Name:	
Class:	

Standardised Competence-Oriented
Written School-Leaving Examination

AHS

20th September 2016

Mathematics

Part 1 Tasks

BMB Bundesministerium für Bildung

Bundesinstitut bifie

Advice for Completing the Tasks

Dear candidate,

The following booklet for Part 1 contains 24 tasks. The tasks can be completed independently of one another. You have *120 minutes* available in which to work through this booklet.

Please use a blue or black pen that cannot be rubbed out. You may use a pencil for tasks that require you to draw a graph, vectors or a geometric construction.

Please do all of your working out solely in this booklet. Write your name on the first page of the booklet in the space provided.

All answers must be written in this booklet. In the assessment of your work, everything that is not crossed out will be considered. Your solutions must be clearly marked. If a solution is not clearly marked or if more than one solution is given, the task will be considered to be unsolved. Draw a line through any notes you make.

You may use a pre-approved formula book as well as your usual electronic device(s).

Please hand in the task booklet at the end of the examination.

Assessment

Every task in Part 1 will be awarded either 0 points or 1 point. Every sub-task in Part 2 will be awarded 0, 1 or 2 points. The tasks marked with an A will be awarded either 0 points or 1 point.

– If at least 16 of the 24 tasks in Part 1 are solved correctly, you will pass the examination.
– If fewer than 16 of the 24 tasks in Part 1 are solved correctly, then the tasks marked with an A from Part 2 may compensate for the shortfall (as part of the "range of essential skills" outlined by the LVBO).
  If, including the tasks marked with an A from Part 2, at least 16 tasks are solved correctly, you will pass the examination.
  If, including the tasks marked with an A from Part 2, fewer than 16 tasks are solved correctly, you will not be awarded enough points to pass the examination.
– If at least 16 tasks are solved correctly (including the compensation tasks marked with an A from Part 2), a grade will be awarded as follows:

Pass	16–23 points
Satisfactory	24–32 points
Good	33–40 points
Very Good	41–48 points

Explanation of the Task Types

Some tasks require a ***free answer***. For these tasks, you should write your answer directly underneath each task in the task booklet. Other task types used in the examination are as follows:

Matching tasks: For this task type you will be given a number of statements, tables or diagrams, which will appear alongside a selection of possible answers. To correctly answer these tasks, you will need to match each statement, table or diagram to its corresponding answer. You should write **the letter of the correct answer** next to the statement, table or diagram in the space provided.

Example:
You are given two equations.

Task:
Match the two equations to their corresponding description (from A to D).

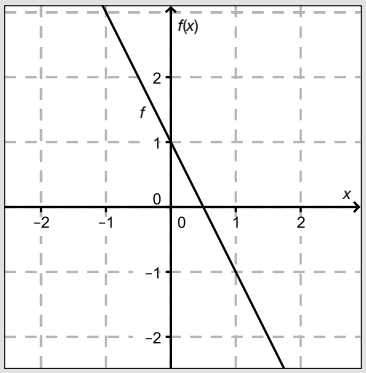
$1 + 1 = 2$	A
$2 \cdot 2 = 4$	C

A	Addition
B	Division
C	Multiplication
D	Subtraction

Construction tasks: This task type requires you to draw points, lines and/or curves in the task booklet.

Example:
Below you will see a linear function f where $f(x) = k \cdot x + d$.

Task:
On the axes provided below, draw the graph of a linear function for which $k = -2$ and $d > 0$.



Multiple-choice tasks of the form "1 out of 6": This task type consists of a question and six possible answers. Only **one answer** should be selected. You should put a cross next to the only correct answer in the space provided.

Example:
Which equation is correct?

Task:
Put a cross next to the correct equation.

1 + 1 = 1	☐
2 + 2 = 2	☐
3 + 3 = 3	☐
4 + 4 = 8	☒
5 + 5 = 5	☐
6 + 6 = 6	☐

Multiple-choice tasks of the form "2 out of 5": This task type consists of a question and five possible answers, of which **two answers** should be selected. You should put a cross next to each of the two correct answers in the space provided.

Example:
Which equations are correct?

Task:
Put a cross next to each of the two correct equations.

1 + 1 = 1	☐
2 + 2 = 4	☒
3 + 3 = 3	☐
4 + 4 = 8	☒
5 + 5 = 5	☐

Multiple-choice tasks of the form "x out of 5": This task type consists of a question and five possible answers, of which **one, two, three, four *or* five answers** may be selected. The task will require you to: "Put a cross next to each correct statement/equation …". You should put a cross next to each correct answer in the space provided.

Example:
Which of the equations given are correct?

Task:
Put a cross next to each correct equation.

1 + 1 = 2	☒
2 + 2 = 4	☒
3 + 3 = 6	☒
4 + 4 = 4	☐
5 + 5 = 10	☒

Gap-fill: This task type consists of a sentence with two gaps, i.e. two sections of the sentence are missing and must be completed. For each gap you will be given the choice of three possible answers. You should put a cross next to **each of the two answers** that are necessary to complete the sentence correctly.

Example:
Below you will see 3 equations.

Task:
Complete the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

The operation in equation _____ ① _____ is known as summation or _____ ② _____ .

①	
1 – 1 = 0	☐
1 + 1 = 2	☒
1 · 1 = 1	☐

②	
Multiplication	☐
Subtraction	☐
Addition	☒

Changing an answer for a task that requires a cross:

1. Fill in the box that contains the cross for your original answer.
2. Put a cross in the box next to your new answer.

1 + 1 = 3	☐
2 + 2 = 4	☒
3 + 3 = 5	☐
4 + 4 = 4	☐
5 + 5 = 9	■

In this instance, the answer "5 + 5 = 9" was originally chosen. The answer was later changed to be "2 + 2 = 4".

Selecting an answer that has been filled in:

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

1 + 1 = 3	☐
2 + 2 = 4	(■)
3 + 3 = 5	☐
4 + 4 = 4	■
5 + 5 = 9	☐

In this instance, the answer "2 + 2 = 4" was filled in and then selected again.

If you still have any questions now, please ask your teacher.

Good Luck!

Task 1

Properties of Numbers

Below you will find statements about numbers and sets of numbers.

Task:

Put a cross next to each of the two correct statements.

The square root of every natural number is an irrational number.	☐
Every natural number can be written as a fraction in the form $\frac{a}{b}$ where $a \in \mathbb{Z}$ and $b \in \mathbb{Z}\backslash\{0\}$.	☐
The product of two rational numbers can be a natural number.	☐
Every real number can be written as a fraction in the form $\frac{a}{b}$ where $a \in \mathbb{Z}$ and $b \in \mathbb{Z}\backslash\{0\}$.	☐
There exists a smallest integer.	☐

Task 2

System of Equations

Below you will see a system of linear equations involving the variables $x, y \in \mathbb{R}$:

I: $x + 4 \cdot y = -8$
II: $a \cdot x + 6 \cdot y = c$ where $a, c \in \mathbb{R}$

Task:

Determine the values for a and c for which the system of equations has infinitely many solutions.

$a =$ _____

$c =$ _____

Task 3

Vectors

In two-dimensional space, the points A, B, C and D are marked on a straight line at equal intervals.

Thus:
$\overrightarrow{AB} = \overrightarrow{BC} = \overrightarrow{CD}$

The coordinates of the points A and C are as follows:

$A = (3, 1)$
$C = (7, 8)$

Task:

Determine the coordinates of D.

$D = (\_\_\_\_\_, \_\_\_\_\_)$

Task 4

Equation of a Line

The line g is represented by the vector equation g: $X = \begin{pmatrix} 2 \\ 6 \end{pmatrix} + t \cdot \begin{pmatrix} 3 \\ -5 \end{pmatrix}$.

Task:

Write down possible values for a and b such that the line h, with equation $a \cdot x + b \cdot y = 1$, is perpendicular to line g.

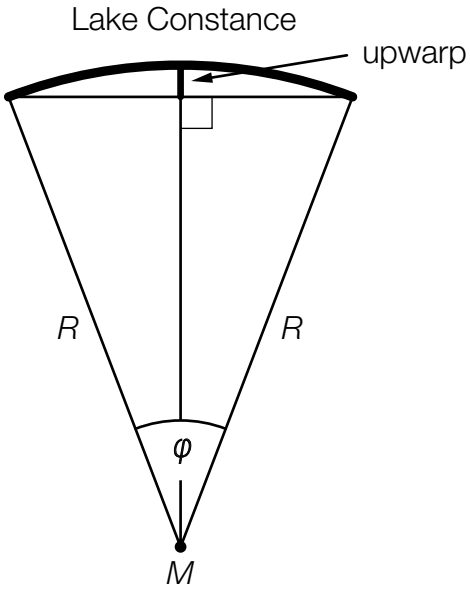
a = _____

b = _____

Task 5

Upwarp of Lake Constance

Due to the curvature of the Earth, the surface of Lake Constance is curved. The Earth can be modelled as a sphere with radius R = 6370 km and midpoint M. The size of the angle $\varphi = 0.5846°$ can be determined from the length of the southeast-northwest span of Lake Constance. Using this information, the height of the upwarp of Lake Constance can be approximated.



Task:

Determine the height of the upwarp of Lake Constance (see diagram above) in metres.

Upwarp: _____ meters

Task 6

Determining an Angle

An angle $\alpha \in [0°, 360°)$ fulfils the following criteria:
$\sin(\alpha) = 0.4$ and $\cos(\alpha) < 0$

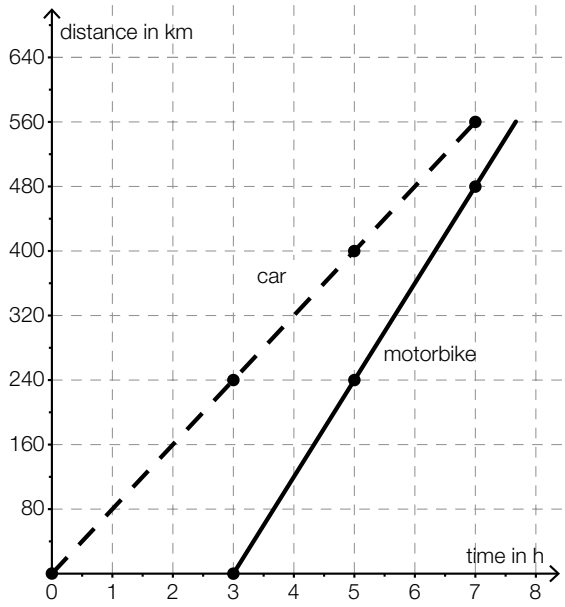
Task:

Determine the size of the angle α.

Task 7

Reading Data from a Diagram

A motorbike travels along the same stretch of road (560 km) as a car. Both journeys are modelled as straight lines on the distance-time-diagram shown below. The points marked in bold have integer coordinates.



Task:

Put a cross next to each of the two statements below that give a correct interpretation of the diagram.

The motorbike sets off three hours after the car leaves.	☐
The motorbike has an average speed of 100 km/h.	☐
When the car reaches its destination, the motorbike is still 120 km away from it.	☐
The average speed of the car is 40 km/h less than that of the motorbike.	☐
The motorbike's total travel time for this section of the road is longer than that of the car.	☐

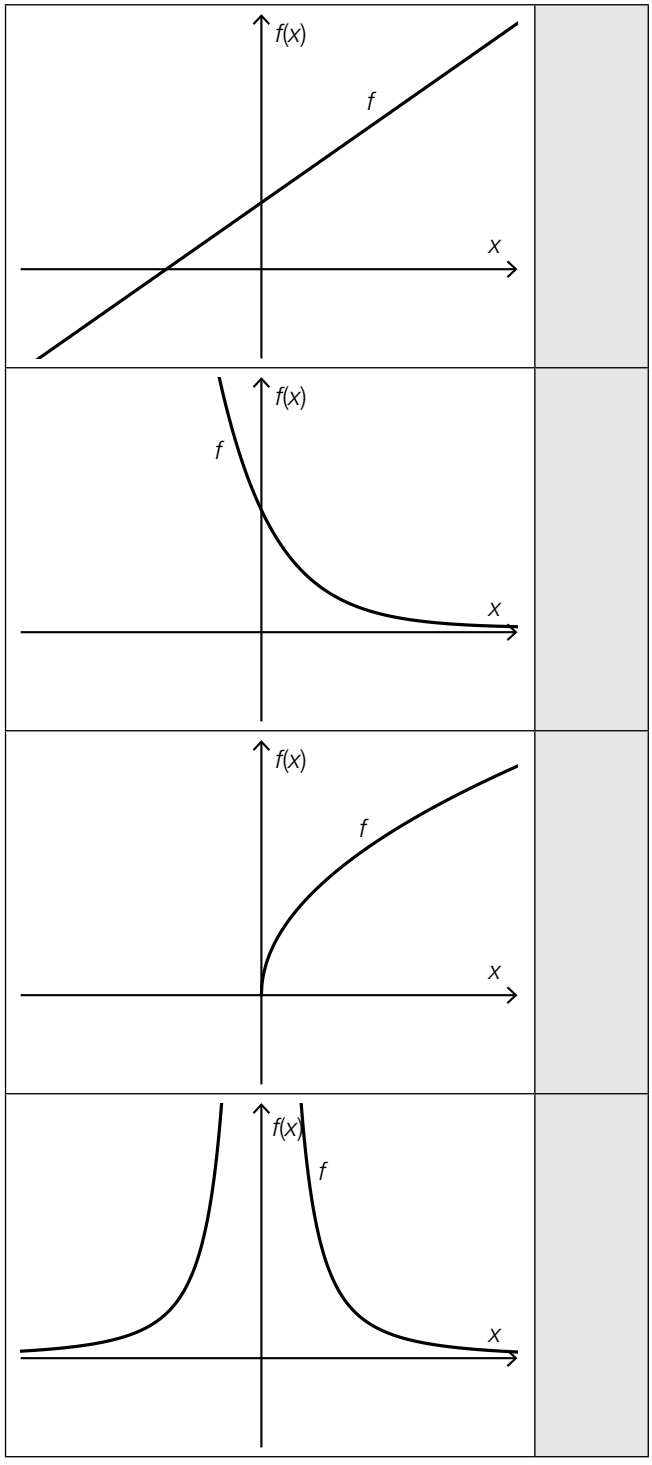
Task 8

Graphs and Types of Functions

The graphs of four functions as well as the equations of six types of functions with parameters $a, b \in \mathbb{R}^+$ are shown below.

Task:

Match each of the four graphs to its corresponding type of function (from A to F).



A	$f(x) = a \cdot b^x$
B	$f(x) = a \cdot x^{\frac{1}{2}}$
C	$f(x) = a \cdot \frac{1}{x^2}$
D	$f(x) = a \cdot x^2 + b$
E	$f(x) = a \cdot x^3$
F	$f(x) = a \cdot x + b$

Task 9

Equation of a Linear Function

A linear function f has the following properties:

- If the argument, x, is increased by 2, then the function value $f(x)$ decreases by 4.
- $f(0) = 1$

Task:

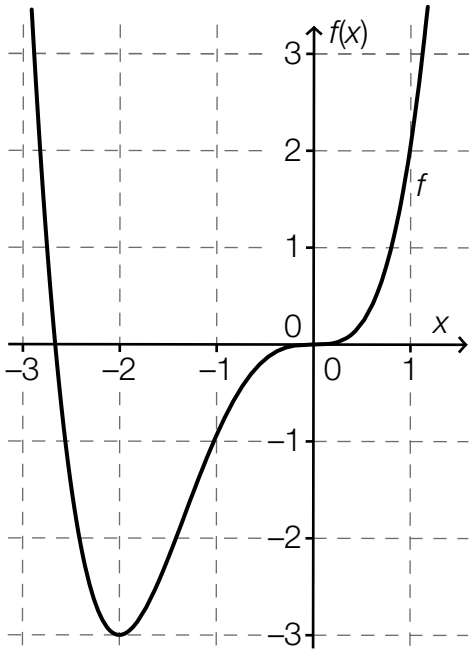
Determine the equation of this linear function f.

$f(x) =$ _____

Task 10

n^{th} Degree Polynomial Function

The diagram below shows the graph of a polynomial function f. All of the characteristic points of the graph (axis intercepts, maxima and minima, points of inflexion) are included on the diagram.



Task:

Complete the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

The polynomial function f has degree _____ ① _____ because f has exactly _____ ② _____.

①	
$n < 3$	☐
$n = 3$	☐
$n > 3$	☐

②	
one maximum or minimum	☐
two points of inflection	☐
two roots	☐

Task 11

Bee Population

Due to environmental poisoning, a beekeeper's population of bees is declining daily by a fixed percentage. The beekeeper has determined that he has suffered a loss of 50 % of the population within 14 days.

Task:

Determine the daily relative population decrease in percent.

Daily relative population decrease: _____ %

Task 12

Periodic Function

A periodic function f has equation $f(x) = \sin(x)$.

Task:

Determine the smallest value $a > 0$ (measured in radians) such that the equation $f(x + a) = f(x)$ holds for all $x \in \mathbb{R}$.

a = _____ rad

Task 13

Share Price

From the time $t = 0$ the price of a share (in euros) is observed and recorded. The price of the share after t days is given by $A(t)$.

Task:

The following value is calculated:

$$\frac{A(10) - A(0)}{10} = 2$$

Write down what this value represents in the context of the development of the price of the share.

Task 14

Differentiation Rules

It is known that $g(x) = 3 \cdot f(x) - 2$ holds for the polynomial functions f and g for all $x \in \mathbb{R}$.

Task:

Which of the statements below is true for all $x \in \mathbb{R}$? Put a cross next to the correct statement.

$g'(x) = f'(x)$	☐
$g'(x) = f'(x) - 2$	☐
$g'(x) = 3 \cdot f'(x)$	☐
$g'(x) = 3 \cdot f'(x) - 2$	☐
$g'(x) = 3 \cdot f'(x) - 2 \cdot x$	☐
$g'(x) = -2 \cdot f'(x)$	☐

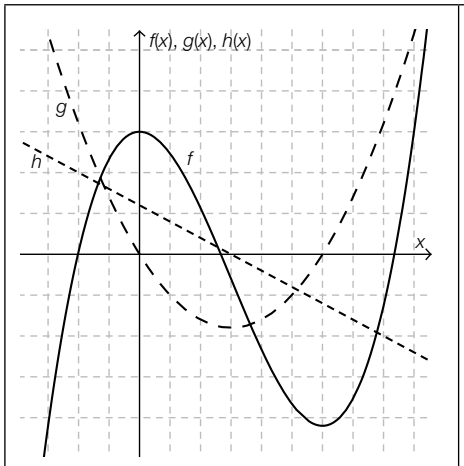
Task 15

Graphs of Derivatives

In the diagrams below the graphs of the functions *f*, *g* and *h* are shown.

Task:

In one of the six diagrams, *g* is the first derivative of *f* and h is the second derivative of *f*. Put a cross next to this diagram.



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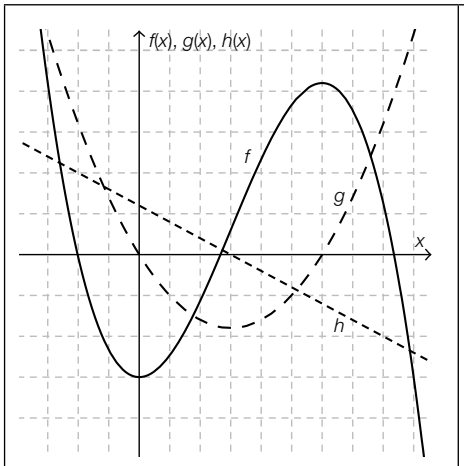

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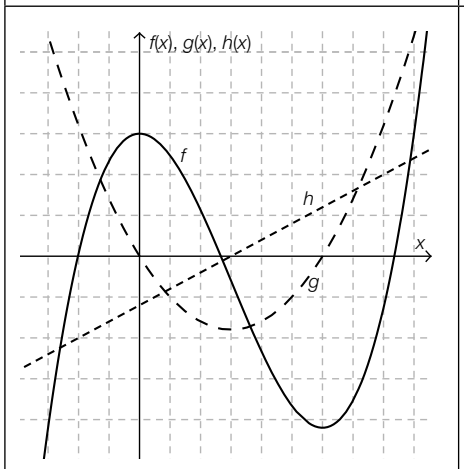

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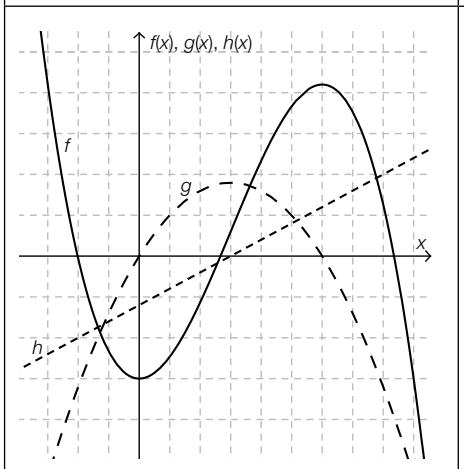

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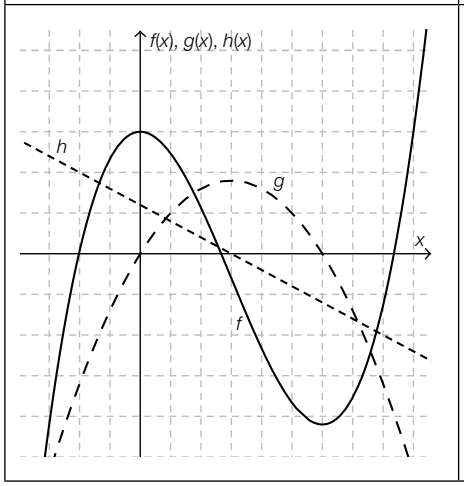

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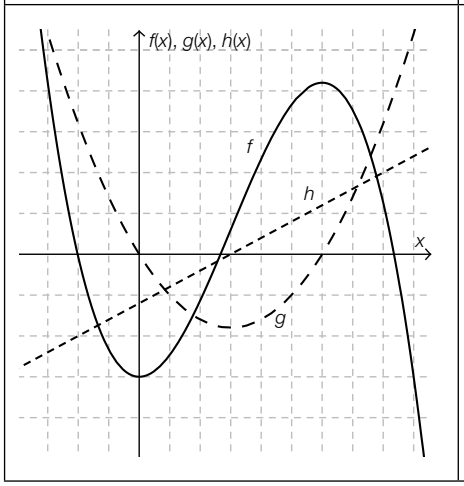
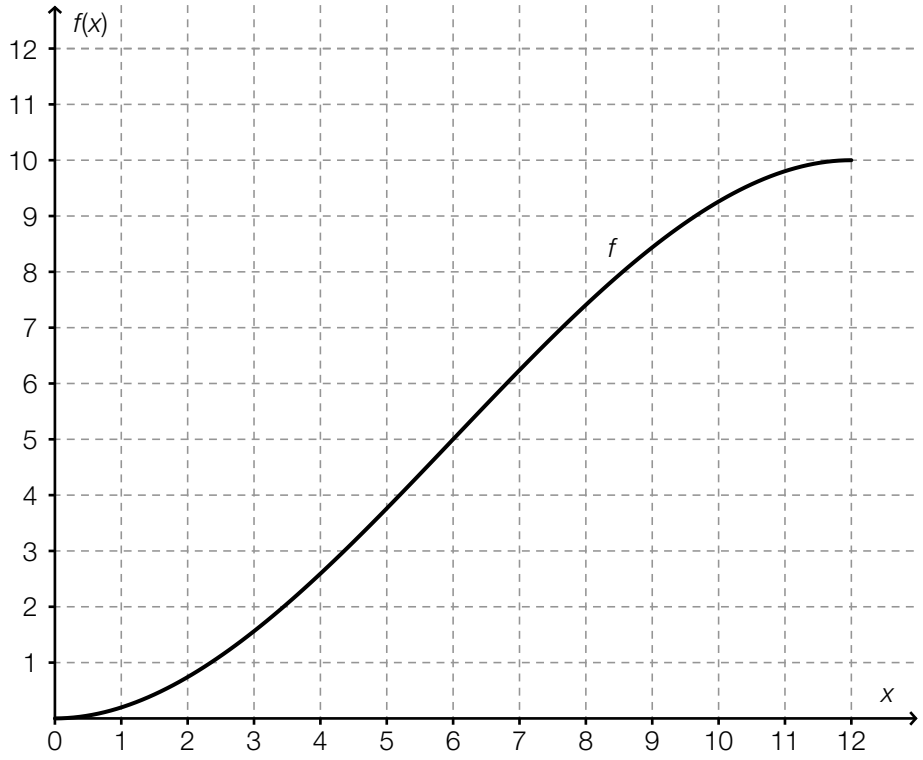


☐

Task 16

Differentiable Function

The diagram below shows a section of a graph of a polynomial function f. The gradient of the tangent is steepest at the point where $x = 6$.



Task:

Put a cross next to each of the two correct statements about the function f.

$f''(6) = 0$	☐
$f''(11) < 0$	☐
$f''(2) < f''(10)$	☐
$f'(6) = 0$	☐
$f'(7) < f'(10)$	☐

Task 17

Integral

The definite integral I is given by $I = \int_0^a (25 \cdot x^2 + 3)\,\mathrm{d}x$ where $a \in \mathbb{R}^+$.

Task:

Put a cross next to each of the two expressions that have the same value as I for all $a > 0$.

$25 \cdot \int_0^a x^2\,\mathrm{d}x + \int_0^a 3\,\mathrm{d}x$	☐
$\int_0^a 25\,\mathrm{d}x \cdot \int_0^a x^2\,\mathrm{d}x + \int_0^a 3\,\mathrm{d}x$	☐
$\int_0^a 25 \cdot x^2\,\mathrm{d}x + 3$	☐
$\frac{25 \cdot a^3}{3} + 3 \cdot a$	☐
$50 \cdot a$	☐

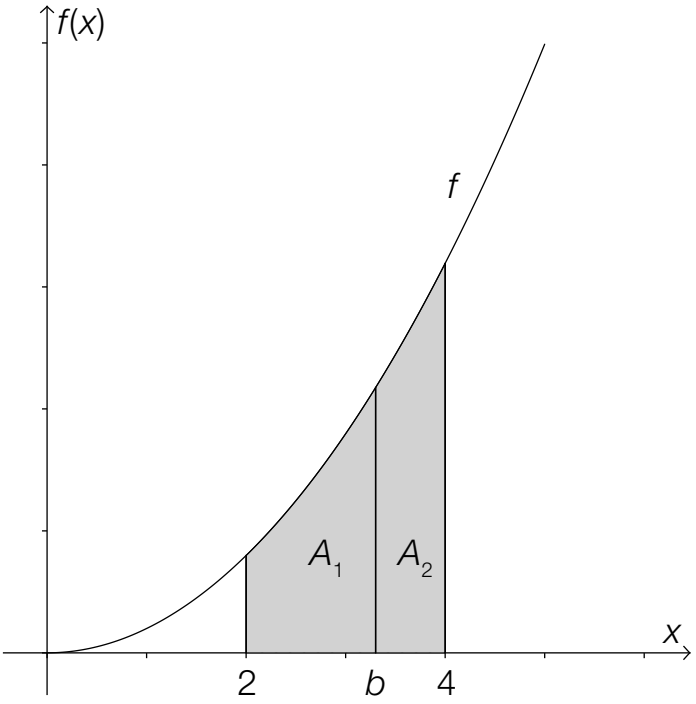
Task 18

Bisecting an Area

The real function f is given by the equation $f(x) = x^2$.

Task:

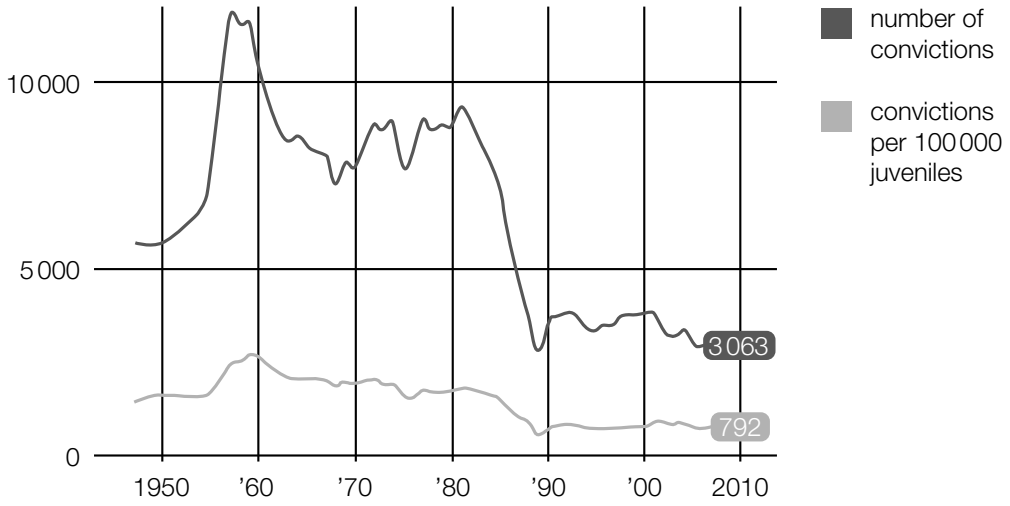
Determine the x-coordinate b such that the area between the x-axis and the graph of the function f in the interval [2, 4] is divided into two areas of equal size, A_1 and A_2 (see diagram below).



Task 19

Convictions of Juveniles

According to the Youth Protection Act of 1988 (version as of 23.03.2016), juveniles are classed as people who are aged between 14 and 17 inclusive. The diagram below shows the absolute number of convictions of juveniles as well as the number of convictions per 100 000 juveniles from 1950 to 2010.



Data source: http://derstandard.at/1371171382188/Jugendkriminalitaet-auf-Rekordtief [04.07.2013].

Task:

Approximately how many juveniles were there in total in Austria in the year 2010?
Put a cross next to the correct number.

792 000	☐
3 063 000	☐
3 863 000	☐
387 000	☐
258 000	☐
2 580 000	☐

Task 20

Probability of the Birth of a Girl

In 2014, of the children born in Austria, 42 162 were boys and 39 560 were girls.

Task:

From this data, determine an approximate value for the probability that a child born in Austria is a girl.

Task 21

Entrance Check

Whilst entering a sporting event a person *P* is carrying a prohibited item. During a security check, prohibited items are discovered with a probability of 0.9. As this sporting event is particularly high-risk, each person must undergo such two independent security checks.

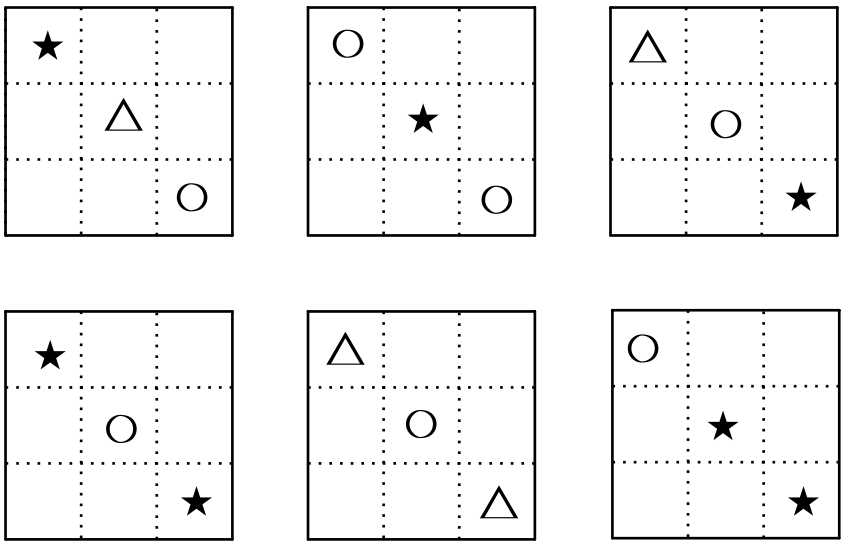
Task:

Determine the probability that person *P*'s prohibited item will be discovered by the security checks.

Task 22

Random Variable

Below you will see the six faces of a fair dice. On each face, three symbols are shown. (A dice is considered to be “fair” if the probability of the dice showing any of its six faces after being thrown is equal for all six faces.)



Task:

In a trial the dice is thrown once. The random variable X represents the number of stars on the face facing upwards. Determine the probability distribution of X, i. e. the possible values of X and their corresponding probabilities.

Task 23

Parameter of a Binomial Distribution

An experiment is described by a binomially distributed random variable X. This random variable has a probability of success of $p = 0.36$ and the standard deviation $\sigma = 7.2$.

Task:

Determine the corresponding parameter n (the number of trials).

$n =$ _____

Task 24

500 Euro Notes in Austria

In a representative survey in Austria, participants were asked for their opinions about the elimination of 500 euro notes. Of the people asked, 234 out of 1 000 participants were for the elimination of 500 euro notes.

Task:

Determine a symmetrical 95 % confidence interval for the relative proportion of Austrians who support the elimination of 500 euro notes.