Competence Check Mathematics (AHS)

January 2016

Part 1 Tasks

Bundesinstitut bifie
Bildungsforschung, Innovation & Entwicklung des österreichischen Schulwesens

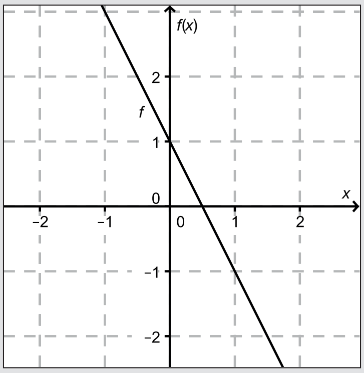
Advice for Completing the Tasks

Some tasks require a *free answer*. For these tasks, you should write your answer directly underneath each task in the task booklet. Other task types used in the examination are as follows:

Construction tasks: This task type requires you to draw points, lines and/or curves in the task booklet.

Example:
Below you will see a linear function f where $f(x) = k \cdot x + d$.

Task:
On the axes provided below, draw the graph of a linear function for which $k = -2$ and $d > 0$.



Matching tasks: For this task type you will be given a number of statements, tables or diagrams, which will appear alongside a selection of possible answers. To correctly answer these tasks, you will need to match each statement, table or diagram to its corresponding answer. You should write **the letter of the correct answer** next to the statement, table or diagram in the space provided.

Example:
You are given two equations.

Task:
Match the two equations to their corresponding description (from A to D).

$1 + 1 = 2$	A
$2 \cdot 2 = 4$	C

A	Addition
B	Division
C	Multiplication
D	Subtraction

Gap-fill: This task type consists of a sentence with two gaps, i.e. two sections of the sentence are missing and must be completed. For each gap you will be given the choice of three possible answers. You should put a cross next to **each of the two answers** that are necessary to complete the sentence correctly.

Example:
Below you will see 3 equations.

Task:
Complete the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

The operation in equation ______①______ is known as summation or ______②______ .

①	
$1 - 1 = 0$	☐
$1 + 1 = 2$	☒
$1 \cdot 1 = 1$	☐

②	
Multiplication	☐
Subtraction	☐
Addition	☒

Multiple-choice tasks of the form "1 out of 6": This task type consists of a question and six possible answers. Only **one answer** should be selected. You should put a cross next to the only correct answer in the space provided.

Example:
Which equation is correct?

Task:
Put a cross next to the correct equation.

1 + 1 = 1	☐
2 + 2 = 2	☐
3 + 3 = 3	☐
4 + 4 = 8	☒
5 + 5 = 5	☐
6 + 6 = 6	☐

Multiple-choice tasks of the form "2 out of 5": This task type consists of a question and five possible answers, of which **two answers** should be selected. You should put a cross next to each of the two correct answers in the space provided.

Example:
Which equations are correct?

Task:
Put a cross next to each of the two correct equations.

1 + 1 = 1	☐
2 + 2 = 4	☒
3 + 3 = 3	☐
4 + 4 = 8	☒
5 + 5 = 5	☐

Multiple-choice tasks of the form "x out of 5": This task type consists of a question and five possible answers, of which **one, two, three, four** *or* **five answers** may be selected. The task will require you to: "Put a cross next to each correct statement/equation …". You should put a cross next to each correct answer in the space provided.

Example:
Which of the equations given are correct?

Task:
Put a cross next to each correct equation.

1 + 1 = 2	☒
2 + 2 = 4	☒
3 + 3 = 6	☒
4 + 4 = 4	☐
5 + 5 = 10	☒

Changing an answer for a task that requires a cross:

1. Fill in the box that contains the cross for your original answer.
2. Put a cross in the box next to your new answer.

1 + 1 = 3	☐
2 + 2 = 4	☒
3 + 3 = 5	☐
4 + 4 = 4	☐
5 + 5 = 9	■

In this instance, the answer "5 + 5 = 9" was originally chosen. The answer was later changed to be "2 + 2 = 4".

Selecting an answer that has been filled in:

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

1 + 1 = 3	☐
2 + 2 = 4	■
3 + 3 = 5	☐
4 + 4 = 4	■
5 + 5 = 9	☐

In this instance, the answer "2 + 2 = 4" was filled in and then selected again.

Good Luck!

Task 1

Statements about Numbers

Below you will see some statements about numbers.

Task:

Which of the following statements are true? Put a cross next to each of the two true statements.

Every real number is also an irrational number.	☐
Every real number is also a complex number.	☐
Every rational number is also an integer.	☐
Every integer is also a natural number.	☐
Every natural number is also a real number.	☐

Task 2

Quadratic Equation

The following quadratic equation with the unknown x has a domain of $\mathbb{R}$:

$4x^2 - d = 2$, where $d \in \mathbb{R}$

Task:

Determine the value of $d \in \mathbb{R}$ such that the equation has exactly one solution.

$d =$ ____

Task 3

Simultaneous Equations

Below you will see two linear equations involving the variables $x, y \in \mathbb{R}$.

$2x + 3y = 7$
$3x + by = c$, where $b, c \in \mathbb{R}$

Task:

Determine the values of the parameters b and c such that this pair of simultaneous linear equations has infinitely many solutions.

b = ____________________

c = ____________________

Task 4

Perpendicular Vectors

$\vec{a}$ is a vector with components $\vec{a} = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$.

Task:

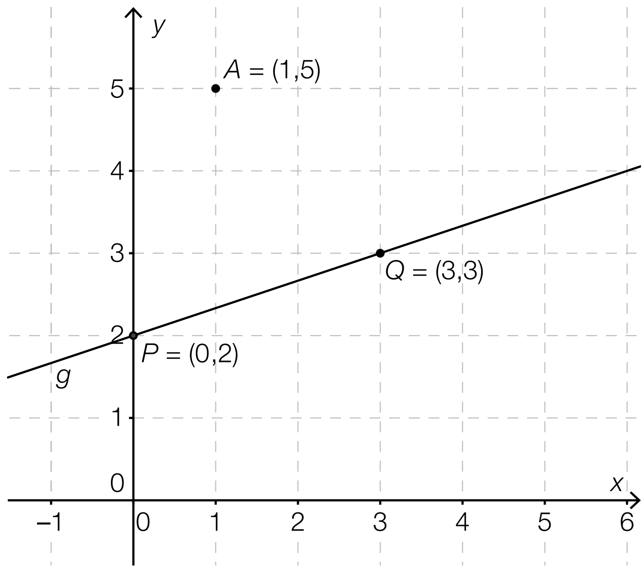
Determine the coordinate z_b of the vector $\vec{b} = \begin{pmatrix} 4 \\ 2 \\ z_b \end{pmatrix}$ such that $\vec{a}$ and $\vec{b}$ are perpendicular.

z_b = ____

Task 5

Equation of a Line

In the following diagram a line, g, which goes through points P and Q, as well as a further point, A, are shown.



Task:

Determine the equation of the line h that goes through the point A and is perpendicular to the line g.

Task 6

Salzburg Funicular Railway

The funicular railway that goes up to the castle in Salzburg has a constant gradient. This railway is the oldest of its kind that is still operational in Austria. The funicular railway travels a distance of 198.5 m over which it scales a vertical height of 96.6 m.

*Image source: https://de.wikipedia.org/wiki/Festungsbahn_Salzburg#/media/File:Festungsbahn_salzburg_20100720.jpg [27.05.2015]
(author: Herbert Ortner, licence: CC BY 3.0)*

Task:

Determine the angle α that the tracks of the railway make with the horizontal.

Task 7

Asymptotes

Below you will see five functions.

Task:

Which of these functions has / have a horizontal asymptote?
Put a cross next to each correct function.

$f_1(x) = \frac{2}{x}$	☐
$f_2(x) = 2^x$	☐
$f_3(x) = \frac{x}{2}$	☐
$f_4(x) = \left(\frac{1}{2}\right)^x$	☐
$f_5(x) = x^{\frac{1}{2}}$	☐

Task 8

Equation of a Function

The graph of a function, f, is a line that goes through the points $P = (2,8)$ and $Q = (4,4)$.

Task:

Determine the equation of the function f.

$f(x) =$ ______________________________

Task 9

Heating System

The number of days that the oil in a tank of a heating system lasts for is indirectly proportional to the average daily oil usage, x (in litres).

Task:

There are 1,500 litres of heating oil in a tank. Write down an expression for the number of days the oil will last, $d(x)$, in terms of the average daily usage, x.

$d(x)$ = ____

Task 10

Properties of a Third Degree Polynomial Function

A third degree polynomial function has the general form $f(x) = ax^3 + bx^2 + cx + d$, where $a, b, c, d \in \mathbb{R}$ and $a \neq 0$.

Task:

Which of the following properties are true for third degree polynomial functions?
Put a cross next to each of the two correct statements.

There exist third degree polynomial functions that have no local maxima or minima.	☐
There exist third degree polynomial functions that have no roots.	☐
There exist third degree polynomial functions that have more than one point of inflection.	☐
There exist third degree polynomial functions that have no points of inflection.	☐
There exist third degree polynomial functions that have exactly two distinct real roots.	☐

Task 11

Properties of an Exponential Function

Let f be a function with equation $f(x) = 50 \cdot 1.97^x$.

Task:

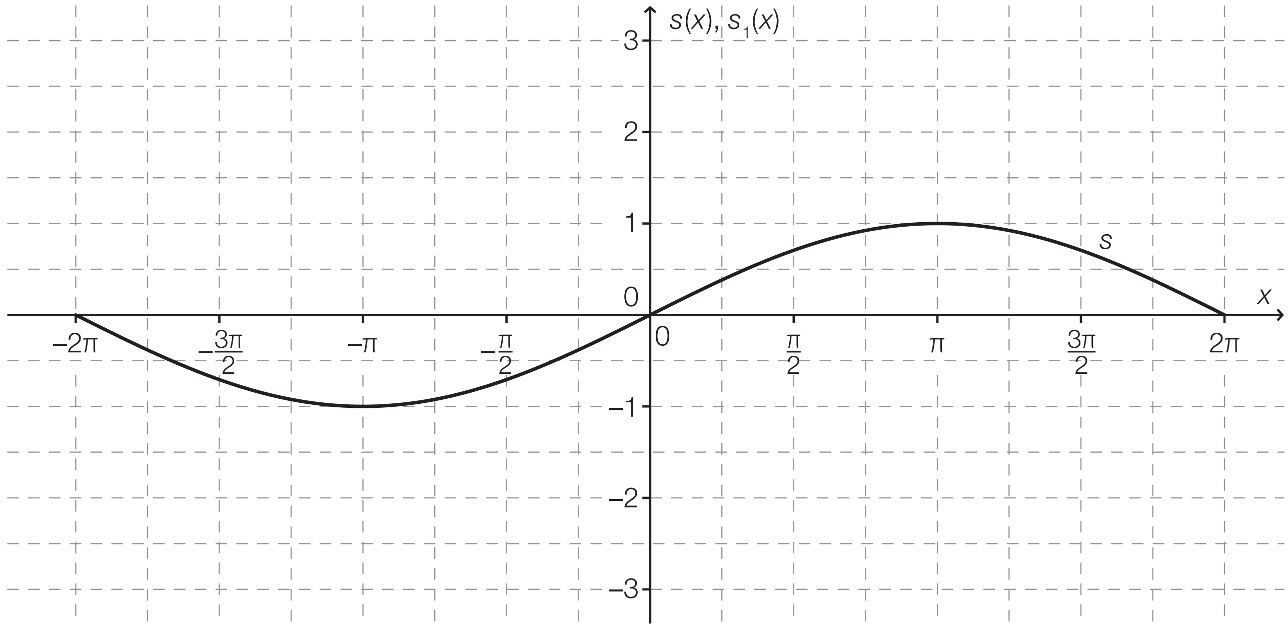
Which of the following statements about this function is/are true?
Put a cross next to each true statement.

The graph of the function f goes through the point $P = (50,0)$.	☐
The function f is strictly monotonically increasing in the interval $[0, 5]$.	☐
If the value of the argument, x, is increased by 5, the value of the function increases by a factor of 50.	☐
The value of the function $f(x)$ is positive for all $x \in \mathbb{R}$.	☐
If the value of the argument, x, is increased by 1, the value of the function increases by 97 %.	☐

Task 12

Parameters of a Sine Wave

The diagram below shows the graph of a function s, where $s(x) = c \cdot \sin(d \cdot x)$ and $c, d \in \mathbb{R}^+$, in the interval $[-2\pi, 2\pi]$.



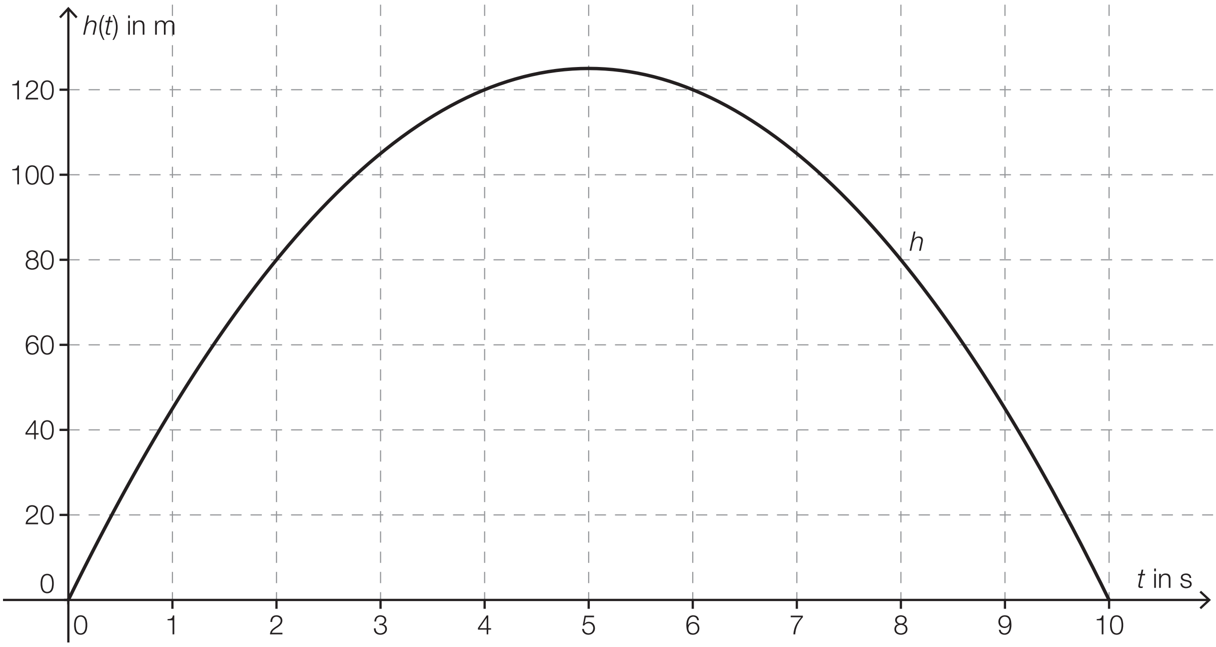
Task:

On the above axes, sketch a possible graph of the function s_1, which has the equation $s_1(x) = 2c \cdot \sin(2d \cdot x)$ in the interval $[-2\pi, 2\pi]$.

Task 13

Average Speed

The function h, whose graph can be seen in the diagram below, roughly describes the height, $h(t)$, in terms of time, t, of a body thrown vertically upwards (t is measured in seconds, $h(t)$ is measured in metres).



Task:

With the help of the graph, determine the average speed of the body in metres per second in the time period [2 s, 4 s].

Task 14

Real Functions

Let f be a real function with equation $f(x) = 4x^3 - 2x^2 + 5x - 2$.

Task:

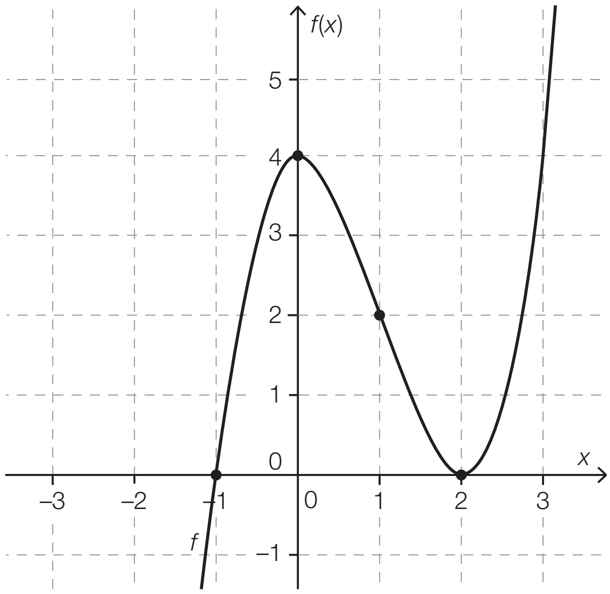
Write down the first derivative, f', of the function f.

$f'(x) =$ ______________________________

Task 15

Properties of the Derivative of a Third Degree Polynomial Function

The following diagram shows the graph of a third degree polynomial function, f. The points shown on the graph have integer coordinates.



Task:

Which of the following statements are true for the first derivative, f', of the function f?
Put a cross next to each of the two correct statements.

The values of the function f' in the interval (0, 2) are negative.	☐
The function f' is strictly monotonically increasing in the interval (–1, 0).	☐
The function f' has a point of inflection at the point $x = 2$.	☐
The function f' has a local maximum at the point $x = 1$.	☐
The function f' has a root at the point $x = 0$.	☐

Task 16

Local Maxima and Minima

In the table below, the values of a third degree polynomial function, f, as well as its first and second derivatives, f' and f'', are given.

x	0	1	2	3	4
$f(x)$	–2	2	0	–2	2
$f'(x)$	9	0	–3	0	9
$f''(x)$	–12	–6	0	6	12

Task:

Write down the points in the interval (0, 4) at which the function f has a local maximum or local minimum point.

Task 17

Antiderivative

Let f be a function with equation $f(x) = e^{2 \cdot x}$.

Task:

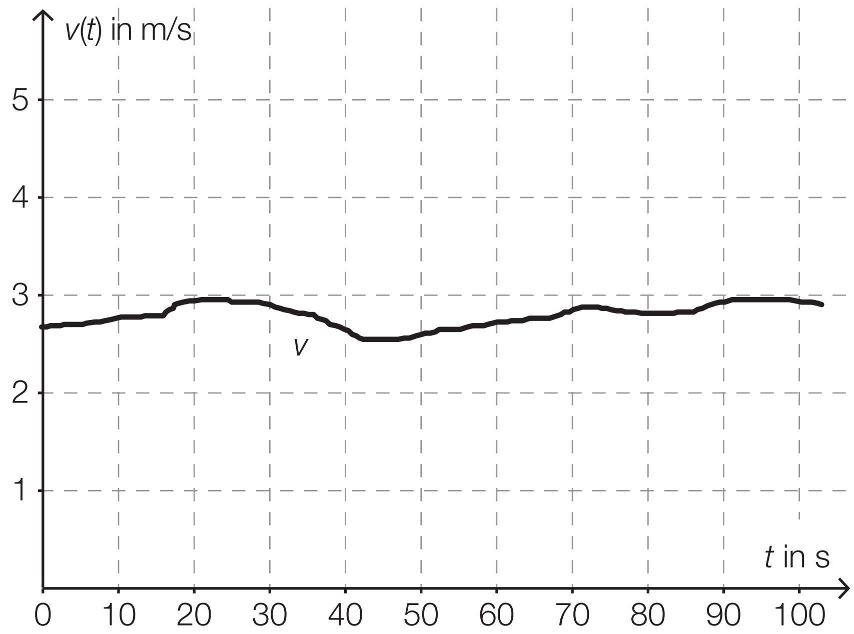
Which of the following functions, F, is an antiderivative of the function f and goes through the point $P = (0,1)$?
Put a cross next to the correct equation.

$F(x) = e^{2 \cdot x} + \frac{1}{2}$	☐
$F(x) = 2 \cdot e^{2 \cdot x} - 1$	☐
$F(x) = 2 \cdot e^{2 \cdot x}$	☐
$F(x) = \frac{e^{2 \cdot x}}{2} + \frac{1}{2}$	☐
$F(x) = e^{2 \cdot x}$	☐
$F(x) = \frac{e^{2 \cdot x}}{2}$	☐

Task 18

Water Supply

Water flows through a pipe. The velocity of the water at time t (measured in seconds) is given by $v(t)$ (measured in m/s). The area of the vertical cross-section of the pipe, Q, is measured in m^2. The diagram below shows how the velocity $v(t)$ depends on time t.



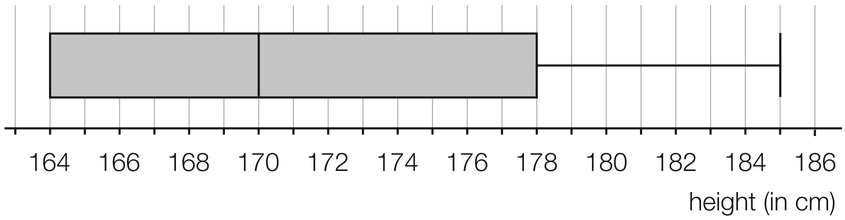
Task:

Write down which value is obtained from calculating the expression $Q \cdot \int_{10}^{40} v(t)\,dt$ in the given context.

Task 19

Heights

The heights of the 450 pupils in a year group in one region were measured in centimetres.
A representation of the results is shown in the boxplot below.



Task:

The following statements about the boxplot have been suggested.
Put a cross next to each of the two statements that are true.

60 % of the pupils are exactly 172 cm tall.	☐
At least one pupil is exactly 185 cm tall.	☐
At most 50 % of the pupils are shorter than 170 cm.	☐
At least 75 % of the pupils are taller than 178 cm.	☐
At most 50 % of the pupils are at least 164 cm and at most 178 cm tall.	☐

Task 20

Median and Mode

Below you see an unordered list of 19 natural numbers:

5, 15, 14, 2, 5, 13, 11, 9, 7, 16, 15, 9, 10, 14, 3, 14, 5, 15, 14

Task:

Write down the median and the mode of this list.

Median: ____

Mode: ____

Task 21

A Pair of Dice

A pair of obviously different, fair dice with the numbers 1, 2, 3, 4, 5, 6 on their faces was thrown. The sum of the numbers shown by both dice was then calculated.
(A dice is considered to be "fair" if the probability of the dice showing any of its six faces after being thrown is the same.)

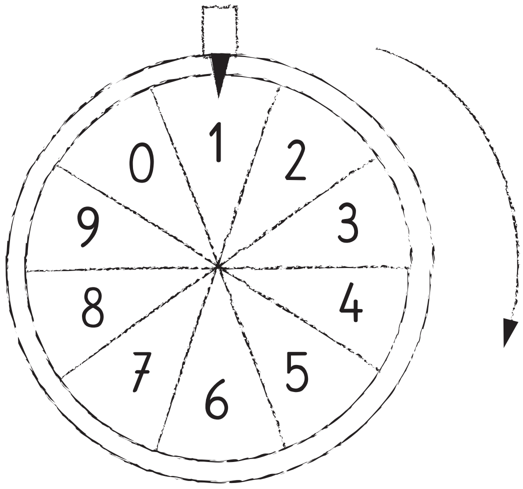
Task:

Someone claims that the dice are equally likely to display a total of 5 as they are to show a total of 9. Determine whether this statement is true or false and justify your answer.

Task 22

Prom Games

At a prom, two different games of chance are offered: a wheel of fortune and a raffle, in which 1 000 tickets are sold. The wheel of fortune is divided up into ten equally large sectors, all of which are equally likely to stop at the pointer. You can win the wheel of fortune if, after the wheel has stopped spinning, the pointer is pointing to a sector that shows the numbers 1 or 6.



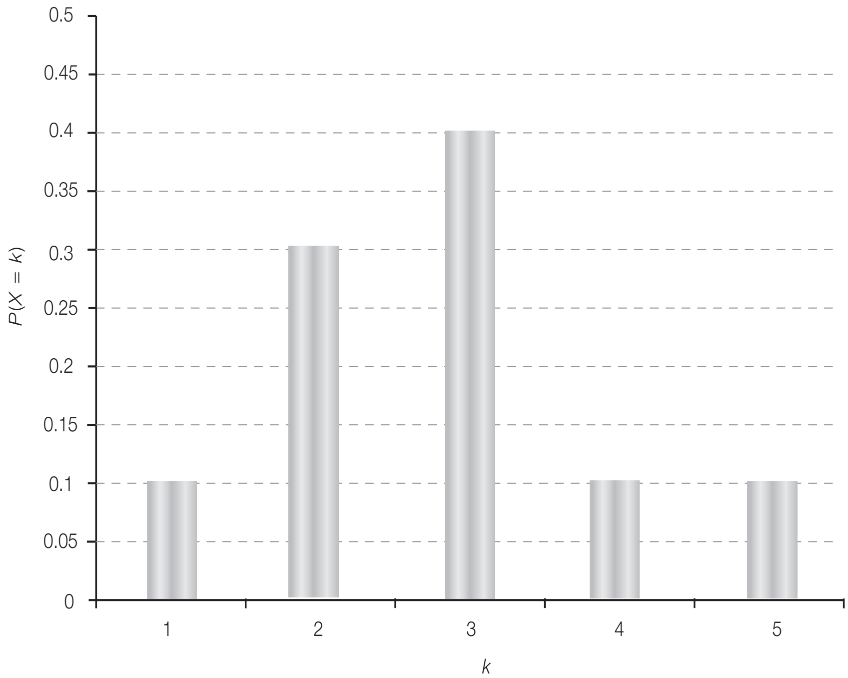
Task:

Max spun the wheel of fortune once and was the first one to buy a raffle ticket. He won both games. The school newspaper reported: “The probability of this event happening is 3 %.” Determine the number of winning raffle tickets.

Task 23

Expectation Value

The following diagram shows the probability distribution of a random variable X, which takes values $k = 1, 2, 3, 4, 5$.



Task:

Determine the expectation value $E(X)$.

Task 24

Width of a Confidence Interval

In an opinion survey, 500 randomly selected inhabitants of a city were asked about their opinion of the creation of a pedestrianised area in the city centre. 60 % of those asked supported the creation of a pedestrianised area, and 40 % said they were against the idea.
A 95 % confidence interval for the proportion of inhabitants of the city who would support the creation of a pedestrianised area in the city centre, was found to be [55.7 %, 64.3 %] using a normal approximation.

Task:

Put a cross next to each of the two true statements.

The confidence interval would be wider if a larger sample of inhabitants had been asked and the relative proportion of those who supported the idea had stayed the same.	☐
The confidence interval would be wider if a higher confidence level (and thus a higher level of certainty) had been chosen.	☐
The confidence interval would be wider if the survey had been conducted in a larger city.	☐
The confidence interval would be wider if the number of inhabitants in the sample who supported the idea had been larger.	☐
The confidence interval would be wider if the proportion of the inhabitants who had supported the idea and the proportion of inhabitants who had been against the idea had been the same size.	☐