

Name:	
Class:	



Standardised Competence-Oriented  
Written School-Leaving Examination

AHS

10<sup>th</sup> May 2016

# Mathematics

Part 1 Tasks



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# Advice for Completing the Tasks

Dear candidate,

The following booklet for Part 1 contains 24 tasks. The tasks can be completed independently of one another. You have *120 minutes* available in which to work through this booklet.

Please use a blue or black pen that cannot be rubbed out. You may use a pencil for tasks that require you to draw a graph, vectors or a geometric construction.

Please do all of your working out solely in this booklet. Write your name on the first page of the booklet in the space provided.

All answers must be written in this booklet. In the assessment of your work, everything that is not crossed out will be considered. Your solutions must be clearly marked. If a solution is not clearly marked or if more than one solution is given, the task will be considered to be unsolved. Draw a line through any notes you make.

You may use a pre-approved formula book as well as your usual electronic device(s).

Please hand in the task booklet at the end of the examination.

## Assessment

Every task in Part 1 will be awarded either 0 points or 1 point. Every sub-task in Part 2 will be awarded 0, 1 or 2 points. The tasks marked with an **A** will be awarded either 0 points or 1 point.

- If at least 16 of the 24 tasks in Part 1 are solved correctly, you will pass the examination.
- If fewer than 16 of the 24 tasks in Part 1 are solved correctly, then the tasks marked with an **A** from Part 2 may compensate for the shortfall (as part of the “range of essential skills” outlined by the LVBO). If, including the tasks marked with an **A** from Part 2, at least 16 tasks are solved correctly, you will pass the examination.  
If, including the tasks marked with an **A** from Part 2, fewer than 16 tasks are solved correctly, you will not be awarded enough points to pass the examination.
- If at least 16 tasks are solved correctly (including the compensation tasks marked with an **A** from Part 2), a grade will be awarded as follows:

Pass	16–23 points
Satisfactory	24–32 points
Good	33–40 points
Very Good	41–48 points

## Explanation of the Task Types

Some tasks require a *free answer*. For these tasks, you should write your answer directly underneath each task in the task booklet. Other task types used in the examination are as follows:

**Matching tasks:** For this task type you will be given a number of statements, tables or diagrams, which will appear alongside a selection of possible answers. To correctly answer these tasks, you will need to match each statement, table or diagram to its corresponding answer. You should write **the letter of the correct answer** next to the statement, table or diagram in the space provided.

### Example:

You are given two equations.

### Task:

Match the two equations to their corresponding description (from A to D).

$1 + 1 = 2$	A
$2 \cdot 2 = 4$	C

A	Addition
B	Division
C	Multiplication
D	Subtraction

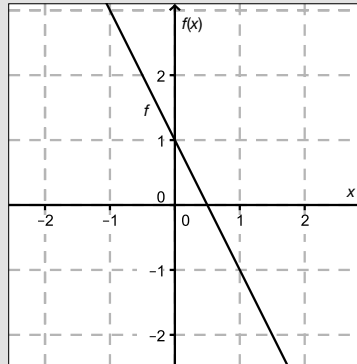
**Construction tasks:** This task type requires you to draw points, lines and/or curves in the task booklet.

**Example:**

Below you will see a linear function  $f$  where  $f(x) = k \cdot x + d$ .

**Task:**

On the axes provided below, draw the graph of a linear function for which  $k = -2$  and  $d > 0$ .



**Multiple-choice tasks of the form “1 out of 6”:** This task type consists of a question and six possible answers. Only one answer should be selected. You should put a cross next to the only correct answer in the space provided.

**Example:**

Which equation is correct?

**Task:**

Put a cross next to the correct equation.

$1 + 1 = 1$	<input type="checkbox"/>
$2 + 2 = 2$	<input type="checkbox"/>
$3 + 3 = 3$	<input type="checkbox"/>
$4 + 4 = 8$	<input checked="" type="checkbox"/>
$5 + 5 = 5$	<input type="checkbox"/>
$6 + 6 = 6$	<input type="checkbox"/>

**Multiple-choice tasks of the form “2 out of 5”:** This task type consists of a question and five possible answers, of which two answers should be selected. You should put a cross next to each of the two correct answers in the space provided.

**Example:**

Which equations are correct?

**Task:**

Put a cross next to each of the two correct equations.

$1 + 1 = 1$	<input type="checkbox"/>
$2 + 2 = 4$	<input checked="" type="checkbox"/>
$3 + 3 = 3$	<input type="checkbox"/>
$4 + 4 = 8$	<input checked="" type="checkbox"/>
$5 + 5 = 5$	<input type="checkbox"/>

**Multiple-choice tasks of the form “x out of 5”:** This task type consists of a question and five possible answers, of which one, two, three, four or five answers may be selected. The task will require you to: “Put a cross next to each correct statement/equation ...”. You should put a cross next to each correct answer in the space provided.

**Example:**

Which of the equations given are correct?

**Task:**

Put a cross next to each correct equation.

$1 + 1 = 2$	<input checked="" type="checkbox"/>
$2 + 2 = 4$	<input checked="" type="checkbox"/>
$3 + 3 = 6$	<input checked="" type="checkbox"/>
$4 + 4 = 4$	<input type="checkbox"/>
$5 + 5 = 10$	<input checked="" type="checkbox"/>

**Gap-fill:** This task type consists of a sentence with two gaps, i.e. two sections of the sentence are missing and must be completed. For each gap you will be given the choice of three possible answers. You should put a cross next to **each of the two answers** that are necessary to complete the sentence correctly.

**Example:**

Below you will see 3 equations.

**Task:**

Complete the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

The operation in equation \_\_\_\_\_<sup>①</sup>\_\_\_\_\_ is known as summation or \_\_\_\_\_<sup>②</sup>\_\_\_\_\_.

①	
$1 - 1 = 0$	<input type="checkbox"/>
$1 + 1 = 2$	<input checked="" type="checkbox"/>
$1 \cdot 1 = 1$	<input type="checkbox"/>

②	
Multiplication	<input type="checkbox"/>
Subtraction	<input type="checkbox"/>
Addition	<input checked="" type="checkbox"/>

**Changing an answer for a task that requires a cross:**

1. Fill in the box that contains the cross for your original answer.
2. Put a cross in the box next to your new answer.

$1 + 1 = 3$	<input type="checkbox"/>
$2 + 2 = 4$	<input checked="" type="checkbox"/>
$3 + 3 = 5$	<input type="checkbox"/>
$4 + 4 = 4$	<input type="checkbox"/>
$5 + 5 = 9$	<input checked="" type="checkbox"/>

In this instance, the answer “ $5 + 5 = 9$ ” was originally chosen. The answer was later changed to be “ $2 + 2 = 4$ ”.

**Selecting an answer that has been filled in:**

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

$1 + 1 = 3$	<input type="checkbox"/>
$2 + 2 = 4$	<input checked="" type="checkbox"/>
$3 + 3 = 5$	<input type="checkbox"/>
$4 + 4 = 4$	<input checked="" type="checkbox"/>
$5 + 5 = 9$	<input type="checkbox"/>

In this instance, the answer “ $2 + 2 = 4$ ” was filled in and then selected again.

If you still have any questions now, please ask your teacher.

**Good Luck!**

# Task 1

## Sets of Numbers

The set  $M = \{x \in \mathbb{Q} \mid 2 < x < 5\}$  is a subset of the rational numbers.

Task:

Put a cross next to each of the two correct statements.

4.99 is the largest number that is an element of set $M$ .	<input type="checkbox"/>
There are infinitely many numbers in the set $M$ that are smaller than 2.1.	<input type="checkbox"/>
Every real number that is greater than 2 and smaller than 5 is contained within the set $M$ .	<input type="checkbox"/>
All elements of the set $M$ can be written in the form $\frac{a}{b}$ , where $a$ and $b$ are integers and $b \neq 0$ .	<input type="checkbox"/>
The set $M$ does not contain any numbers from the set of complex numbers.	<input type="checkbox"/>

## Task 2

### Equivalency

When rearranging an equation, not every rearrangement gives rise to an equivalent equation.

#### Task:

With specific reference to the example given below, explain why the rearrangement shown does not give rise to an equivalent equation. The equation is defined on the set of all real numbers.

$$\begin{array}{l} x^2 - 5x = 0 \quad | \div x \\ x - 5 = 0 \end{array}$$

## Task 3

### Fuel Costs

The average car requires  $y$  litres of fuel for each 100 km driven. The cost of the fuel is  $a$  euros per litre.

Task:

Write down an expression that gives the average fuel cost,  $K$  (in euros), for a journey of  $x$  km.

$K =$  \_\_\_\_\_

## Task 4

### Quadratic Equation

Let  $x^2 + p \cdot x - 12 = 0$  be a quadratic equation.

**Task:**

Determine the value of  $p$  for which the equation has the solution set  $L = \{-2, 6\}$ .



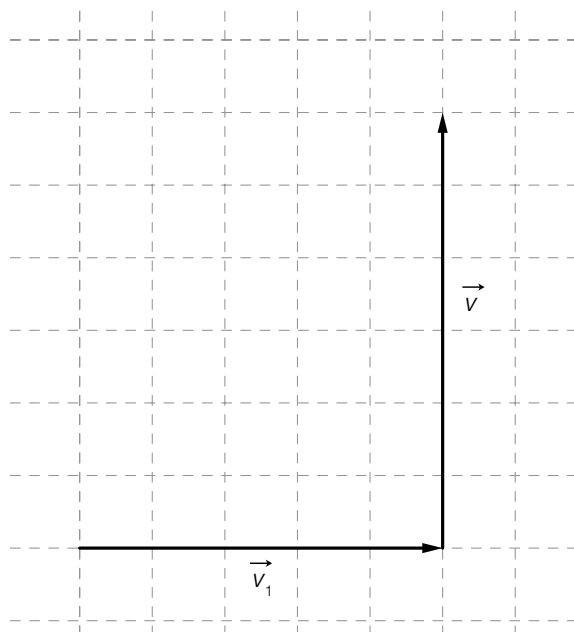
## Task 5

### Addition of Vectors

The diagram below shows two vectors  $\vec{v}_1$  and  $\vec{v}$ .

Task:

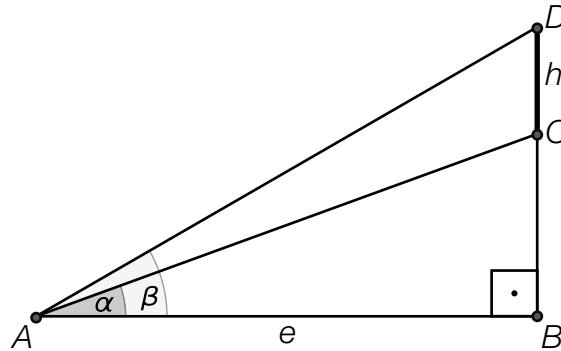
On the diagram below, draw a third vector,  $\vec{v}_2$ , such that  $\vec{v}_1 + \vec{v}_2 = \vec{v}$ .



## Task 6

### Measuring an Inaccessible Steep Wall

A section of a steep wall,  $CD$ , is inaccessible. It has a height of  $h = \overline{CD}$ . To be able to determine the height of the section of the wall, the angles  $\alpha = 24^\circ$  and  $\beta = 38^\circ$  and the distance  $e = 6$  metres were measured. The situation is represented in the diagram below. The diagram is not to scale.



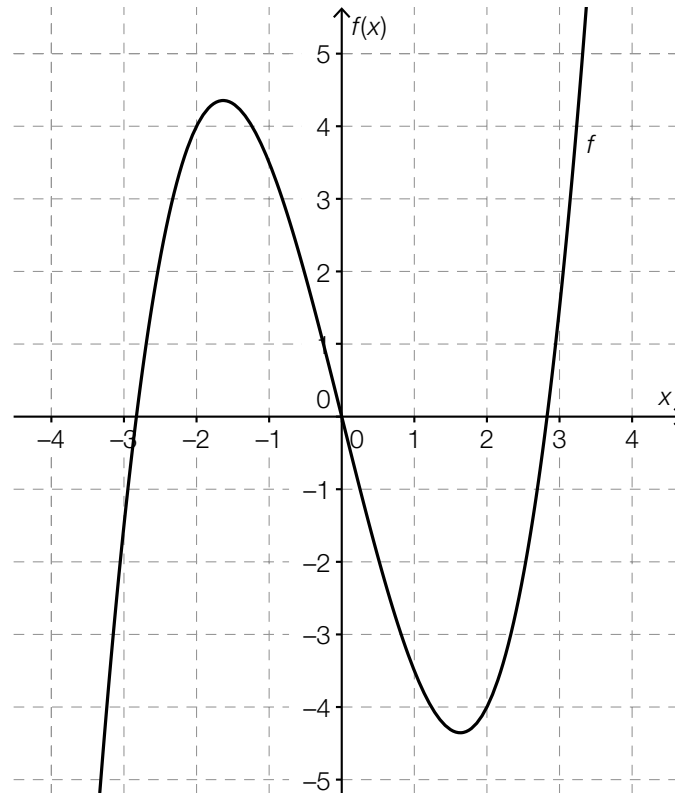
Task:

Calculate the height,  $h$ , of the inaccessible section of the steep wall in metres.

# Task 7

## Recognising the Properties of a Function

Below you will find a diagram of a third degree polynomial function,  $f$ .



Task:

Put a cross next to the true statement(s) about the graph of  $f$ .

The function $f$ is monotonically increasing in the interval $(2, 3)$ .	<input type="checkbox"/>
The function $f$ has a local maximum in the interval $(1, 2)$ .	<input type="checkbox"/>
The concavity of the function $f$ changes in the interval $(-1, 1)$ .	<input type="checkbox"/>
The graph of $f$ is symmetrical about the vertical axis.	<input type="checkbox"/>
The monotonicity of the function changes in the interval $(-3, 0)$ .	<input type="checkbox"/>

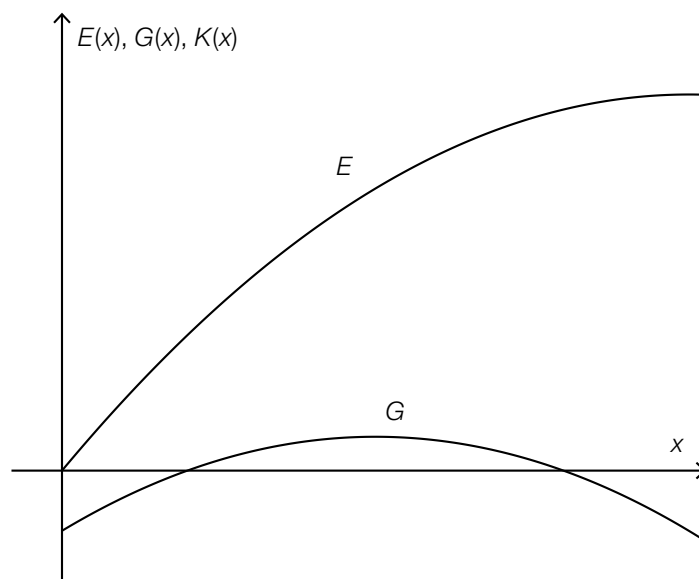
# Task 8

## Cost, Revenue and Profit

The function  $E$  describes the revenue (in €) that is generated from the sale of  $x$  units of a product. The function  $G$  describes the corresponding profit in €. This is defined as the difference “revenue – costs”.

### Task:

Complete the diagram below by drawing the graph of the corresponding cost function,  $K$ . Assume that  $K$  is linear. (The model in this task assumes that all manufactured products are sold.)



## Task 9

### Heating Water

During an experiment, a specified amount of water is warmed in a microwave for a time  $t$ . The energy level of the microwave remains constant during the experiment. The initial temperature of the water and its temperature after 30 seconds are measured.

Time (in seconds)	$t = 0$	$t = 30$
Temperature (in °C)	35.6	41.3

Task:

Complete the equation of the corresponding linear function that gives the temperature  $T(t)$  at time  $t$ .

$$T(t) = \underline{\hspace{2cm}} \cdot t + 35.6$$

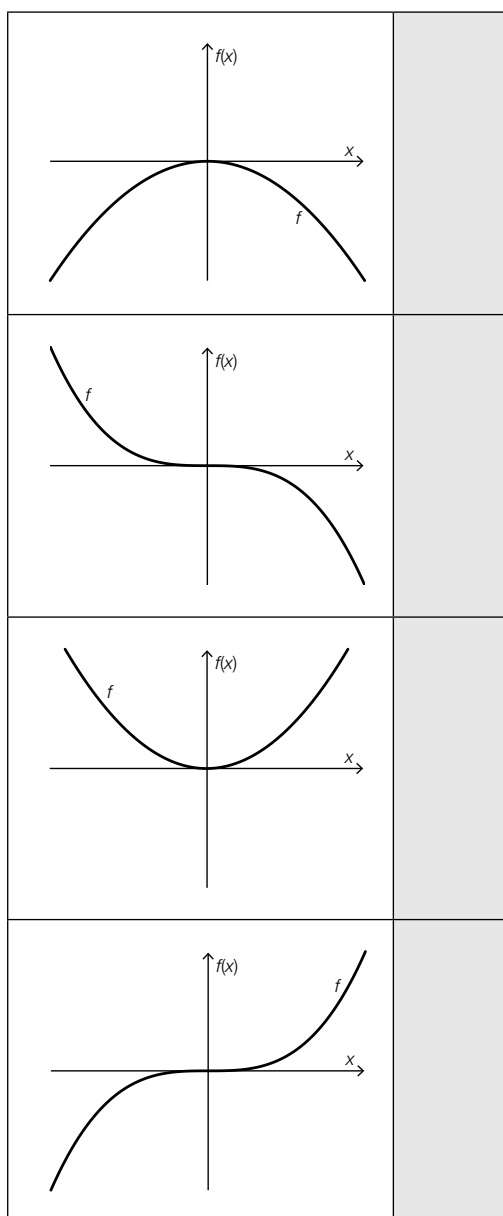
# Task 10

## Power Functions

The graphs of four different power functions,  $f$ , where  $f(x) = a \cdot x^z$ , are shown below along with six conditions for the parameter  $a$  and the exponent  $z$ . The parameter  $a$  is a real number, and the exponent  $z$  is a natural number.

**Task:**

Match each of the four graphs with the corresponding conditions for the parameter  $a$  and the exponent  $z$  of the equation of the function (from A to F).



A	$a > 0, z = 1$
B	$a > 0, z = 2$
C	$a > 0, z = 3$
D	$a < 0, z = 1$
E	$a < 0, z = 2$
F	$a < 0, z = 3$

# Task 11

## Expansion of an Oil Slick

The area of an oil slick is currently  $1.5 \text{ km}^2$ , and it is expanding at a rate of 5 % per day.

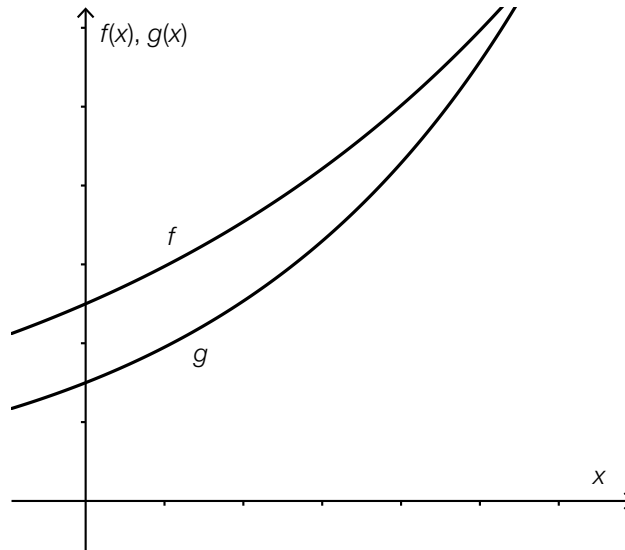
**Task:**

Determine the number of days after which the oil slick is larger than  $2 \text{ km}^2$  for the first time.

# Task 12

## Parameters of Exponential Functions

The diagram below shows the graphs of two exponential functions,  $f$  and  $g$ , with equations  $f(x) = c \cdot a^x$  and  $g(x) = d \cdot b^x$ , where  $a, b, c, d \in \mathbb{R}^+$ .



Task:

Complete the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

For the parameters  $a, b, c, d$  of the two exponential functions given above, the relationships ① and ② hold.

①	
$c < d$	<input type="checkbox"/>
$c = d$	<input type="checkbox"/>
$c > d$	<input type="checkbox"/>

②	
$a < b$	<input type="checkbox"/>
$a = b$	<input type="checkbox"/>
$a > b$	<input type="checkbox"/>



## Task 13

### Interpreting the Average Rate of Change

Let  $f$  be a third degree polynomial function. The average rate of change of  $f$  in the interval  $[x_1, x_2]$  is 5.

Task:

Which of the following statements are definitely true for the function  $f$ ?

Put a cross next to each of the two correct statements.

In the interval $[x_1, x_2]$ there is at least one point $x$ for which $f(x) = 5$ .	<input type="checkbox"/>
$f(x_2) > f(x_1)$	<input type="checkbox"/>
The function $f$ is monotonically increasing in the interval $[x_1, x_2]$ .	<input type="checkbox"/>
$f'(x) = 5$ for all $x \in [x_1, x_2]$	<input type="checkbox"/>
$f(x_2) - f(x_1) = 5 \cdot (x_2 - x_1)$	<input type="checkbox"/>

# Task 14

## Savings Account

Mrs. Fröhlich has a savings account in which she deposits the same amount of money in euros on the first working day of each year. On this day her bank also credits the account with the interest earned over the previous year. Afterwards, the new statement showing the balance of the account is printed.

The relationship between the balance from the previous year,  $K_{i-1}$ , and the balance of the current year,  $K_i$ , can be described by the following formula:

$$K_i = 1.03 \cdot K_{i-1} + 5000$$

### Task:

Based on the information given above, which of the following statements are correct?  
Put a cross next to each of the two correct statements.

Mrs. Fröhlich pays € 5,000 a year into her savings account.	<input type="checkbox"/>
The balance of the savings account increases by € 5,000 per year.	<input type="checkbox"/>
The relative annual growth of the balance of the savings account is larger than 3 %.	<input type="checkbox"/>
The difference between the balances of two consecutive years is always the same.	<input type="checkbox"/>
The balance of the savings account increases at a linear rate.	<input type="checkbox"/>

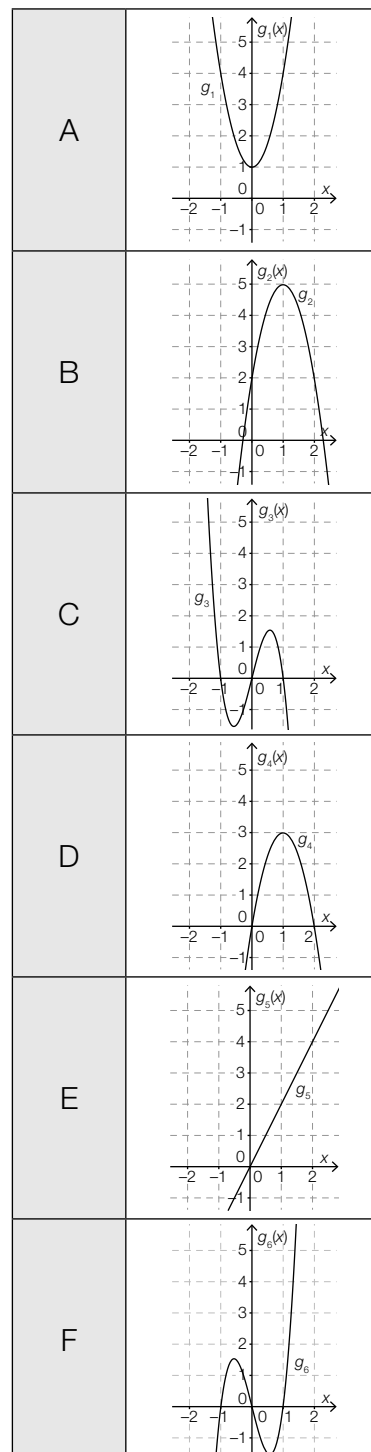
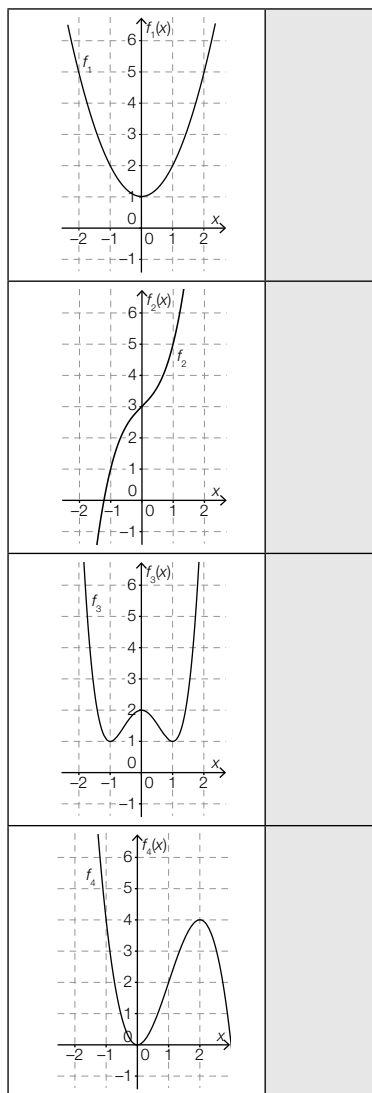
# Task 15

## Functions and Derivatives

The graphs of four polynomial functions ( $f_1, f_2, f_3, f_4$ ) are shown on the left and the graphs of six more functions ( $g_1, g_2, g_3, g_4, g_5, g_6$ ) are shown on the right.

Task:

Match each of the polynomial functions,  $f_1$  to  $f_4$ , to its corresponding derivative  $g_1$  to  $g_6$  (from A to F).



## Task 16

### Local Minimum

Let  $p$  be a polynomial function with  $p(x) = x^3 - 3 \cdot x + 2$ . The first derivative  $p'$ , where  $p'(x) = 3 \cdot x^2 - 3$ , has the value zero at the point where  $x = 1$ .

Task:

Demonstrate by means of calculation that  $p$  has a local minimum at this point (i. e. that the graph of  $p$  has a local minimum there).

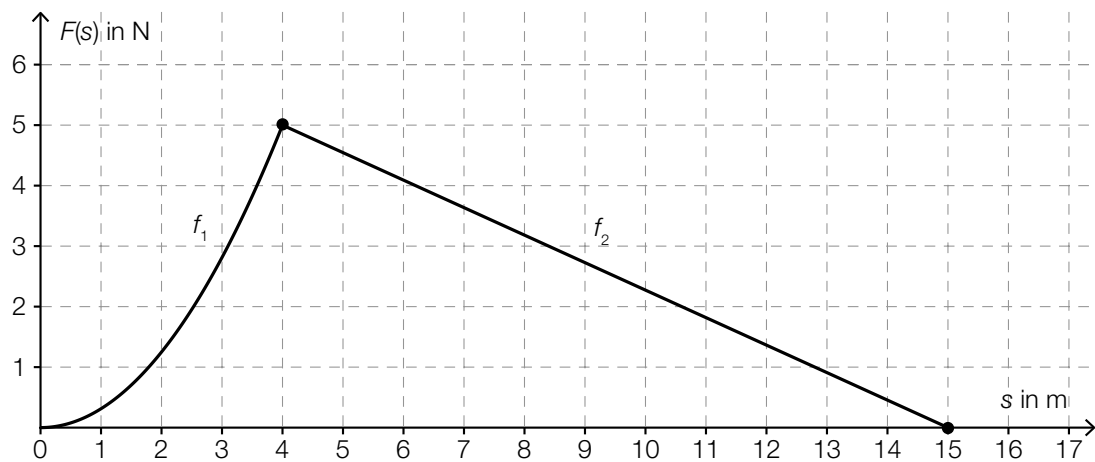
# Task 17

## Work Done in Moving a Body

A body is moved in a straight line under the influence of a force. The required component of the force in the direction of motion as a function of the distance covered is shown in the diagram below. The distance,  $s$ , is measured in metres, and the force,  $F(s)$ , is measured in Newtons (N).

In the first section,  $F(s)$  is described by  $f_1$ , where  $f_1(s) = \frac{5}{16} \cdot s^2$ . In the second section ( $f_2$ ), the value decreases in a linear way until it reaches 0.

The points on the graph shown in bold have integer coordinates.



**Task:**

Determine the work done,  $W$ , in joules (J), that this force exerts on the body if it is moved from  $s = 0$  m to  $s = 15$  m.

$W =$  \_\_\_\_\_ J

## Task 18

### Integral

Let  $f$  be a power function, where  $f(x) = x^3$ .

Task:

Determine the values of the boundary conditions  $b$  and  $c$  ( $b \neq c$ ) such that  $\int_b^c f(x) dx = 0$  holds.

# Task 19

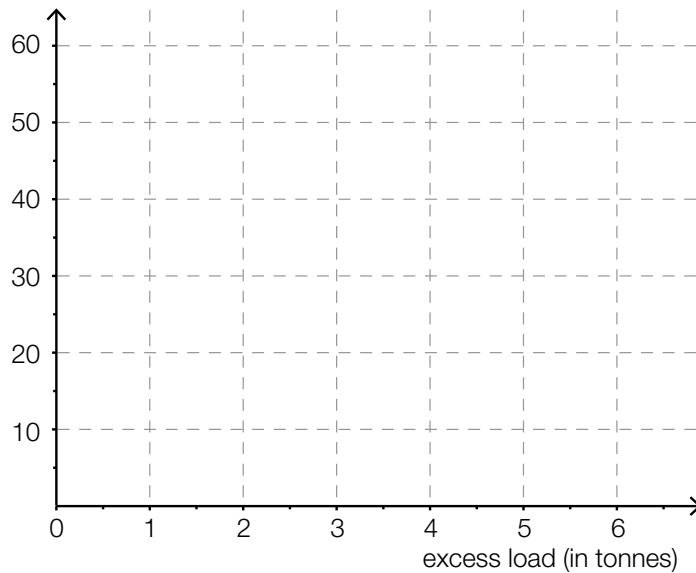
## Truck Loads

During a traffic inspection, the loads carried by trucks were checked. 140 of the trucks checked were overloaded. Details of the check are summarised in the table below.

Excess load ( $E$ ) in tonnes	$E < 1\text{t}$	$1\text{t} \leq E < 3\text{t}$	$3\text{t} \leq E < 6\text{t}$
Number of trucks	30	50	60

Task:

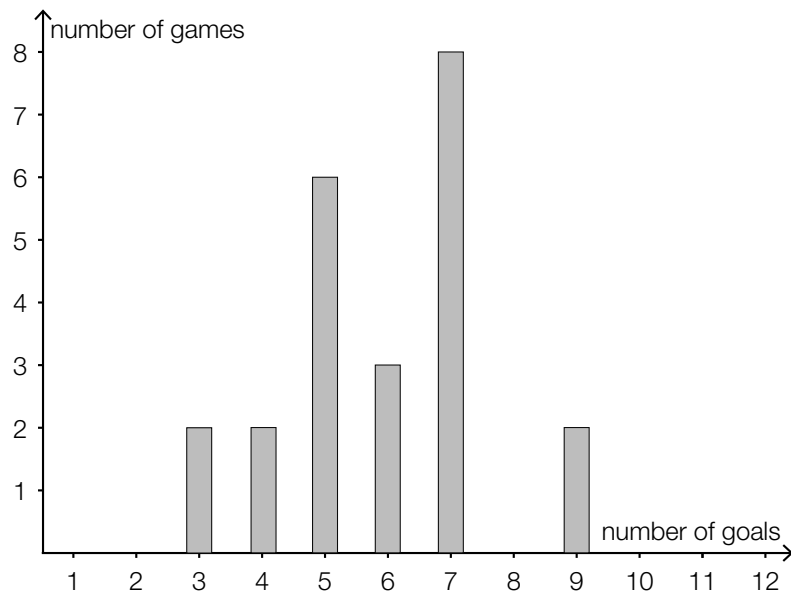
Represent the data in the table above as a histogram. The absolute frequencies should be represented by the areas of rectangles.



## Task 20

### Ice Hockey Goals

The results of all games played in the Austrian Ice Hockey League are evaluated statistically. In the 2012/13 season, the number of games in which a particular number of goals were scored over a determined period of time was counted. The bar chart below shows the results of the evaluation.



#### Task:

Using the bar chart, determine the median value of this data set.



## Task 21

### Customs Check

A group of ten people are crossing a border between two countries. Two people have contraband items with them. While crossing the border three members of the group are randomly selected and checked by customs officials.

#### Task:

Determine the probability that both of the two smugglers happen to be in the selected group of the three people to be checked.

## Task 22

### Probability Distribution

The range of values of a random variable  $X$  is comprised of the values  $x_1, x_2, x_3$ .

The probability  $P(X = x_1) = 0.4$  is known. Also, it is known that  $x_3$  is twice as likely to occur as  $x_2$ .

Task:

Determine  $P(X = x_2)$  and  $P(X = x_3)$ .

$$P(X = x_2) = \underline{\hspace{15em}}$$

$$P(X = x_3) = \underline{\hspace{15em}}$$

# Task 23

## Coloured Balls

There is a box on a table with three red and twelve black balls inside. Three balls are randomly selected and removed from the box one after the other. After each ball has been selected, it is placed back into the box.

### Task:

Consider the following expression:

$$3 \cdot 0.8^2 \cdot 0.2$$

Put a cross next to the event whose probability can be calculated with this expression.

At most one black ball is selected.	<input type="checkbox"/>
Exactly two black balls are selected.	<input type="checkbox"/>
Two red balls and one black ball are selected.	<input type="checkbox"/>
Only red balls are selected.	<input type="checkbox"/>
At least one red ball is selected.	<input type="checkbox"/>
No red balls are selected.	<input type="checkbox"/>

# Task 24

## Comparison of Two Confidence Intervals

Based on a random sample of size  $n_1$ , an opinion research institute gives the confidence interval of the current share of the vote of a political party as  $[0.23, 0.29]$ . The corresponding confidence level is  $\gamma_1$ .

A different institute asks  $n_2$  randomly chosen eligible voters and gives the according confidence interval  $[0.24, 0.28]$  at a confidence level of  $\gamma_2$ . Both institutes use the same method of calculation.

### Task:

Complete the following sentence by putting a cross next to one of the given possibilities for each gap, so that the sentence becomes a correct statement.

Given that  $n_1 = n_2$ , it can be concluded that \_\_\_\_\_ ① \_\_\_\_\_;  
given that  $\gamma_1 = \gamma_2$ , it can be concluded that \_\_\_\_\_ ② \_\_\_\_\_.

①	
$\gamma_1 < \gamma_2$	<input type="checkbox"/>
$\gamma_1 = \gamma_2$	<input type="checkbox"/>
$\gamma_1 > \gamma_2$	<input type="checkbox"/>

②	
$n_1 < n_2$	<input type="checkbox"/>
$n_1 = n_2$	<input type="checkbox"/>
$n_1 > n_2$	<input type="checkbox"/>







