

Task:

Write down the values of a and z.

$z =$ __________

$a =$ __________

[0/1 p.]

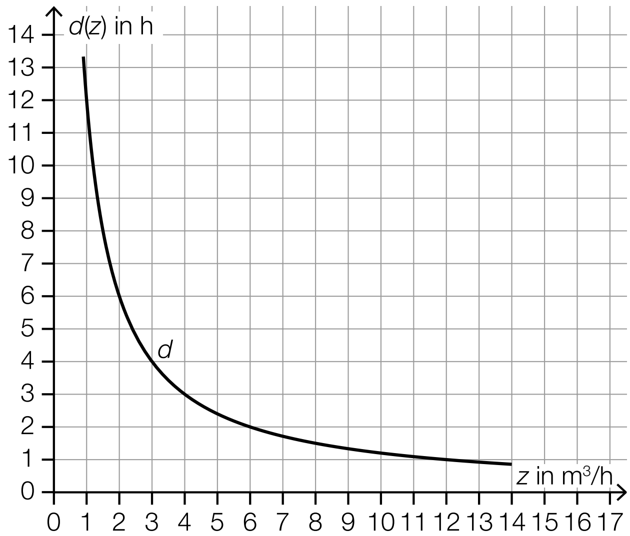
Task 10

Filling a Water Tank

An empty water tank is filled completely with water.

The function $d\colon \mathbb{R}^+ \to \mathbb{R}^+$ gives the filling time in terms of the inflow rate z (z in m³/h, $d(z)$ in h).

The diagram below shows the graph of d.



Task:

Write down the volume V of the water tank.

V = ____________________ m³

[0/1 p.]

Task 11

Half-Life

The mass of a radioactive substance can be described by an exponential function in terms of the time t. The following statements hold:

$N(t) = N_0 \cdot e^{-k \cdot t}$

$N(t)$ ... mass of the radioactive substance at time t
N_0 ... mass of the radioactive substance at time $t = 0$
$k \in \mathbb{R}^+$ ... decay constant

The half-life of the radioactive substance is given by τ.
An arbitrary time is given by t^*.
The following statements hold: $t^* \neq \tau$ and $t^* > 0$

Task:

Put a cross next to the expression that is equivalent to $N(t^* + \tau)$. *[1 out of 6]*

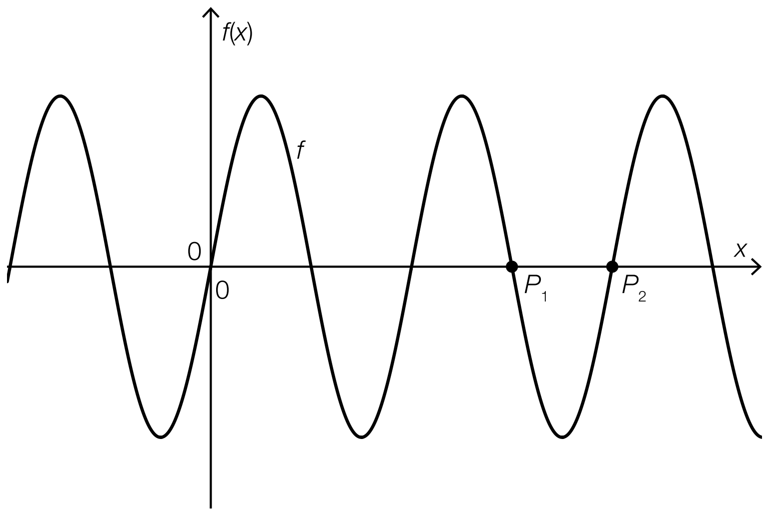
$2 \cdot N_0$	☐
$N(\tau)$	☐
$N\left(\frac{1}{2} \cdot t^*\right)$	☐
$2 \cdot N(\tau)$	☐
$N(2 \cdot t^*)$	☐
$\frac{1}{2} \cdot N(t^*)$	☐

[0/1 p.]

Task 12

Sine Function

The diagram below shows the graph of the function f with $f(x) = a \cdot \sin(b \cdot x)$ with $a, b \in \mathbb{R}^+$.



The points $P_1 = (x_1, 0)$ and $P_2 = (x_2, 0)$ with $x_1 = \frac{3 \cdot \pi}{4}$ and $x_2 = \pi$ lie on the graph of f.

The following statement holds: $f\left(\frac{x_1 + x_2}{2}\right) = -3$

Task:

Determine a and b.

a = ____________________

b = ____________________

[0/½/1 p.]

Task 13

CO_2 Emissions

The table below shows the level of CO_2 emissions in Austria for selected years.

year	1990	2005	2017	2018
CO_2 emissions in millions of tonnes	78.5	92.5	82.0	79.0

Task:

Determine the following values:

absolute change in CO_2 emissions from 2017 to 2018: _______________ millions of tonnes

relative change in CO_2 emissions from 1990 to 2005: _______________

[0/½/1 p.]

Task 14

Movement of a Cyclist

The polynomial function s approximates the distance covered by a cyclist in terms of the time t (t in h, $s(t)$ in km).

Task:

Complete the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

The expression $\lim\limits_{t \to 2} \frac{s'(t) - s'(2)}{t - 2}$ describes ______①______ and the expression $\frac{s(t_2) - s(t_1)}{t_2 - t_1}$ describes ______②______.

①	
the instantaneous acceleration at time $t = 2$	☐
the instantaneous velocity at time $t = 2$	☐
the distance covered until time $t = 2$	☐

②	
the average acceleration in the time interval $[t_1, t_2]$	☐
the average velocity in the time interval $[t_1, t_2]$	☐
the distance covered in the time interval $[t_1, t_2]$	☐

[0/½/1 p.]

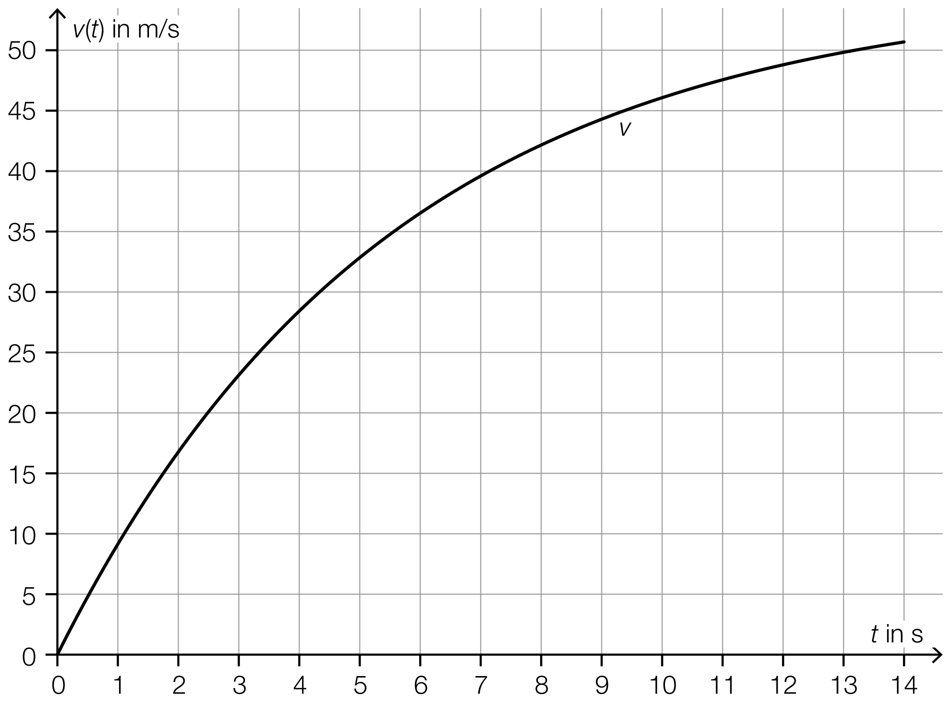
Task 15

Parachute Jump

The velocity of a parachutist during a particular parachute jump can be modelled in terms of the time t over the interval [0, 14] by the differentiable function v (t in s, $v(t)$ in m/s). The diagram below shows the graph of v.

Task:

Label the time t_1 at which the acceleration of the parachutist is 5 m/s^2 on the diagram below.



[0 / 1 p.]

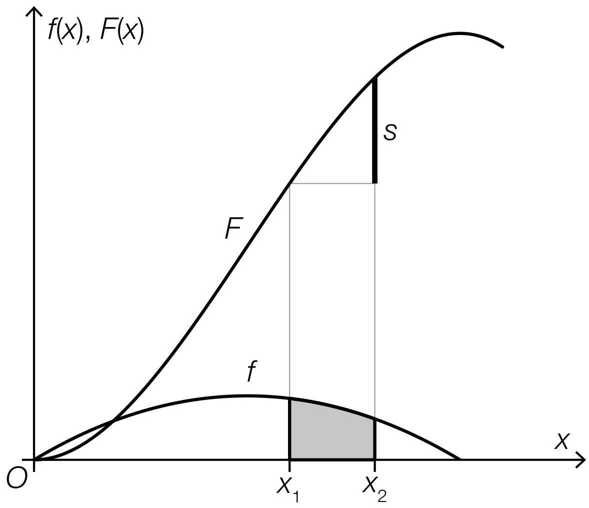
Task 16

Function and Antiderivative

The diagram below shows the graph of the function *f* and the graph of one of its antiderivatives *F*.

An area under the graph of the function *f* has been shaded in grey in the interval $[x_1, x_2]$.

A line segment of length *s* has been drawn under the graph of *F*.



Task:

Put a cross next to the equation that correctly describes the relationship between *s* and the area shaded in grey. *[1 out of 6]*

$s = F(x_1) - F(x_2)$	☐
$s = f(x_2) - f(x_1)$	☐
$s = \frac{F(x_1) + F(x_2)}{2}$	☐
$s = \int_{x_1}^{x_2} f(x)\,\mathrm{d}x$	☐
$s = \int_{x_2}^{x_1} f(x)\,\mathrm{d}x$	☐
$s = \int_{x_1}^{x_2} F(x)\,\mathrm{d}x$	☐

[0/1 p.]

Task 17

Properties of Quadratic Functions

Let f and h be two quadratic functions.
For all $x \in \mathbb{R}$, the following statements hold: $f'(x) = h'(x)$ and $f(x), h(x) > 0$

Task:

Put a cross next to each of the two statements that are definitely true. *[2 out of 5]*

For all $x \in \mathbb{R}$, $h''(x) < 0$ holds.	☐
h' is strictly monotonically decreasing.	☐
There exists a number $c \in \mathbb{R}$ such that for all $x \in \mathbb{R}$, $f(x) - h(x) = c$ holds.	☐
h' is a linear function whose graph goes through the point (0, 0).	☐
f' has a zero.	☐

[0/1 p.]

Task 18

Area between the Graphs of Two Functions

The graphs of the quadratic function *f* and the linear function *g* are shown in diagram 1.
The graphs of the functions *F* and *G* are shown in diagram 2.

The following statements hold:
F is an antiderivative of *f*.
G is an antiderivative of *g*.

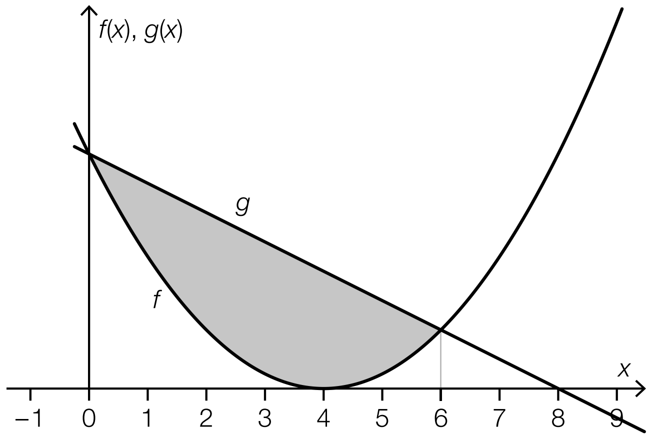


diagram 1

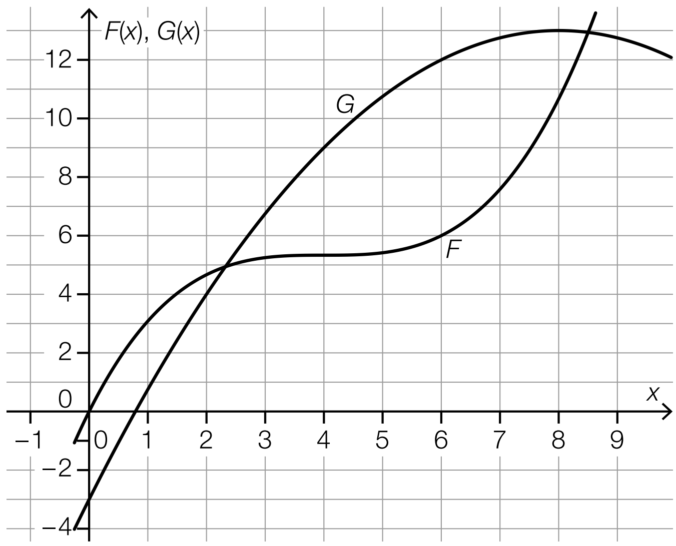


diagram 2

Task:

Using the diagrams, determine the area *A* of the grey shaded area.

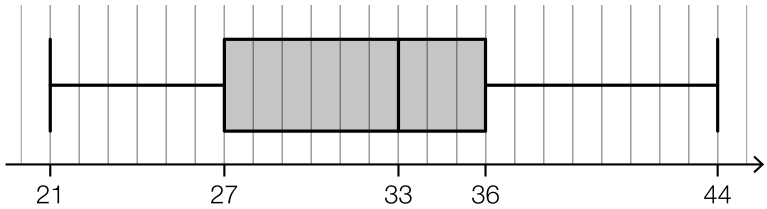
[0/1 p.]

Task 19

Comparison of Two Diagrams

In the diagrams below, the list of data A is represented by a stem and leaf diagram, and the list of data B is represented by a boxplot.

2	1	7	7	9
3	1	3	6	6
4	3			



Task:

Put a cross next to each of the two statistics that are different for the lists of data A and B.
[2 out of 5]

1st quartile	☐
range	☐
3rd quartile	☐
minimum	☐
median	☐

[0/1 p.]

Task 20

Mean

A particular list of data consists of 20 values and has the mean $\bar{x} = 15.5$.

The values 4, 6, 13, and 27 are removed from this list of data. The remaining list of data with 16 values has the mean $\bar{x}_1$.

Task:

Determine the mean $\bar{x}_1$.

[0/1 p.]

Task 21

Solution Times for Sudoku

6 games of online sudoku are played. The table below shows the solution times for the first 5 games.

game	solution time in s
1	356
2	321
3	378
4	450
5	298
6	t

The median of all 6 solution times is 350 s.

Task:

Determine t.

t = ____________________ s

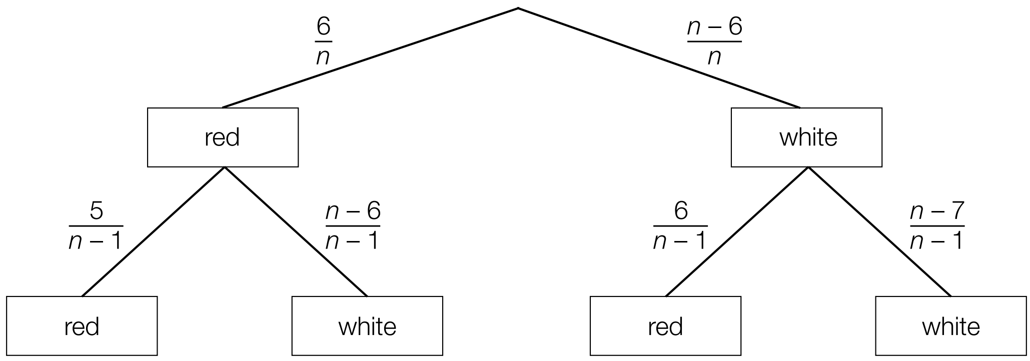
[0/1 p.]

Task 22

Removing Balls

There are n balls in an urn. Of these n balls, 6 balls are red and the rest are white. 2 balls are selected at random and removed from the urn without replacement.

The corresponding probabilities are shown in the tree diagram below.



The probability that both of the balls removed are red is p.

Task:

Write down an equation in terms of n that can be used to calculate p.

$p =$ ____________________

[0/1 p.]

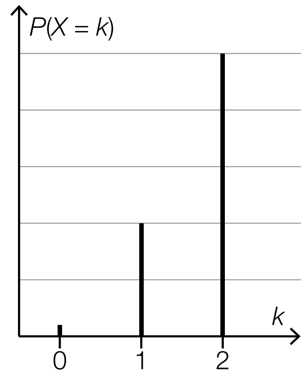
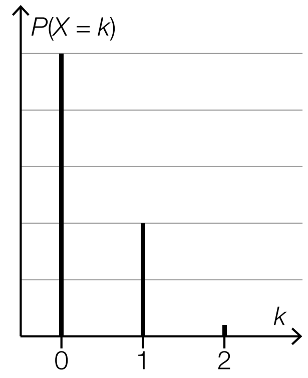
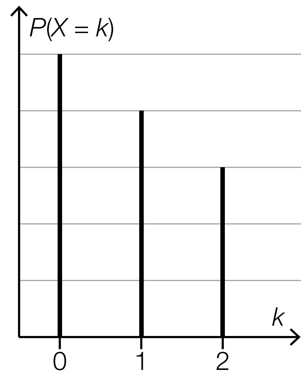
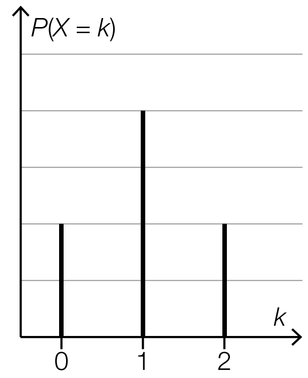
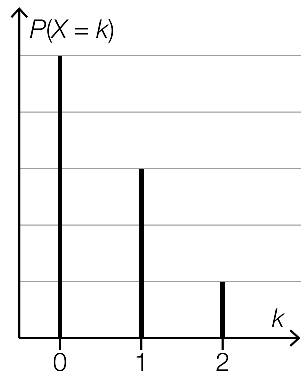
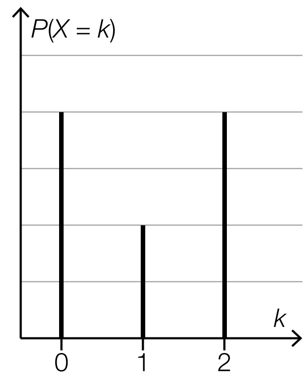
Task 23

Probability Distribution

A fair 6-sided dice with faces numbered 1, 2, 3, 4, 5, and 6 is thrown 2 times.
The random variable X describes how often a 6 is thrown in these 2 throws.

Task:

Put a cross next to the diagram that correctly represents the probability distribution of X.
[1 out of 6]

	☐		☐
	☐		☐
	☐		☐

[0/1 p.]

Task 24

Computer Game

A particular computer game consists of a number of rounds to play.
In each round, there are 5 questions each with 4 possible answers of which only 1 possible answer is correct.
A round can be won if more than half of the questions are answered correctly.
The 4 possible answers for each question are arranged at random.

In a particular round, Gerhard chooses the first possible answer for each question without reading the questions.

Task:

Determine the probability that Gerhard wins this round.

[0/1 p.]

Task 25 (Part 2)

Archery

On the grounds of a particular 3D archery facility, figures are shot at using a bow and arrow.

Task:

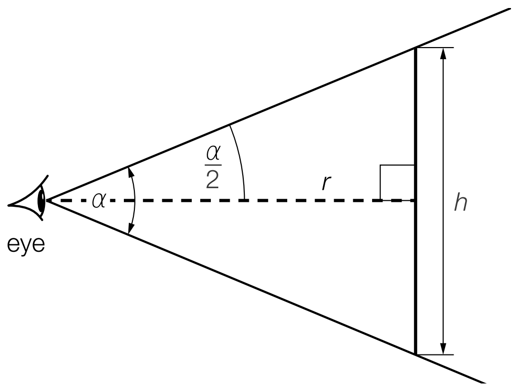
a) Paul shoots an arrow at a figure. The flight path of the tip of the arrow from its start at point S to its target at point Z can be modelled by the line g.

The following statements hold: $S = (0, 0, 1.8)$, $Z = (-5, 7, 8.5)$

1) Write down a vector equation of g.

$g: X =$ ____ *[0/1 p.]*

b) Lara can see a particular figure in an angle of vision α. The relationship between the angle of vision α, the distance r, and the height h are represented in the not-to-scale diagram below.



1) Using α and r, write down a formula that can be used to calculate h.

$h =$ ____ *[0/1 p.]*

c) During training, Paul shoots at 3 targets A, B, and C. His probability of hitting the target for each shot is independent from every other shot and is given in the table below.

target	A	B	C
probability	$\frac{2}{5}$	$\frac{7}{10}$	$\frac{1}{4}$

Paul shoots 1 time at each of the targets A, B, and C in that order.

1) Determine the probability that Paul hits at least 1 of these 3 targets. *[0/1 p.]*

Paul shoots 10 times at target A. The binomially distributed random variable X gives the number of hits.

2) Determine the expectation value $E(X)$. *[0/1 p.]*

Task 26 (Part 2, Best-of Assessment)

Bungee Jumping

Bungee jumping is an extreme sport in which a person is attached to an elastic rope and jumps off of a platform at a great height.

Task:

a) Sabine undertakes a bungee jump. During the jump, she bounces up and down a number of times.

Her height above the ground can be modelled in terms of the time t by the function h: $\mathbb{R}_0^+ \to \mathbb{R}^+$.

$$h(t) = a \cdot \left(e^{-0.03 \cdot t} \cdot \cos\left(\frac{\pi \cdot t}{6}\right) + 1\right)$$

t ... time after jumping in s
$h(t)$ ... height above the ground at time t in m
a ... positive parameter

At time $t = 0$, Sabine jumps from a platform at a height of 90 m above the ground.

1) Determine the parameter a. *[0/1 p.]*

The total length of time for which Sabine is at a height of more than 70 m above the ground during the bungee jump is given by d.

2) Determine d in seconds. *[0/1 p.]*

After reaching the lowest point, Sabine is pulled up by the rope before falling again.

3) Determine by how many metres Sabine is pulled up. *[0/1 p.]*

At time t_1, Sabine's (vertical) falling velocity is maximal.

4) Complete the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement. *[0/½/1 p.]*

At the time t_1, ___①___ holds; the falling velocity can be calculated using ___②___.

①	
$h''(t_1) > 0$	☐
$h''(t_1) < 0$	☐
$h''(t_1) = 0$	☐

②	
$h(t_1)$	☐
$\lvert h'(t_1) \rvert$	☐
$\int_0^{t_1} h(t)\,\mathrm{d}t$	☐

Task 27 (Part 2, Best-of Assessment)

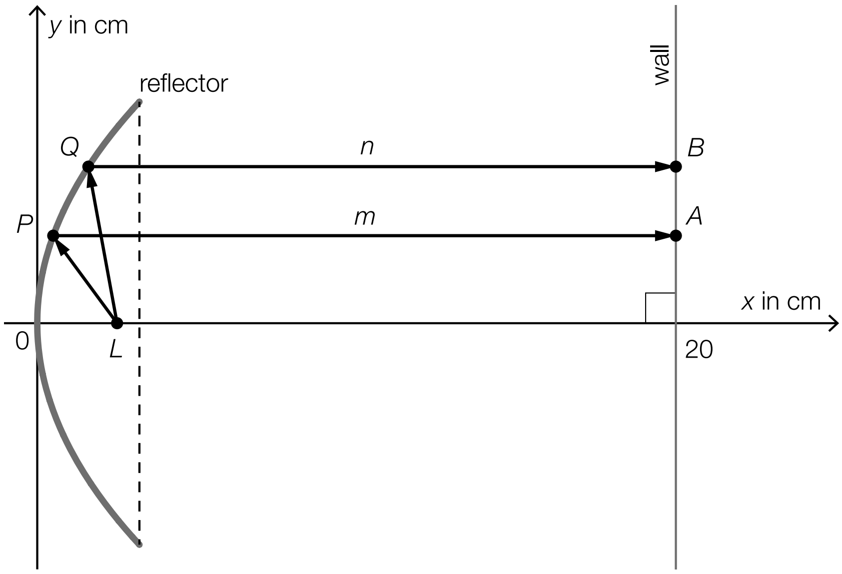
Torches

A company produces and sells torches.

Task:

a) The front part of a particular torch comprises a point that gives the light source L and a reflector that surrounds the light source.

A model of the cross-section of the front part of this torch is shown in the not-to-scale diagram below in a coordinate system.



Two straight rays of light leave the light source L and are deflected at points P and Q on the reflector parallel to the x-axis towards a wall. They hit the wall at points A and B.

$L = (2.5, 0)$
$\overline{LP} = 3$ cm and $\overline{LQ} = 4.1$ cm
$A = (20, y_A)$ and $B = (20, y_B)$
$m = 19.5$ cm

$\overline{LP} + m = \overline{LQ} + n$

1) Determine y_B. *[0/1 p.]*

b) During a quality control procedure, torches are checked for faults F_1, F_2 and F_3. These 3 faults occur independently.

The table below shows these faults along with their corresponding probabilities.

fault	description	probability
F_1	The torch is defective.	p_1
F_2	The torch is the wrong colour.	0.02
F_3	The torch has no storage bag.	0.01

A torch is selected at random and checked.

1) Match each of the four events to the probability they definitely correspond to from A to F.

[0/½/1 p.]

The torch is defective and is the wrong colour.	☐
The torch is the correct colour.	☐
The torch is not defective, it is the correct colour, and it does not have a storage bag.	☐
The torch exhibits at least 1 of these 3 faults.	☐

A	0.98
B	$1 - (1 - p_1) \cdot 0.98 \cdot 0.99$
C	$p_1 \cdot 0.02$
D	$1 - p_1 \cdot 0.02 \cdot 0.01$
E	$p_1 \cdot 0.02 \cdot 0.01$
F	$(1 - p_1) \cdot 0.98 \cdot 0.01$

c) The total costs for the production of the torches in terms of the amount produced x can be modelled by the differentiable cost function K.

x ... amount produced in units of quantity (ME)
$K(x)$ ... total costs for the production of x units in monetary units (GE)

The corresponding marginal cost function K' has the equation
$K'(x) = 0.33 \cdot x^2 - 1.8 \cdot x + 3$.

$K(1) = 44.21$ holds.

1) Write down an equation of the function K.

$K(x) =$ ____ *[0/1 p.]*

It can be assumed that every torch produced is also sold.

The revenue from the sale of these torches can be modelled by the function E in terms of the amount produced x.

$E(x) = a \cdot x$

x ... amount produced in ME
$E(x)$ ... revenue for the amount produced x in GE
a ... price in GE/ME

The profit is modelled by the profit function G (x in ME, $G(x)$ in GE).

The business goal is to generate a profit of at least 100 GE for the production and sale of 5 ME of torches.

2) Determine the lowest possible price with which this business goal can be achieved.

[0/1 p.]

Task 28 (Part 2, Best-of Assessment)

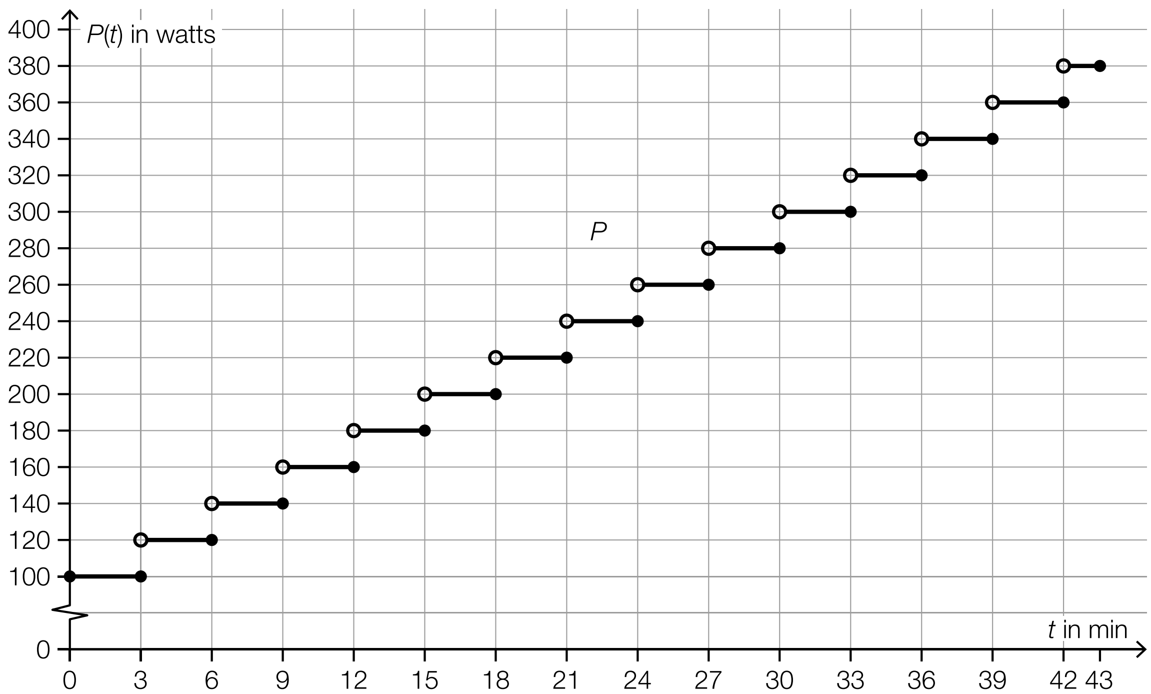
Stress Tests

Lactic acid is a metabolic product. With increasing physical stress, more lactic acid is produced by the body.
As part of stress tests, the heart frequency and the concentration of lactic acid in the blood (in mmol/l) are measured.

Task:

a) Katharina undertakes a stress test. During this test, the physical stress is increased in stages until Katharina terminates the test after 43 min.

The function $P\colon [0, 43] \to \mathbb{R}^+,\ t \mapsto P(t)$ models Katharina's power output in terms of the time t from the start of the stress test (t in min, $P(t)$ in watts).
The graph of P is shown in the diagram below.



For the work done W (in joules) in the time interval $[t_A, t_B]$ (in min), the following statement holds:

$$W = 60 \cdot \int_{t_A}^{t_B} P(t)\,\mathrm{d}t$$

1) Determine the work done by Katharina in joules in the time interval [30, 43]. *[0/1 p.]*

As part of the stress test, the lactic acid concentration in Katharina's blood is measured. The function c_1: $[0, 43] \to \mathbb{R}^+$ with $c_1(t) = 1.13 + 4 \cdot 10^{-8} \cdot t^5$ models the concentration of lactic acid in terms of the time t from the start of the stress test (t in min, $c_1(t)$ in mmol/l).

2) Determine the power (in watts) during this stress test for which the concentration of lactic acid is 1.95 mmol/l. *[0/1 p.]*

During this stress test, Katharina's heart frequency is also measured.
The function H: $[0, 43] \to \mathbb{R}^+$ with $H(t) = 2 \cdot t + 85$ models the heart frequency in terms of the time t from the start of the stress test (t in min, $H(t)$ in beats/min).

3) Explain the meaning of the numbers 2 and 85 in the given context. Write down each of the corresponding units.

meaning of the number 2:

____

meaning of the number 85:

____

[0/½/1 p.]

b) Katharina takes another stress test. In this test, the concentration of lactic acid in her blood is measured at the beginning of, during, and after a period of intense physical stress.

The function c_2: $[0, 30] \to \mathbb{R}^+$ with $c_2(t) = 31.2 \cdot (e^{-0.066 \cdot t} - e^{-0.325 \cdot t}) + 1.13$ models the lactic acid concentration in terms of the time t from the start of the stress test (t in min, $c_2(t)$ in mmol/l).

At the time t_1, the concentration of lactic acid has reduced to half of the maximum value reached.

1) Determine t_1. *[0/1 p.]*