Name:	Date:
Class:	

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

June 2016

Mathematics

Supplementary Examination 9
Candidate's Version

BM Bundesministerium für Bildung und Frauen
Bundesinstitut bifie

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Instructions for the supplementary examination

Dear candidate,

The following supplementary examination is comprised of five tasks that can be completed independently of one another.

Each task contains two parts: The statement of the task requires you to demonstrate core competencies, and the guiding question that follows it requires you to demonstrate your ability to communicate your ideas.

You will be given preparation time of at least 30 minutes, and the examination will last at the most 25 minutes.

Assessment

Each task can be awarded zero, one or two points. There is one point available for each demonstration of core competencies as well as for each guiding question. A maximum of 10 points can be achieved.

For the grading of the examination the following scale will be used:

Grade	Minimum number of points
Pass	4 points for the core competencies + 0 points for the guiding questions 3 points for the core competencies + 1 point for the guiding questions
Satisfactory	5 points for the core competencies + 0 points for the guiding questions 4 points for the core competencies + 1 point for the guiding questions 3 points for the core competencies + 2 points for the guiding questions
Good	5 points for the core competencies + 1 point for the guiding questions 4 points for the core competencies + 2 points for the guiding questions 3 points for the core competencies + 3 points for the guiding questions
Very good	5 points for the core competencies + 2 points for the guiding questions 4 points for the core competencies + 3 points for the guiding questions

The examination board will decide on the final grade based on your performance in the supplementary examination as well as the result of the written examination.

Good Luck!

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Task 1

Lines in $\mathbb{R}^3$

Let g and h be two lines in $\mathbb{R}^3$.

The line g goes through the point $P = (3,1,5)$ and is parallel to the y-axis.

Task:

Determine a vector equation of the line g.

Explain why it is not possible to find a point Q, where $Q = (1,y_Q,z_Q)$, such that the point Q lies on the line g.

Guiding question:

Describe all possible relative positions of two lines in $\mathbb{R}^3$.

The line h is described by the vector equation $X = \begin{pmatrix} x_h \\ 1 \\ 3 \end{pmatrix} + s \cdot \begin{pmatrix} 2 \\ y_h \\ 1 \end{pmatrix}$ where $s, x_h, y_h \in \mathbb{R}$.

Is it possible to determine values for x_h and y_h such that the two lines g and h are perpendicular to one another and meet at point P?
If not, justify by means of calculation why this is not possible.
If so, determine the relevant values of x_h and y_h.

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Task 2

Quadratic Function

Let f be a function with $f(x) = r \cdot x^2 + s$ where $r, s \in \mathbb{R}, r \neq 0$.

Task:

Explain the effects that changes of the values of the parameters r and s have on the behaviour of the graph of f.

Guiding question:

The graph of the function f goes through the two points $B = (a,b)$ and $E = (0,e)$, where $a \neq 0$.

Determine expressions for r and s in terms of the coordinate values a, b, and e.

State the value of b for which the function f is not a quadratic function.

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Task 3

Spring Force

If a spring is stretched, then the force required to stretch the spring is directly proportional to the amount by which the spring is stretched. The function F describes the force required in terms of the amount by which the spring is stretched, x.

Therefore: $F(x) = k \cdot x$.

The distance x is measured in metres (m) and $F(x)$ is measured in Newtons (N). The constant k is known as the spring constant and describes the "strength" of a spring.

Task:

Sketch a possible graph of F and label k on your sketch.

Guiding question:

Write down an expression in terms of k that could be used to calculate the work required to stretch the spring by a distance of x_0.

Explain how the work done changes if the spring is stretched by the length $2 \cdot x_0$.

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Task 4

Marginal Costs

For a business, the cost function, K, where $K(x) = 4 \cdot x^3 - 60 \cdot x^2 + 400 \cdot x + 1\,000$, gives the cost of manufacturing x units of a product. The production costs, $K(x)$, are given in monetary units (GE), and the number of units manufactured is given in production units (ME).

The marginal costs (in GE/ME) are the additional costs accrued from increasing the number of units manufactured by 1 ME.

Task:

The approximate calculation of the marginal costs for a particular production amount, x_0, can be approximated by the first derivative $K'(x_0)$.

Using the derivative of the function K, determine the marginal costs for a production volume of 15 ME.

Guiding question:

Determine the difference in GE between the approximated marginal costs for a production volume of 15 ME and the actual increase in cost when the production volume is increased from 15 ME to 16 ME.

The first derivative K' is strictly monotonically increasing from $x = 5$ ME. Explain the implications of this statement for the production costs if the production volume increases.

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Task 5

Discrete Random Variable

For a discrete random variable, X, the following table shows all of its possible values as well as their corresponding probabilities. The parameter n is a natural number with $n \neq 0$.

k	1	4	7	10	15
$P(X = k)$	0.2	$\frac{2}{n}$	$\frac{6}{n}$	0.1	0.3

Task:

Determine the value of the parameter n and the expectation value of the random variable X. Explain your method.

Guiding question:

The standard deviation σ for the random variable described above has a value of $\sigma = 5.2$.

In the table, change the given probabilities $P(X = k)$ for at least two values of k such that the standard deviation becomes smaller and the table still shows a valid probability distribution. The values of the random variable (in the first row of the table) should remain unchanged. Explain your method.

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