| Name: | |
|---|---|
| Class: | |

Standardised Competence-Oriented
Written School-Leaving Examination

AHS

10th May 2016

# Mathematics

## Part 2 Tasks

BM | BF Bundesministerium für Bildung und Frauen

Bundesinstitut bifie

# Advice for Completing the Tasks

Dear candidate,

The following booklet for Part 2 contains four tasks, each of which contains between two and four sub-tasks. All sub-tasks can be completed independently of one another. You have *150 minutes* available in which to work on these tasks.

Please use a blue or black pen that cannot be rubbed out. You may use a pencil for tasks that require you to draw a graph, vectors or a geometric construction.

When completing these tasks please use this booklet and the paper provided. Write your name on each piece of paper you use as well as on the first page of this task booklet in the space provided. Please show clearly which sub-task each answer relates to.

In the assessment of your work, everything that is not crossed out will be considered. Your solutions must be clearly marked. If a solution is not clearly marked or if more than one solution is given, the task will be considered to be unsolved. Draw a line through any notes you make.

You may use a pre-approved formula book as well as your usual electronic device(s).

Please hand in both the task booklet and the separate sheets you have used at the end of the examination.

## Assessment

Every task in Part 1 will be awarded either 0 points or 1 point. Every sub-task in Part 2 will be awarded 0, 1 or 2 points. The tasks marked with an $\boxed{\text{A}}$ will be awarded either 0 points or 1 point.

– If at least 16 of the 24 tasks in Part 1 are solved correctly, you will pass the examination.
– If fewer than 16 of the 24 tasks in Part 1 are solved correctly, then the tasks marked with an $\boxed{\text{A}}$ from Part 2 may compensate for the shortfall (as part of the “range of essential skills” outlined by the LVBO).
  If, including the tasks marked with an $\boxed{\text{A}}$ from Part 2, at least 16 tasks are solved correctly, you will pass the examination.
  If, including the tasks marked with an $\boxed{\text{A}}$ from Part 2, fewer than 16 tasks are solved correctly, you will not be awarded enough points to pass the examination.
– If at least 16 tasks are solved correctly (including the compensation tasks marked with an $\boxed{\text{A}}$ from Part 2), a grade will be awarded as follows:

| | |
|---|---|
| Pass | 16–23 points |
| Satisfactory | 24–32 points |
| Good | 33–40 points |
| Very Good | 41–48 points |

## Explanation of the Task Types

Some tasks require a *free answer*. For these tasks, you should write your answer directly underneath each task in the task booklet or on the paper provided. Other task types used in the examination are as follows:

*Matching tasks:* For this task type you will be given a number of statements, tables or diagrams, which will appear alongside a selection of possible answers. To correctly answer these tasks, you will need to match each statement, table or diagram to its corresponding answer. You should write the letter of the correct answer next to the statement, table or diagram in the space provided.

Example:
You are given two equations.

Task:
Match the two equations to their corresponding description (from A to D).

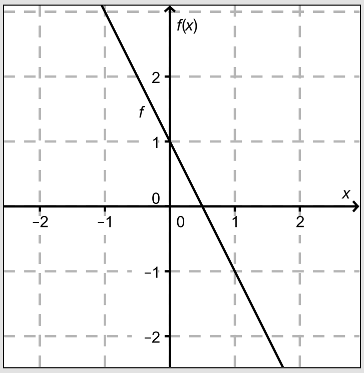
| | |
|---|---|
| $1 + 1 = 2$ | A |
| $2 \cdot 2 = 4$ | C |

| | |
|---|---|
| A | Addition |
| B | Division |
| C | Multiplication |
| D | Subtraction |

*Construction tasks:* This task type requires you to draw points, lines and/or curves in the task booklet.

Example:
Below you will see a linear function $f$ where $f(x) = k \cdot x + d$.

Task:
On the axes provided below, draw the graph of a linear function for which $k = -2$ and $d > 0$.




*Multiple-choice tasks of the form "1 out of 6":* This task type consists of a question and six possible answers. Only one answer should be selected. You should put a cross next to the only correct answer in the space provided.

Example:
Which equation is correct?

Task:
Put a cross next to the correct equation.

| | |
|---|---|
| 1 + 1 = 1 | ☐ |
| 2 + 2 = 2 | ☐ |
| 3 + 3 = 3 | ☐ |
| 4 + 4 = 8 | ☒ |
| 5 + 5 = 5 | ☐ |
| 6 + 6 = 6 | ☐ |

*Multiple-choice tasks of the form "2 out of 5":* This task type consists of a question and five possible answers, of which two answers should be selected. You should put a cross next to each of the two correct answers in the space provided.

Example:
Which equations are correct?

Task:
Put a cross next to each of the two correct equations.

| | |
|---|---|
| 1 + 1 = 1 | ☐ |
| 2 + 2 = 4 | ☒ |
| 3 + 3 = 3 | ☐ |
| 4 + 4 = 8 | ☒ |
| 5 + 5 = 5 | ☐ |

*Multiple-choice tasks of the form "x out of 5":* This task type consists of a question and five possible answers, of which one, two, three, four *or* five answers may be selected. The task will require you to: "Put a cross next to each correct statement/equation …". You should put a cross next to each correct answer in the space provided.

Example:
Which of the equations given are correct?

Task:
Put a cross next to each correct equation.

| | |
|---|---|
| 1 + 1 = 2 | ☒ |
| 2 + 2 = 4 | ☒ |
| 3 + 3 = 6 | ☒ |
| 4 + 4 = 4 | ☐ |
| 5 + 5 = 10 | ☒ |

*Gap-fill:* This task type consists of a sentence with two gaps, i.e. two sections of the sentence are missing and must be completed. For each gap you will be given the choice of three possible answers. You should put a cross next to **each of the two answers** that are necessary to complete the sentence correctly.

Example:
Below you will see 3 equations.

Task:
Complete the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

The operation in equation ___①___ is known as summation or ___②___ .

| ① | |
|---|---|
| 1 – 1 = 0 | ☐ |
| 1 + 1 = 2 | ☒ |
| 1 · 1 = 1 | ☐ |

| ② | |
|---|---|
| Multiplication | ☐ |
| Subtraction | ☐ |
| Addition | ☒ |

*Changing an answer for a task that requires a cross:*

1. Fill in the box that contains the cross for your original answer.
2. Put a cross in the box next to your new answer.

| | |
|---|---|
| 1 + 1 = 3 | ☐ |
| 2 + 2 = 4 | ☒ |
| 3 + 3 = 5 | ☐ |
| 4 + 4 = 4 | ☐ |
| 5 + 5 = 9 | ■ |

In this instance, the answer “5 + 5 = 9” was originally chosen. The answer was later changed to be “2 + 2 = 4”.

*Selecting an answer that has been filled in:*

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

| | |
|---|---|
| 1 + 1 = 3 | ☐ |
| 2 + 2 = 4 | ■ |
| 3 + 3 = 5 | ☐ |
| 4 + 4 = 4 | ■ |
| 5 + 5 = 9 | ☐ |

In this instance, the answer “2 + 2 = 4” was filled in and then selected again.

If you still have any questions, please ask your teacher.

**Good Luck!**

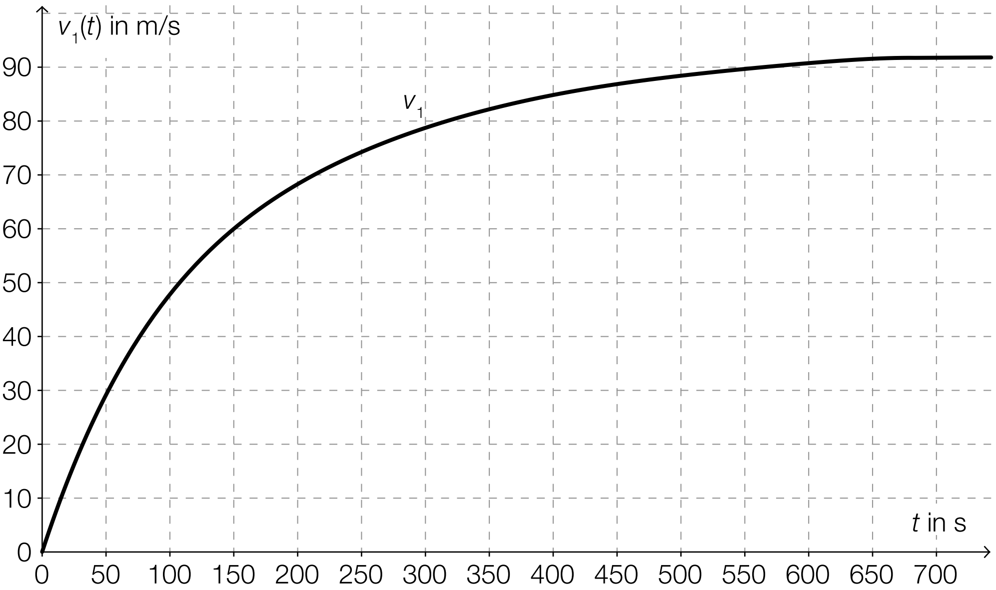
# Task 1

## Intercity-Express (ICE)

The German rail service has a fleet of ICE trains, which is a collection of high speed trains. With a maximum speed of up to 330 km/h (approximately 91.7 m/s) these are Germany's fastest trains. They are approximately 200 metres long and weigh around 400 tonnes. Each train is made up of eight carriages. Trial journeys are carried out for acceleration and braking tests. The results of these tests can be represented graphically.

Task:

a) The data collected during an acceleration test from rest to the maximum speed (where the velocity, $v_1(t)$, is measured in metres per second and the time, $t$, is measured in seconds) is approximately represented by the velocity-time graph below.




Determine the average rate of change of the velocity in the time interval [0 s, 700 s] and write down a time at which the instantaneous rate of change of the velocity is higher than the average rate of change.

$\boxed{A}$ Interpret the definite integral $\int_0^{700} v_1(t)\,\mathrm{d}t$ in the given context.

b) Data was collected during a braking test. This data can be described by the distance covered, $s(t)$, as a function of time, $t$, as $s(t) = 70 \cdot t - 0.25 \cdot t^2$, where $t$ is in seconds and $s(t)$ is in metres from the start of braking.

Determine the time-velocity function, $v_2$, for the braking test in the form $v_2(t) = m \cdot t + c$ and explain the meaning of the parameters $m$ and $c$ in terms of the context described above.

Determine the distance the ICE covered from the start of braking until the standstill.

# Task 2

## ZAMG Weather Balloon

A weather balloon is a balloon filled with helium or hydrogen that is used in meteorology to transport radiosondes (measurement instruments). The Centre for Meteorology and Geodynamics (ZAMG) releases a weather balloon from the *Hohe Warte* weather station twice a day on 365 days of the year. During the ascent, measurements of temperature, humidity, air pressure, wind direction, and wind speed are taken continually.

The values recorded on one ascent of a weather balloon for air pressure and the temperature at the height, $h$, above sea level are shown in the table below.

| Height $h$ of the balloon above sea level (in m) | Air pressure $p$ (in hPa) | Temperature (in °C) |
|---|---|---|
| 1000 | 906 | 1.9 |
| 2000 | 800 | –3.3 |
| 3000 | 704 | –8.3 |
| 4000 | 618 | –14.5 |
| 5000 | 544 | –21.9 |
| 6000 | 479 | –30.7 |
| 7000 | 421 | –39.5 |
| 8000 | 370 | –48.3 |

Task:

a) $\boxed{\text{A}}$ Determine the relative rate (percentage) of change of the air pressure during the ascent of the weather balloon from 1000 m to 2000 m. Give your answer as a percentage.

The relationship between the air pressure and the height can be approximated by an exponential function. Explain how this can be justified based on the table above.

b) In the interval [5000 m, 8000 m] the temperature is dependent on the height. This relationship can be described by a linear function, $T$.

Explain how this can be justified based on the values given in the table above.

For this function $T$, where $T(h) = m \cdot h + c$, determine the values of the parameters $m$ and $c$.

c) The volume of the weather balloon is approximately indirectly proportional to the air pressure, $p$. At a height of 1 000 m, the weather balloon has a volume of 3 $m^3$.

Express the relationship between the volume (in $m^3$) and the air pressure (in hPa) in an equation.

$V(p) =$ ________________________________________

Determine the absolute value of the change in the volume of the balloon in the interval [1 000 m, 2 000 m].

# Task 3

## Income Tax

Employees have to pay part of their income to the state in the form of income tax. In the tax model for 2015 there are four tax brackets with the tax rates: 0 %, 36.5 %, 43.2 % and 50 %.

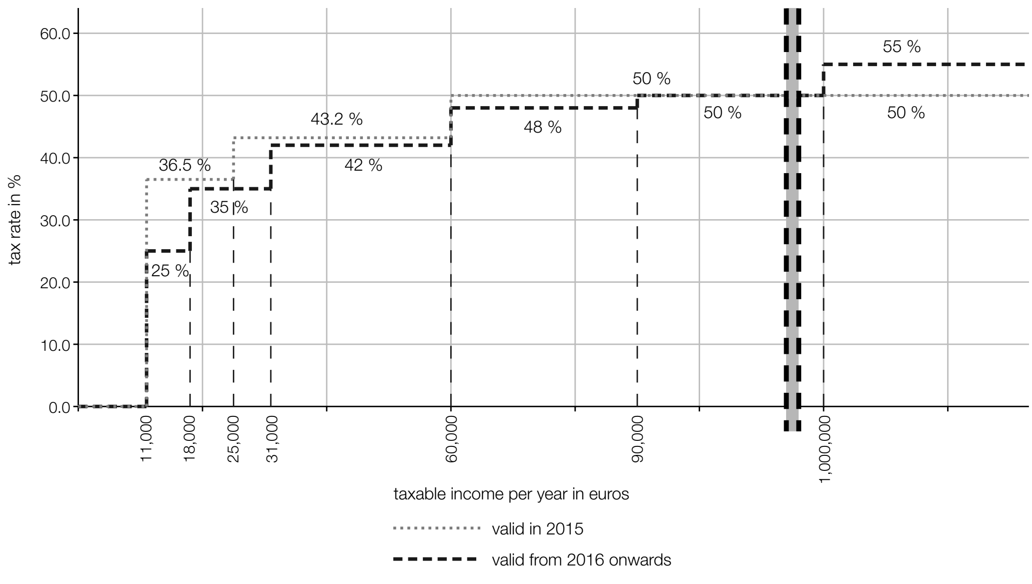
A person’s net income can be calculated as follows:

Net income for the year = taxable income for the year – income tax

An income tax calculation is based on the taxable income for the year. For the year 2015 income tax should be calculated as follows:

- Any income up to an amount of € 11,000 is tax exempt.
- Any income that exceeds € 11,000 up to an amount of € 25,000 is taxed at a rate of 36.5 %. This means that if the income is above € 11,000, then the first € 11,000 earned is tax exempt and the income up to € 25,000 is taxed at a rate of 36.5 %.
- Any income that exceeds € 25,000 up to an amount of € 60,000 is taxed at a rate of 43.2 % (or, more precisely $43\frac{3}{14}$ %).
- Any income that exceeds € 60,000 is taxed at a rate of 50 %.

On July 7th 2015, a tax reform law was passed by the National Assembly. This tax model came into force on January 1st 2016, and has seven tax brackets. The model used in 2015 (with four tax brackets) and the model that is used in 2016 (with seven tax brackets) are represented on the diagram below.




*Data source: http://www.parlament.gv.at/ZUSD/BUDGET/BD_-_Steuerreform_2015_und_2016.pdf, p. 15 [11.11.2015]*

Task:

a) $\boxed{\text{A}}$ Using the tax rates for 2015, calculate the net income for the year for a person whose taxable income was € 20,000.

For the year 2015, write down a formula that gives the net income for the year, $N$, for a person whose taxable income, $E$, is between € 11,000 and € 25,000.

b) The so-called composite tax rate is defined as follows:

$$\text{composite tax rate} = \frac{\text{income tax paid}}{\text{taxable income for the year}}$$

In the year 2015, a person registered a taxable income of € 40,000. Determine this person's composite tax rate for the year 2015.

Using the diagram given, explain what can be calculated with the following expression:
$7000 \cdot 0.115 + 7000 \cdot 0.015 + 6000 \cdot 0.082 + 9000 \cdot 0.012$

c) A person states:

(1) "Despite the change in law, there would be no change in the amount of tax paid on a taxable income of € 100,000."

(2) "The tax rate for a taxable income of between € 11,000 and € 18,000 has changed by 11.5 %."

Are these statements true? Demonstrate mathematically why each statement is true or false.

d) The Ministry of Finance has the following formula on its website for the calculation of the income tax (ESt) for 2015 for the tax bracket from € 25,000 to € 60,000:

$$\text{ESt} = \frac{(\text{taxable income for the year} - 25\,000) \cdot 15\,125}{35\,000} + 5\,110$$

Explain the meaning of the factor $\frac{15\,125}{35\,000}$ and the number 5 110 based on the income tax calculation.

Write down a formula to calculate the income tax ($\text{ESt}_{new}$) for a taxable income between € 31,000 and € 60,000 for the 2016 tax model.

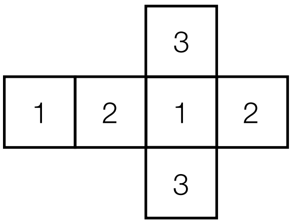
$\text{ESt}_{new}$ = ____________________________________________

# Task 4

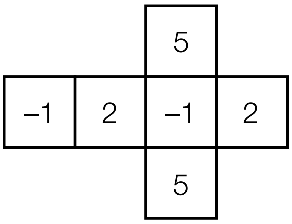
## Dice with Different Numbers

The nets of three fair dice are given below. The dice have various numbers written on their sides in different ways. (A dice is considered to be “fair” if the probability of the dice showing any of its six faces after being thrown is the same.)

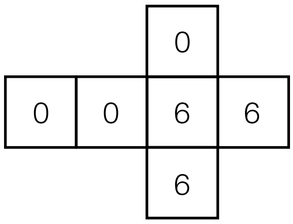
Dice *A*




Dice *B*




Dice *C*




Task:

a) Mr. Fischer throws dice *A* twice. The random variable *X* represents the sum of the two numbers that come up. The random variable *X* can take the values 2, 3, 4, 5 and 6. Mrs. Fischer throws dice *A* and dice *B*. The random variable *Y* represents the sum of the two numbers that come up.

$\boxed{\text{A}}$ Write down all the possible values that the random variable *Y* can take.

possible values of *Y*: ____________________________________________ .

There are values of the random variables that are more likely to arise for Mr. Fisher than for Mrs. Fisher. Determine the value for which the difference between the two probabilities is the greatest and find this difference.

b) During a game, dice *B* is thrown three times. To play the game, a player has to pay € 2. The prize a player receives depends on the sum of the three numbers that come up. The prize money a player receives is outlined in the table below.

| Sum of the three numbers | Prize |
|---|---|
| positive | 0 |
| zero | 2 |
| negative | ? |

A person plays this game five times. Determine the probability that the sum of the three numbers that come up is exactly zero in exactly two out of the five games.

Determine the maximum amount the vendor can pay out for a negative sum so that he/she would not make a loss in the long run.

c) Peter throws dice *C* 100 times. The random variable *Z* represents the number of sixes thrown.

Determine the expectation value and the standard deviation of *Z*.

Determine the probability that the sum of the numbers that come up is greater than 350.