Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

June 2024

Mathematics

Supplementary Examination 1
Examiner's Version

Bundesministerium
Bildung, Wissenschaft
und Forschung

Instructions for the standardized implementation of the supplementary examination

The following supplementary examination booklet contains four tasks that can be completed independently of one another as well as the corresponding solutions.

Each task comprises three competencies to be demonstrated.

The preparation time is to be at least 30 minutes; the examination time is at most 25 minutes.

The use of the official formula booklet that has been approved by the relevant government authority for use in the standardized school-leaving examination in mathematics is allowed. Furthermore, the use of electronic devices (e.g. graphic display calculators or other appropriate technology) is allowed provided there is no possibility to communicate (e.g. via the internet, intranet, Bluetooth, mobile networks etc.) and there is no access to an individual's data stored on the device.

After the examination, all materials (tasks, extra sheets of paper etc.) from the candidates are to be collected in. The examination materials (tasks, extra sheets of paper, data that has been produced digitally etc.) may only be made public after the time period allocated for the examination has passed.

Evaluation grid for the supplementary examination

The evaluation grid below may be used to assist in assessing the candidates' performances.

	Candidate 1			Candidate 2			Candidate 3			Candidate 4			Candidate 5		
Task 1															
Task 2															
Task 3															
Task 4															
Total															

Explanatory notes on assessment

Each task can be awarded zero, one, two or three points. A maximum of twelve points can be achieved.

Assessment scale for the supplementary examination

Total number of competencies demonstrated	Assessment of the oral supplementary examination
12	Very good
10–11	Good
8–9	Satisfactory
6–7	Pass
0–5	Fail

Task 1

Railways

a) The rail network of the Austrian Federal Railways comprises a length of around 3686 km of single-track routes. This length corresponds to 65.37 % of the total length of all routes of the Austrian Federal Railways.

1) Determine the total length of all routes of the Austrian Federal Railways.

b) The Mittenwald Railway has a gradient of 3.8 % at its steepest point.

1) Show that if the gradient of 3.8 % is doubled, then this means that the angle of the slope will also be approximately doubled.

c) A group of 9 people buys tickets for a train trip. This group comprises 3 adults, 2 seniors and 4 children; they pay a total of g euros.
The price of 1 adult ticket is twice the price of 1 child ticket.
The price of 1 senior ticket is 25 % less than the price of 1 adult ticket.

a ... price of 1 adult ticket in euros
s ... price of 1 senior ticket in euros
c ... price of 1 child ticket in euros

1) Using g, write down a system of equations that can be used to determine a, s and c.

Solution to Task 1

Railways

a1) $\frac{3\,686}{0.6537} = 5\,638.6...$

The total length of all routes of the Austrian Federal Railways is around 5 639 km.

b1) angle of the slope for a gradient of 3.8 %:

$\alpha = \arctan(0.038) = 2.17...°$

angle of the slope for a doubled gradient of 7.6 %:

$\beta = \arctan(0.076) = 4.34...°$

Therefore $\beta \approx 2 \cdot \alpha$ holds.

c1) $3 \cdot a + 2 \cdot s + 4 \cdot c = g$
$a = 2 \cdot c$
$s = 0.75 \cdot a$

Task 2

Rare Breed

In a particular area, animals of a rare breed are observed over a certain time period.

The table below shows the number of animals for the years 2010 and 2020.

year	number of animals
2010	600
2020	300

a) It can be assumed that the number of animals decayed exponentially in the time period from 2010 to 2020.
The exponential function f models the number of animals in terms of time.

t ... time in years with $t = 0$ for the year 2010
$f(t)$ ... number of animals at time t

1) Write down an equation of the exponential function f.

b) 1) Interpret the result of the calculation shown below in the given context.

$$\frac{300 - 600}{2020 - 2010} = -30$$

c) In another model, the number of animals in the time period from 2010 to 2020 is given by the function g.

$g(t) = \frac{c}{t}$ with $10 \leq t \leq 20$
t ... time in years with $t = 0$ for the year 2000
$g(t)$ ... number of animals at time t
c ... positive parameter

1) Determine the number of animals in the year 2015 according to this model.

Solution to Task 2

Rare Breed

a1) $f(t) = a \cdot b^t$
$f(0) = 600$
$f(10) = 300$

$a = 600$
$b = \sqrt[10]{\frac{300}{600}} = 0.9330...$

$f(t) = 600 \cdot 0.933^t$

or:

$f(t) = a \cdot e^{\lambda \cdot t}$
$f(0) = 600$
$f(10) = 300$

$a = 600$
$\lambda = \ln(0.9330...) = -0.0693...$

$f(t) = 600 \cdot e^{-0.0693... \cdot t}$

b1) In the time period from 2010 to 2020, the number of animals reduced by an average of 30 animals per year.

c1) $g(10) = 600$

$c = 10 \cdot 600 = 6\,000$

$g(15) = \frac{6\,000}{15} = 400$

In the year 2015, the number of animals according to this model was 400.

Task 3

Wine Cellar

a) The air temperature in a wine cellar is measured regularly (see table below).

time in days	0	60	100
air temperature in °C	8	13	17

1) Show by calculation that the three pairs of values shown in the table above are points that do not lie on a line.

b) The temperature over time in a different cellar can be modelled by the function T.

$T(t) = 0.0005 \cdot t^3 - 0.02 \cdot t^2 + 0.23 \cdot t + 8 \quad \text{with} \quad 0 \leq t \leq 24$

t ... time in h with $t = 0$ for the start of the measurements
$T(t)$ ... temperature at time t in °C

The average temperature in a time interval $[t_1, t_2]$ can be calculated using the expression below.

$$\frac{1}{t_2 - t_1} \cdot \int_{t_1}^{t_2} T(t)\,\mathrm{d}t$$

1) Determine the average temperature in this cellar in the time interval [0, 24].

c) In a wine cellar, there is a dehumidifier that collects water from the air in the form of condensation.

At an air temperature of 10°C, the volume of condensation collected per day is 5 l. At an air temperature of 20°C, the volume of condensation collected per day is 7 l.
At an air temperature of 11.25°C, the volume of condensation collected per day is lowest.
The volume of condensation collected per day in terms of the air temperature is to be modelled by the quadratic function V.

$V(T) = a \cdot T^2 + b \cdot T + c$

T ... air temperature in °C
$V(T)$ ... volume of condensation collected per day at an air temperature of T in l

1) Write down a system of linear equations that can be used to determine the coefficients a, b and c.

Solution to Task 3

Wine Cellar

a1) $\frac{13-8}{60-0} = 0.083...$

$\frac{17-13}{100-60} = 0.1$

$\frac{17-8}{100-0} = 0.09$

As the difference quotients are not equal, the three points do not lie on a line.

For the point to be awarded, it is not necessary for all 3 difference quotients to be calculated.
A justification that uses the reciprocal values of the difference quotients given is also correct.

b1) $\frac{1}{24-0} \cdot \int_0^{24} T(t)\,\mathrm{d}t = 8.648$

The average temperature in this cellar in the time interval [0, 24] is around 8.65 °C.

c1) $V'(T) = 2 \cdot a \cdot T + b$

I: $V(10) = 5$
II: $V(20) = 7$
III: $V'(11.25) = 0$

or:

I: $100 \cdot a + 10 \cdot b + c = 5$
II: $400 \cdot a + 20 \cdot b + c = 7$
III: $22.5 \cdot a + b = 0$

Task 4

Cinema

7 friends are going to the cinema to watch a film together.

a) At this cinema, there are reduced ticket prices for members of the bonus club and for school pupils.

All of the prices are shown in the table below.

	price per cinema ticket in €
normal price	15
member of the bonus club	13.50
school pupil	12

The 7 friends buy 2 tickets at the normal price, 1 ticket for a member of the bonus club and 4 tickets at the school pupil price.

1) Interpret the result of the calculation shown below in the given context.

$$\frac{2 \cdot 15 + 13.50 + 4 \cdot 12}{7} \approx 13.07\ldots$$

b) At this cinema, there is the opportunity to win vouchers. Every person receives exactly one chance. The probability of winning a voucher is the same for each person.

The binomially distributed random variable X describes how many of the 7 friends win exactly one voucher.

$P(X = 0) = 0.3206$ holds.

1) Determine the probability that at least 3 out of the 7 friends win exactly one voucher.

c) The number of the 7 friends who enjoyed the film is given by a.
After going to the cinema, 2 out of the 7 friends are chosen at random to participate in a questionnaire.
The event that these 2 friends enjoyed the film is given by E.

1) Using a, write down a formula that can be used to calculate the probability $P(E)$.

$P(E) =$ ____

Solution to Task 4

Cinema

a1) The 7 friends spend on average around 13.07 euros on a ticket.

or:

The mean of the prices of the tickets for these 7 friends is around 13.07 euros.

b1) $\binom{7}{0} \cdot p^0 \cdot (1 - p)^7 = (1 - p)^7 = 0.3206$
$p = 1 - \sqrt[7]{0.3206}$
$p = 0.1499...$

calculation using technology:

$P(X \geq 3) = 0.0737...$

The probability that at least 3 out of the 7 friends win exactly one voucher is around 7.4 %.

c1) $P(E) = \frac{a}{7} \cdot \frac{a - 1}{6}$