Name:	
Class:	

Standardised Competence-Oriented
Written School-Leaving Examination

AHS

20th September 2016

Mathematics

Part 2 Tasks

BMB Bundesministerium für Bildung

Bundesinstitut bifie

Advice for Completing the Tasks

Dear candidate,

The following booklet for Part 2 contains five tasks, each of which contains between two and four sub-tasks. All sub-tasks can be completed independently of one another. You have *150 minutes* available in which to work on these tasks.

Please use a blue or black pen that cannot be rubbed out. You may use a pencil for tasks that require you to draw a graph, vectors or a geometric construction.

When completing these tasks please use this booklet and the paper provided. Write your name on each piece of paper you use as well as on the first page of this task booklet in the space provided. Please show clearly which sub-task each answer relates to.

In the assessment of your work, everything that is not crossed out will be considered. Your solutions must be clearly marked. If a solution is not clearly marked or if more than one solution is given, the task will be considered to be unsolved. Draw a line through any notes you make.

You may use a pre-approved formula book as well as your usual electronic device(s).

Please hand in both the task booklet and the separate sheets you have used at the end of the examination.

Assessment

Every task in Part 1 will be awarded either 0 points or 1 point. Every sub-task in Part 2 will be awarded 0, 1 or 2 points. The tasks marked with an [A] will be awarded either 0 points or 1 point.

– If at least 16 of the 24 tasks in Part 1 are solved correctly, you will pass the examination.
– If fewer than 16 of the 24 tasks in Part 1 are solved correctly, then the tasks marked with an [A] from Part 2 may compensate for the shortfall (as part of the "range of essential skills" outlined by the LVBO).
  If, including the tasks marked with an [A] from Part 2, at least 16 tasks are solved correctly, you will pass the examination.
  If, including the tasks marked with an [A] from Part 2, fewer than 16 tasks are solved correctly, you will not be awarded enough points to pass the examination.
– If at least 16 tasks are solved correctly (including the compensation tasks marked with an [A] from Part 2), a grade will be awarded as follows:

Pass	16–23 points
Satisfactory	24–32 points
Good	33–40 points
Very Good	41–48 points

Explanation of the Task Types

Some tasks require a *free answer*. For these tasks, you should write your answer directly underneath each task in the task booklet or on the paper provided. Other task types used in the examination are as follows:

Matching tasks: For this task type you will be given a number of statements, tables or diagrams, which will appear alongside a selection of possible answers. To correctly answer these tasks, you will need to match each statement, table or diagram to its corresponding answer. You should write the letter of the correct answer next to the statement, table or diagram in the space provided.

Example:
You are given two equations.

Task:
Match the two equations to their corresponding description (from A to D).

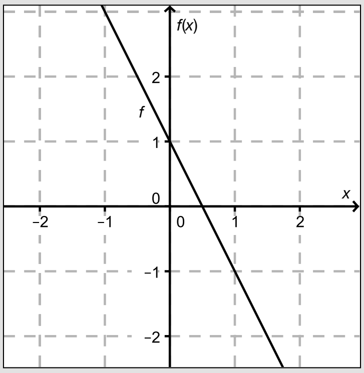
$1 + 1 = 2$	*A*
$2 \cdot 2 = 4$	*C*

A	Addition
B	Division
C	Multiplication
D	Subtraction

Construction tasks: This task type requires you to draw points, lines and/or curves in the task booklet.

Example:
Below you will see a linear function *f* where $f(x) = k \cdot x + d$.

Task:
On the axes provided below, draw the graph of a linear function for which $k = -2$ and $d > 0$.



Multiple-choice tasks of the form "1 out of 6": This task type consists of a question and six possible answers. Only one answer should be selected. You should put a cross next to the only correct answer in the space provided.

Example:
Which equation is correct?

Task:
Put a cross next to the correct equation.

1 + 1 = 1	☐
2 + 2 = 2	☐
3 + 3 = 3	☐
4 + 4 = 8	☒
5 + 5 = 5	☐
6 + 6 = 6	☐

Multiple-choice tasks of the form "2 out of 5": This task type consists of a question and five possible answers, of which two answers should be selected. You should put a cross next to each of the two correct answers in the space provided.

Example:
Which equations are correct?

Task:
Put a cross next to each of the two correct equations.

1 + 1 = 1	☐
2 + 2 = 4	☒
3 + 3 = 3	☐
4 + 4 = 8	☒
5 + 5 = 5	☐

Multiple-choice tasks of the form "x out of 5": This task type consists of a question and five possible answers, of which one, two, three, four *or* five answers may be selected. The task will require you to: "Put a cross next to each correct statement/equation …". You should put a cross next to each correct answer in the space provided.

Example:
Which of the equations given are correct?

Task:
Put a cross next to each correct equation.

1 + 1 = 2	☒
2 + 2 = 4	☒
3 + 3 = 6	☒
4 + 4 = 4	☐
5 + 5 = 10	☒

Gap-fill: This task type consists of a sentence with two gaps, i.e. two sections of the sentence are missing and must be completed. For each gap you will be given the choice of three possible answers. You should put a cross next to **each of the two answers** that are necessary to complete the sentence correctly.

Example:
Below you will see 3 equations.

Task:
Complete the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

The operation in equation ______①______ is known as summation or ______②______ .

①	
1 – 1 = 0	☐
1 + 1 = 2	☒
1 · 1 = 1	☐

②	
Multiplication	☐
Subtraction	☐
Addition	☒

Changing an answer for a task that requires a cross:

1. Fill in the box that contains the cross for your original answer.
2. Put a cross in the box next to your new answer.

1 + 1 = 3	☐
2 + 2 = 4	☒
3 + 3 = 5	☐
4 + 4 = 4	☐
5 + 5 = 9	■

In this instance, the answer “5 + 5 = 9” was originally chosen. The answer was later changed to be “2 + 2 = 4”.

Selecting an answer that has been filled in:

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

1 + 1 = 3	☐
2 + 2 = 4	(■)
3 + 3 = 5	☐
4 + 4 = 4	■
5 + 5 = 9	☐

In this instance, the answer “2 + 2 = 4” was filled in and then selected again.

If you still have any questions, please ask your teacher.

Good Luck!

Task 1

Training for Skiers

A group of skiers is completing training runs on prepared routes. The trainer is focussing her analysis on a 240 m long section from the starting block, A, to a point B. By using video analysis, the trainer determines the distance covered by the skiers in dependence to the time taken.

For a particular training run, a skier's distance covered can be modelled as a function, s, of the time taken for her journey from A to B. This function is as follows: $s(t) = -\frac{1}{144} \cdot t^4 + \frac{8}{3} \cdot t^2$.
The skier leaves the starting block at time $t = 0$. The time t is measured in seconds and the function $s(t)$ gives the distance covered up to time t in metres.

The questions below refer to the distance-time function s described above.

Task:

a) In order to be able to check the efficacy of the skier's start, her average speed, $\overline{v}$, in the time period [0 s, 3 s] is calculated.

Determine the skier's average speed $\overline{v}$ in m/s.

Find the time required for the skier to travel from A to B.

b) [A] Find the time t_1 at which $s''(t_1) = 0$.

Interpret t_1 within the context of the skier's journey from A to B.

c) Determine the instantaneous speed of the skier at time $t_2 = 6$.

Assume that the skier's speed remains constant from time t_2. Determine after how many seconds from the point of time t_2 the skier would reach point B.

d) In a mathematical model of the distance covered by the skier dependent on the time taken, the following conditions are valid:

(1) At time $t = 0$ the skier's instantaneous speed is 0 m/s.
(2) During the journey from A to B, the distance covered is strictly monotonically increasing.

Write down the mathematical properties of a differentiable distance-time function, s_1, that are guaranteed by these conditions.

Task 2

Population Growth in the USA

The first census in the USA took place in 1790. Since this time censuses have been held at ten year intervals. Between censuses the number of inhabitants is calculated using public records.

The information below shows an overview of the population development in the USA from 1790 to 1890 (in the table) and from 2003 to 2013 (in the diagram).

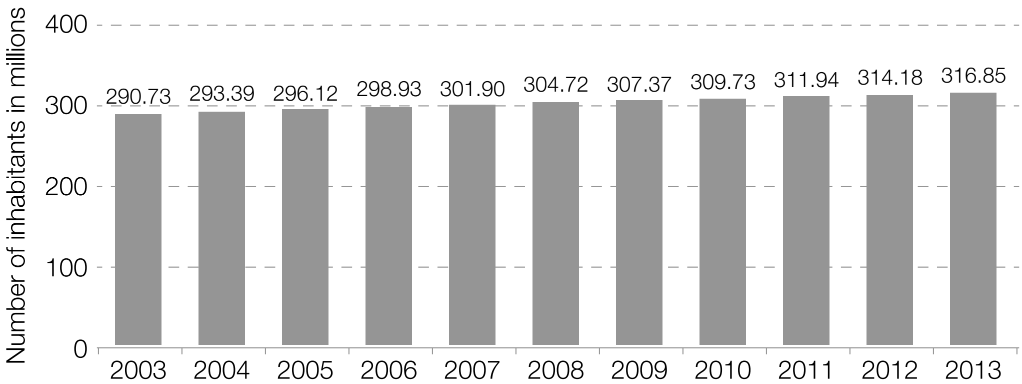
Table: Population development in the USA from 1790 to 1890

Year	Number of inhabitants in millions
1790	3.9
1800	5.2
1810	7.2
1820	9.6
1830	12.9
1840	17.1

Year	Number of inhabitants in millions
1850	23.2
1860	31.4
1870	38.6
1880	49.3
1890	62.9

Source: Keller, G. (2011). Mathematik in den Life Sciences. Stuttgart: Ulmer, p. 55.

Diagram: Population development in the USA from 2003 to 2013



Data source: http://de.statista.com/statistik/daten/studie/19320/umfrage/gesamtbevoelkerung-der-usa/ [19.09.2013] (adapted).

For the time period from 1790 to 1890, the development of the number of inhabitants in the USA can be approximated by an exponential function, B, where $B(t) = B_0 \cdot a^t$. The time, t, is measured in years elapsed since 1790 and $B(t)$ is measured in millions of inhabitants.

Task:

a) Determine an equation of the function B by using the data from the years 1790 and 1890.

Interpret the integral $\int_0^{50} B'(t)\,\mathrm{d}t$ in the given context.

b) The first derivative of the function B is given by $B'(t) = B_0 \cdot \ln(a) \cdot a^t$.

Write down the value of t^* such that $B'(t^*) = B_0 \cdot \ln(a)$ holds.
Interpret $B'(t^*)$ in the context of the population growth in the USA.

c) [A] Justify why population growth in the USA in the time period from 2003 to 2013 can be approximated by a linear function N, where $N(t) = k \cdot t + d$ (where t is the time in years that have elapsed since 2003).

Interpret the meaning of the parameter k of this linear function. A calculation of the parameter k is not required.

Task 3

Pong

The first globally popular video game was *Pong*, which was released in 1972 by the company Atari. (Source: http://de.wikipedia.org/wiki/Pong)

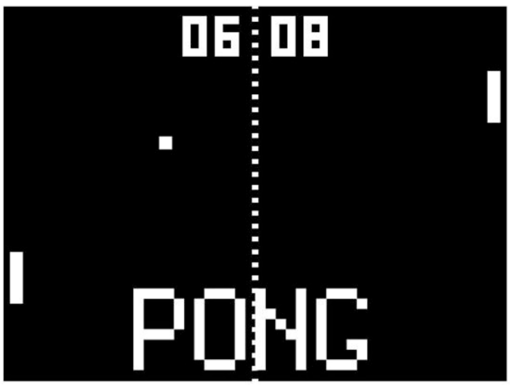
The principle of Pong is as follows: A point (or “ball”) moves in straight lines backwards and forwards across the screen. Each of the two players controls a vertical line (or “paddle”) which she/he can move upwards and downwards using a joystick. If the ball misses a player’s paddle and travels off the screen, then the player’s opponent gets one point.

The playing field in which the ball and paddle move has a width of 800 pixels and a height of 600 pixels (a pixel is a square point on the screen). As a simplification, it will be assumed that the ball can be represented as a pixel.

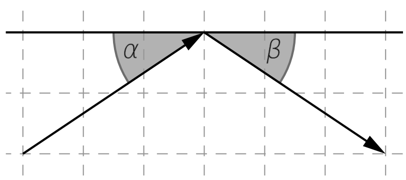
When the ball hits either the upper or lower edges of the playing field or a paddle, it is reflected back. The law of reflection, which says that $\alpha = \beta$ (see diagram below), holds in this case.

The playing field can be imagined as a plane with a coordinate system. The point (1, 1) can be found in the bottom left-hand corner and the point (800, 600) can be found in the top right-hand corner.

The image on the screen is refreshed every 0.02 seconds. The velocity vector $\vec{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$ of the ball describes the number of pixels by which the ball has moved from one image to the next in the horizontal direction (v_x) and the vertical direction (v_y).



Picture source: http://www.overclockers.at/games_forum/euer-erstes-computerspiel_237146/page_2 [15.10.2015].



Task:

a) In a particular game, the ball has a velocity vector of $\vec{v} = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$ pixels per image when it hits the upper edge of the playing field.

[A] Determine the angle α at which the ball hits the edge of the playing field.

α = ______________________

The velocity vector always has integer components. Assume that the sum of the absolute values of the components can never be greater than 20.

The ball is reflected by the upper edge of the playing field at an angle of β. What is the smallest value, β_{min}, that the angle β can have under these conditions? Determine β_{min}.

β_{min} = ______________________

b) In another game, the ball is at point (401, 301) and its velocity vector is $\vec{v} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ pixels per image.

Determine how many *seconds* it takes for the ball to be reflected by the lower edge of the playing field.

After this reflection the ball moves in a straight line until it hits either a paddle or the edge of the playing field.

Determine the vector equation of this straight line.

c) Two children, Nicola and Florian, have been playing *Pong* against each other for a long time. Out of 45 games, Nicola has won 31 times and Florian has won 14 times.

On the basis of this information, find a symmetrical 95 % confidence interval for the probability that Nicola wins.

Explain why it does not make sense to find a 100 % confidence interval.

Task 4

Roulette

In the game *roulette*, players try to guess either the number or group of numbers that will come up when a ball is thrown into the roulette machine.
In *French roulette*, the roulette machine is comprised of a wheel that sits within a bowl. This wheel is divided into 36 numbered sections that are alternately coloured red and black and a 37th green section for the number zero (see Diagram 1). The roulette wheel is spun and the ball is thrown into the wheel in the direction opposite to the direction of rotation. Through this process, the probability of the ball landing in any of the numbered sections is equal and there is no possibility to influence the outcome of the game (for example by throwing the ball in a special manner).
The aim of the game is to guess in advance which numbered section the ball will land in.
The playing tokens (chips) are placed on the roulette table layout (see Diagram 2). The areas containing, for example, the “1” and the “7” are red (“rouge”) and those containing the “4” and the “6” are black (“noir”).

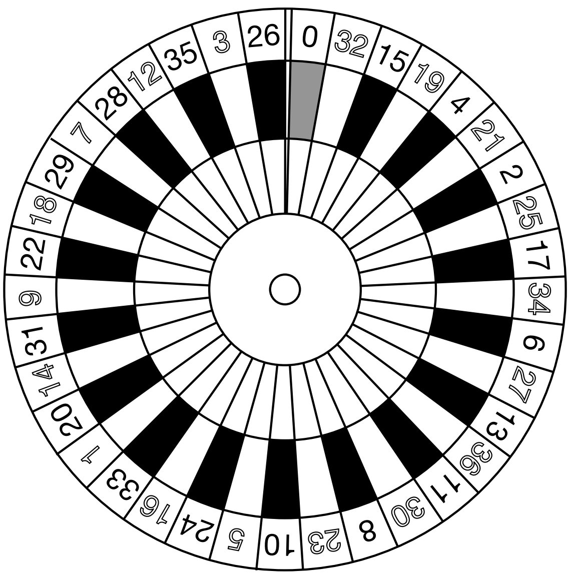


Diagram 1
Source: http://www.rouletteplay.com/images/software_logos_small/european-roulette-wheel.gif [23.03.2016].

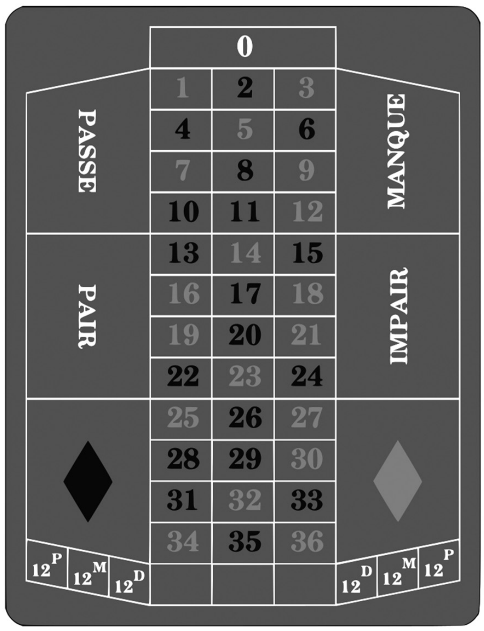


Diagram 2
Source: https://commons.wikimedia.org/wiki/File:Roulette_frz.png [23.03.2016].

Task:

a) A person argues: “If in five games the ball lands on a red section five times in a row, then on the sixth game the ball is more likely to land on a black section than a red one because ‘rouge’ and ‘noir’ should occur with the same frequency over the course of many games.” Write down whether or not this explanation is correct and justify your answer.

On one evening 100 games are played at a roulette table.
[A] Find the probability that the ball lands in a red section at most 40 times.

b) In the table below some of the betting possibilities as well as their corresponding pay-outs are listed.

Type of bet	Pay-out
Rouge: The ball lands in a red section.	1 : 1
Noir: The ball lands in a black section.	1 : 1
12^P: first dozen (numbers 1 to 12)	2 : 1
12^M: middle dozen (numbers 13 to 24)	2 : 1
12^D: final dozen (numbers 25 to 36)	2 : 1
Straight: The chip is placed on one of the 37 numbers.	35 : 1
Split: The chip is placed across two horizontally or vertically adjacent numbers, e.g. 2 and 5 or 8 and 9.	17 : 1

A pay-out of 2 : 1 means, for example, that if a player wins then they receive both the amount that was bet as well as double that amount. If the player loses, then the bet is lost and they receive no money.

In gambling, the house advantage is the expected loss of the player in relation to his/her bet.
A player places a bet of € 10 on 12^M.
Determine the house advantage as a percentage of the bet.

Demonstrate whether the house advantage (as a percentage of the bet) will be bigger, smaller or stay the same if the player places a split bet of € a. Justify your answer.